SCTE | STANDARDS

Network Operations Subcommittee

SCTE OPERATIONAL PRACTICE

SCTE 270 2021r1

Mathematics of Cable

NOTICE

Document Types and Tags

Document Type: Operational Practice

Document Tags:

☐ Test or Measurement	☐ Checklist	☐ Facility
☐ Architecture or Framework	☐ Metric	☐ Access Network
☒ Procedure, Process or Method	☐ Cloud	☐ Customer Premises

Table of Contents

Title **Page Number**

List of Figures

Title **Page Number**

List of Tables

Title **Page Number**

1. Introduction

1.1. Executive summary

Mathematical calculations and conversions often are a necessary part of effectively managing cable systems and the technology that makes our networks function. This Operational Practice contains an assortment of cable-related mathematical formulas covering a broad range of topics. Most formulas are accompanied by a summary of the formula and its application, and one or more examples utilizing the formula. Readers should find this Operational Practice a wealth of practical information.

1.2. Scope

This Operational Practice includes a wide variety of cable-related mathematical formulas and examples of how to use them, as well as references to additional information, such as derivations and theory, for those who would like to learn more.

1.3. Benefits

The formulas and examples in this Operational Practice are intended to serve as a handy reference and teaching tool for the cable industry.

1.4. Intended audience

This Operational Practice is intended for cable system technical personnel such as installers, service and maintenance technicians, engineers, and others who have an interest in mathematical formulas for use in the cable industry.

1.5. Areas for further investigation or to be added in future versions

Areas for further investigation include the possible addition of mathematical formulas not yet part of this Operational Practice, and updates and/or corrections to existing formulas and examples.

2. Normative References

The following documents contain provisions, which, through reference in this text, constitute provisions of this document. At the time of Subcommittee approval, the editions indicated were valid. All documents are subject to revision; and while parties to any agreement based on this document are encouraged to investigate the possibility of applying the most recent editions of the documents listed below, they are reminded that newer editions of those documents might not be compatible with the referenced version.

2.1. SCTE references

- No normative references are applicable.

2.2. Standards from other organizations

- No normative references are applicable.

2.3. Published materials

- No normative references are applicable.

3. Informative References

The following documents might provide valuable information to the reader but are not required when complying with this document.

3.1. SCTE references

[SCTE 209] SCTE 209 2015, Technical Report UHF Leakage, Ingress, Direct Pickup

[SCTE 222] SCTE 222 2020, Useful Signal Leakage Formulas

[SCTE 258] SCTE 258 2020, DOCSIS 3.1 Downstream OFDM Power Definition, Calculation, and Measurement Techniques

[SCTE 07] ANSI/SCTE 07 2018, Digital Transmission Standard for Cable Television

3.2. Standards from other organizations

[IEEE 802.5] IEEE Std 802.5, 1998 Edition (ISO/IEC 8802-5:1998)

[IEEE 100] ANSI/IEEE Std 100-1984 IEEE Standard Dictionary of Electrical and Electronics Terms

3.3. Published materials

Note: References to the following in the main text of this document are denoted by [*n*], where *n* is the number of the listed item below.

1. *The ARRL Handbook for Radio Amateurs* (any recent edition), American Radio Relay League, Newington, CT; www.arrl.org
2. Kraus, J. (1988). *Antennas, Second Edition*. New York: McGraw-Hill
3. Johnson, R., Jasik, H. (1984). *Antenna Engineering Handbook, Second Edition*. New York: McGraw-Hill
4. Large, D., Farmer, J. (2009). *Broadband Cable Access Networks: The HFC Plant*. Morgan Kaufmann Publishers
5. *Broadband Data Book*, Revision 21, August 2019. Cisco Systems
6. Currivan, B., Hranac, R. (2006). "Digital Transmission: Carrier-to-Noise Ratio, Signal-to-Noise Ratio, and Modulation Error Ratio" (white paper published by Broadcom and Cisco; content also presented at SCTE Cable-Tec Expo)
7. Ciciora, W., Farmer, J., Large, D., Adams, M. (2004). *Modern Cable Television Technology, 2nd Edition*. Morgan Kaufmann Publishers
8. National Institute of Standards and Technology Special Publication 811, 2008 Edition, Guide for the Use of the International System of Units (SI)
9. NIST Special Publication 330 The International System of Units (SI).
10. Multiple contributors (1975). *Reference Data for Radio Engineers*. Indianapolis, IN: Howard W. Sams & Co., Inc.
11. Multiple contributors (1989). *Reference Data for Engineers: Radio, Electronics, Computer, and Communications, Seventh Edition*. Indianapolis, IN: Howard W. Sams & Co., Inc.
12. "HFTA-010.0: Physical Layer Performance: Testing the Bit Error Ratio (BER)" Maxim Integrated (first appeared in *Lightwave Magazine*, September, 2004, "Explaining those BER testing mysteries.")
13. CATV Systems Reference Data RD-13 pocket technical guide; Jerrold Electronics Corporation (circa 1979-1982)

14. Zahn, S. "Log Periodic Antennas for CATV." *Communications Technology* magazine, April 1988
15. *The ARRL Antenna Book* (any recent edition), American Radio Relay League, Newington, CT; www.arrl.org
16. Hranac, R. "A Quick Look at S-Parameters" *Broadband Library*, Winter 2019 issue.
17. Kuhns, J. (1997) *Satellite Calculations Handbook*. Published by NCTI, Inc. ISBN 0-88683-001-X
18. Gray, D. *Handbook of Coaxial Microwave Measurements* (Gilbert Engineering, now Corning-Gilbert; originally published by GenRad, Inc. in 1968)
19. Kizer, G. *Digital Microwave Communication: Engineering Point-to-Point Microwave Systems*. John Wiley & Sons, Inc. (2013), Hoboken, New Jersey
20. *AML Seminar Maintenance Manual*, Hughes Microwave Communications Products (circa 1979).
21. *Fundamentals of RF and Microwave Noise Figure Measurement*, Application Note 57-1, Hewlett-Packard, July 1983.
22. Freeman, R. *Telecommunications Transmission Handbook, 2nd Ed.*, John Wiley & Sons, 1981.
23. *Engineering Considerations for Microwave Communications Systems*, GTE Lenkurt, Inc., fourth printing, June 1970.
24. Hranac, R., Catter, B. "Techbook." *Communications Technology* magazine, February 1987
25. Jerrold CATV Reference Guide RD-15 pocket technical guide; General Instrument, Jerrold Division, Third Edition, February 1989.
26. IEEE Std 802.5, 1998 Edition (ISO/IEC 8802-5:1998)
27. ANSI/IEEE Std 100-1984 *IEEE Standard Dictionary of Electrical and Electronics Terms*
28. Prodan, R., "Full Duplex DOCSIS PHY Layer Design and Analysis for the Fiber Deep Architecture," SCTE Cable-Tec Expo 2017
29. Al-Banna, A. and Cloonan, T., "The Spectral Efficiency of DOCSIS 3.1 Systems," SCTE Cable-Tec Expo 2014, Denver, CO
30. ANSI/SCTE 07 2018 Digital Transmission Standard for Cable Television
31. "Communication in the Presence of Noise," by Claude E. Shannon, from PROCEEDINGS OF THE IRE, vol. 37, no. 1, pp. 10–21, Jan. 1949
32. Smith, Phillip H. (1995, 2000) *Electronic Applications of the Smith Chart.* Noble Publishing Corporation
33. *ITU: ITU-T Recommendation J.83 (2007),* "Cable networks and transmission of television, sound programme and other multimedia signals," International Telecommunications Union, Geneva

4. Compliance Notation

shall	This word or the adjective "***required***" means that the item is an absolute requirement of this document.
shall not	This phrase means that the item is an absolute prohibition of this document.
forbidden	This word means the value specified shall never be used.
should	This word or the adjective "*recommended*" means that there may exist valid reasons in particular circumstances to ignore this item, but the full implications should be understood and the case carefully weighted before choosing a different course.
should not	This phrase means that there may exist valid reasons in particular circumstances when the listed behavior is acceptable or even useful, but the full implications should be understood and the case carefully weighed before implementing any behavior described with this label.
may	This word or the adjective "*optional*" means that this item is truly optional. One vendor may choose to include the item because a particular marketplace requires it or because it enhances the product, for example; another vendor may omit the same item.
deprecated	Use is permissible for legacy purposes only. Deprecated features may be removed from future versions of this document. Implementations should avoid use of deprecated features.

5. Abbreviations and Definitions

5.1. Abbreviations

2sb	second most significant bit
3sb	third most significant bit
AC (ac)	alternating current
ADC	analog to digital converter
A_e	effective aperture
A_{em}	maximum effective aperture
AF	antenna factor
AM	amplitude modulation
AMP	1) ampere; 2) amplifier
AMSL	above mean sea level
APL	average optical power level
ASE	amplified spontaneous emission
atan	arctangent
avg	average
A/W	ampere per watt
AWGN	additive white Gaussian noise
BER	1) bit error rate; 2) bit error ratio
bps	bits per second
bps_{gross}	bits per second (gross)
BPSK	binary phase shift keying
bps_{net}	bits per second (net)
BSE	bit stream efficiency

b/s/Hz (bps/Hz)	bits per second per hertz
Btu/h	British thermal units per hour
BW	bandwidth
c	speed of light in a medium other than a vacuum
c_0	speed of light in a vacuum
˚C	degree Celsius
CARS	Cable Television Relay Service
CATV	cable television (originally community antenna television)
CCAP	converged cable access platform
CCN	carrier-to-composite noise
CEA	Consumer Electronics Association (now Consumer Technology Association)
ceil	ceiling [function]
CER	codeword error ratio
CHR	carrier-to-hum ratio
CID	composite intermodulation distortion
Cin	cosine integral
CIN	composite intermodulation noise
CLI	cumulative leakage index
cm	centimeter
cm^2	square centimeter
CMTS	cable modem termination system
C/N	carrier-to-noise ratio
CNR	carrier-to-noise ratio
cos	cosine
$\cos^{-1}$	inverse cosine or arccosine
CPD	common path distortion
CSO	composite second order
CTA	Consumer Technology Association (formerly Consumer Electronics Association)
CTB	composite triple beat
CW	continuous wave
DAC	digital to analog converter
dB	decibel
dBc	decibel carrier
dBc/Hz	decibel carrier per hertz
dBd	decibel dipole
dBi	decibel isotropic
dBm	decibel milliwatt
dB/m	decibel/meter
dB/m^2	decibel per square meter
dBmV	decibel millivolt
dBµV	decibel microvolt
dBµV/m	decibel microvolt per meter
dBV	decibel volt
dBW	decibel watt
dBW/m^2	decibel watt per square meter
DC (dc)	direct current
DOCSIS	Data-Over-Cable Service Interface Specifications
DRFI	DOCSIS Downstream Radio Frequency Interface Specification
DS	1) downstream; 2) double-square

DSO	discrete second order
E	voltage
E_b/N_0	energy-per-bit to noise power spectral density ratio
EDFA	erbium doped fiber amplifier
e.g.	for example (*exempli gratia*)
EHz	exahertz
EIRP	equivalent isotropic radiated power
$E_{\mu V/m}$	field strength in microvolts per meter
EMC	electromagnetic compatibility
EMF	electromotive force
EMI	electromagnetic interference
EOL	end-of-line
E_s/N_0	energy-per-symbol to noise-density ratio
EVM	error vector magnitude
f	frequency
F	noise factor
˚F	degree Fahrenheit
FCC	Federal Communications Commission
FEC	forward error correction
FM	frequency modulation
FSPL	free space path loss
ft	foot
ft^2	square foot
fW	femtowatt
GD	group delay
GDV	group delay variation
GHz	gigahertz
G/T	gain to noise temperature ratio
HFC	hybrid fiber/coax
Hz	hertz
HPBW	half power beamwidth
HVAC	heating, ventilation, and air conditioning
I	1) in-phase; 2) current
ICFR	in-channel frequency response
i.e.	that is (*id est*)
IID	independent and identically distributed
IMN	intermodulation noise
in	inch
in^2	square inch
IRE	information rate efficiency
J/s	joules per second
K	kelvin
kg	kilogram
kHz	kilohertz
km	kilometer
km^2	square kilometer
ksym/s	kilosymbols per second
kWh	kilowatt-hour
lb	pound

lb·ft (lbf·ft)	pound·foot
lb·in (lbf·in)	pound·inch
ln (LN)	natural logarithm
LNA	low noise amplifier
LNB	low noise block converter
log	logarithm (base 10 unless otherwise stated)
LSB	least significant bit
m	meter
m^2	square meter
MAC	media access control
Mbps	megabits per second
ME	modulation efficiency
MER	modulation error ratio
MHz	megahertz
mi	mile
mi^2	square mile
min/yr	minutes per year
mm	millimeter
mm^2	square millimeter
ms	millisecond
MSB	most significant bit
Msym/s	megasymbols per second
MTA	maximum-to-average [constellation power ratio]
MTBF	mean time between failure
MTTR	1) mean time to restore; 2) mean time to repair
mV	millivolt
mW	milliwatt
NCP	next codeword pointer
NF	noise figure
N·m	newton·meter
NRZ	non-return to zero
ns	nanosecond
NIST	National Institute of Standards and Technology
NTSC	National Television System Committee
nW	nanowatt
OFDM	orthogonal frequency division multiplexing
OMI	optical modulation index
oz	ounce
P	power
p	peak
pA	picoampere
PAM	pulse amplitude modulation
PAPR	peak-to-average power ratio
PAR	peak-to-average ratio
P.C.	picture carrier
PDE	powered data efficiency
PEP	peak envelope power
PER	packet error ratio
PEV	peak envelope voltage

PF	power factor
PHY	physical [layer]
PHz	petahertz
PLC	physical layer link channel (also PHY link channel)
PNM	proactive network maintenance
p-p	peak-to-peak
PSD	power spectral density
PRBS	pseudo random binary sequency
Q	quadrature
QAM	quadrature amplitude modulation
QPSK	quadrature phase shift keying
R	resistance
rad	radian
rad/s	radians per second
ref	reference
RF	radio frequency
RIN	relative intensity noise
RMS (rms)	root mean square
RxMER	receive modulation error ratio
S-CDMA	synchronous code division multiple access
SCTE	Society of Cable Telecommunications Engineers
SC-QAM	single carrier quadrature amplitude modulation
SE	1) spectral efficiency; 2) system efficiency
SI	International System of Units (*Le Système International d' Unités*)
sin	sine
SNR	signal-to-noise ratio
SSB	single-sideband
SRL	structural return loss
SUE	symbol utilization efficiency
SWR	standing wave ratio
sym/s/Hz	symbols per second per hertz
tan	tangent
tan^{-1}	inverse tangent or arctangent
TE	transverse electric
TEM	transverse electromagnetic
TFE	time-frequency efficiency
THz	terahertz
TV	television
TVRO	television receive only
TxMER	transmit modulation error ratio
RMS	root mean square
US	upstream
VA	volt-ampere
VF	velocity factor
VoP (VP)	velocity of propagation
W	watt
Wh	watt-hour
W/m^2	watt per square meter
XMOD	cross modulation

yd	yard
yd^2	square yard
Z_0	characteristic impedance
μH	microhenry
μm	micrometer
μs	microsecond
μV	microvolt
μV/m	microvolt per meter
μW	microwatt
λ	wavelength
Ω	ohm
Ω·m	ohm-meter

5.2. Definitions

alternating current (AC or ac)	An electric current that periodically reverses direction and whose instantaneous magnitude varies continuously over time.
ampere (A)	A measure of electric current, where 1 ampere equals 1 coulomb of charge flowing past a given point in 1 second. Analogous to a given volume of water (e.g., 1 gallon) flowing through a garden hose per second. Note: Coulomb is a unit of measure of electrically charged particles, where 1 coulomb = 6.242×10^{18} electrons.
amplitude modulation (AM)	A means of conveying information by varying the amplitude of a carrier wave in proportion to a baseband modulating signal, such as audio, video, or digital data.
analog intensity modulation	A means of conveying information through an optical fiber link by varying the intensity of the transmitted light in proportion to an analog electrical signal, such as a cable network's active downstream or upstream RF spectrum.
antenna	A transducer that converts RF current to electromagnetic waves in transmit applications, or converts electromagnetic waves to RF current in receive applications.
antenna factor	The ratio of the field strength of an electromagnetic field incident upon an antenna to the voltage produced by that field across a load of impedance Z_0 connected to the antenna's terminals.
apparent power	The product of voltage and current in an alternating current circuit. Expressed in units of volt-ampere (VA).
attenuation	see *loss*
availability	The ratio of time that a service, device, or network is available for use to total time, usually expressed as a percentage of the total time.
average (avg)	1) The arithmetic mean, or the sum of a set of numbers (or values) divided by the count of numbers (or values) in the set. 2) The average (or mean) value of a symmetrical alternating quantity such as a sine wave is defined here as the average value of the (full wave) rectified quantity measured over one complete cycle. Mathematically, the average value of a sinusoidal AC waveform is $avg = p * (2/\pi)$.
bandwidth (BW)	1) The amount of spectrum, measured in units of hertz, that an electromagnetic signal significantly occupies. 2) The operating passband of a device or system, typically expressed in units of hertz.

bit error ratio (BER)	The ratio of bits in error to the total number of bits transmitted, received, or processed. Often referred to as bit error rate, though non-temporal.
British thermal units per hour (Btu/h)	A unit of power used for heating and cooling systems, where 1 Btu/h is equal to approximately 0.2931 watt, or 1 watt is equal to approximately 3.4121 Btu/h.
capacitive reactance	The opposition to alternating current by a capacitor (or capacitance). Represented by the symbol X_C and expressed in ohms.
carrier-to-noise ratio (CNR or C/N)	The ratio of carrier or signal power to the thermal noise power in a specified bandwidth, as measured on an RF spectrum analyzer or similar equipment. Note that "noise" can also refer other types of noise, such as relative intensity noise, shot noise, etc., but does not refer to transient noise.
ceiling (ceil)	A mathematical function that returns the lowest-valued integer that is greater than or equal to a given value.
channel	1) A portion of the electromagnetic spectrum used to convey one or more RF signals between a transmitter and receiver. Can be specified by parameters such as center frequency, bandwidth, CTA or other designated channel number. 2) An RF signal (or signals) carried on a cable network.
common path distortion (CPD)	Second and/or third order intermodulation products that appear in the upstream spectrum of a cable network. Unlike distortions produced in active devices, CPD is usually produced at a corrosion-related diode-like interface where downstream and upstream signals are present.
composite intermodulation noise (CIN)	A combination of thermal noise and noise-like second and third order intermodulation distortion.
composite second order (CSO) distortion	Composite beat clusters generated by the direct addition or subtraction of fundamental visual carrier frequencies. These products normally fall at ±750 kHz and/or ±1.25 MHz from the visual carriers.
composite triple beat (CPD) distortion	Composite beat clusters generated by third order distortions resulting from the addition and subtraction of fundamental visual carriers with the second harmonic of other visual carriers or the addition and subtraction of a combination of three visual carriers. These products normally fall within ±15 kHz of the visual carriers.
crest factor	A characteristic of a waveform defined as the ratio of its peak to effective value. Expressed mathematically, $C = \|\chi_{peak}\|/\chi_{rms}$, where C is crest factor, χ_{peak} is the waveform's peak value and χ_{rms} is the waveform's effective or root mean square value.
cross modulation (XMOD) distortion	A distortion phenomenon in which modulation from other carriers is impressed on the carrier of interest (e.g., a test carrier). Cross modulation is defined as the difference between the detected cross modulation level and the detected level that would correspond to 100% modulation, expressed in dB. It mathematically derives from the third order term in the power series model of distortion in linear amplifiers.
CTA channel	A 6 MHz portion of a cable network's RF spectrum based upon *CTA-542-D R-2018 Cable Television Channel Identification Plan* (formerly CEA 542-D).

current (I)	A flow of charged particles per unit of time, measured in units of amperes. Analogous to the volume of water flowing through a garden hose.
decibel (dB)	A logarithmic-based expression of the ratio between two values of a physical quantity, typically power or intensity. The decibel provides an efficient way to express ratios which span one or more powers of the logarithmic base, most commonly 10. Mathematically, the ratio of two power levels P_1 and P_2 in decibels is dB = $10\log_{10}(P_1/P_2)$.
decibel microvolt (dBμV)	Unit of RF power expressed in terms of voltage, defined as decibels relative to 1 microvolt, where 1 microvolt equals 13.33 femtowatts in a 75 ohms impedance. Mathematically, dBμV = $20\log_{10}$(value in μV/1 μV).
decibel microvolt per meter (dBμV/m)	An RF signal's power density expressed in terms of voltage, defined as decibels relative to 1 microvolt per meter, where 1 microvolt per meter equals 1 microvolt delivered to a receiving antenna's terminals recovered from an imaginary 1 meter x 1 meter square in free-space or air. Mathematically, dBμV/m = $20\log_{10}$(μV/m).
decibel millivolt (dBmV)	Unit of RF power expressed in terms of voltage, defined as decibels relative to 1 millivolt, where 1 millivolt equals 13.33 nanowatts in a 75 ohms impedance. Mathematically, dBmV = $20\log_{10}$(value in mV/1 mV).
decibel milliwatt (dBm)	Unit of power, defined as decibels relative to 1 milliwatt, where 0 dBm equals 1 milliwatt. Mathematically, dBm = $10\log_{10}$(value in mW/1 mW).
decibel volt (dBV)	Unit of power expressed in terms of voltage, in particular a logarithmic-based expression of the ratio of a value in volts to 1 volt. The 0 dB reference for decibel volt is 0 dBV, which equals 1 volt.
decibel watt (dBW)	Unit of power, in particular a logarithmic-based expression of the ratio of a value in watts to 1 watt (W). The 0 dB reference for decibel watt is 0 dBW, which equals 1 watt.
direct current (DC or dc)	An electric current that is unidirectional.
directivity (antenna)	A characteristic that is a measure of the concentration of an antenna's radiated energy in a single direction, compared to an isotropic source.[1]
effective aperture (A_e)	The geometric area over which an antenna receives power from an incident RF signal and delivers that power to a connected load. Mathematically, $A_e = \lambda^2 G/4\pi$, where λ is the wavelength of the RF signal, G is the receiving antenna's linear ("numerical") power gain (e.g., 1.64 for a half-wave dipole), and π = 3.14. If the antenna is considered lossless, effective aperture is called maximum effective aperture (A_{em}).
electromotive force (EMF)	The force of electrical attraction between two points of different charge potential. EMF is more commonly known as voltage (technically speaking, the volt is a measure of electromotive force), and is analogous to water pressure in a garden hose. 1 volt is the potential difference between two points on a wire carrying 1 ampere of current when the power dissipated between the points is 1 watt.

[1] The term directivity also is a characteristic of RF passive devices such as directional couplers. However, that context of the term is not used in this document.

error vector magnitude (EVM)	The ratio of RMS constellation error magnitude to peak constellation symbol magnitude, stated in percent.
far-field	The region of an antenna's radiation pattern in which the angular distribution of radiated energy is largely independent of distance from the antenna, and in which the power varies inversely with the square of distance. The approximate distance from the antenna to the beginning of the far-field is generally accepted to be $R = 2D^2/\lambda$, where R is distance from the antenna, D is the largest linear dimension of the antenna effective aperture, and λ is wavelength. Signal leakage field strength measurements are made in the far-field. See also *near-field*.
field strength	An RF signal's power density within an imaginary 1 meter x 1 meter square (that is, watts per square meter) in free space or in the air. Usually expressed in units of voltage, for example, microvolts per meter.
free space path loss	The attenuation, typically in decibels, of an electromagnetic signal traveling over an unobstructed line-of-sight path through free space (usually air) between two points. Mathematically, $Loss_{dB} = 20\,log_{10}(f_{MHz}) + 20\,log_{10}(d_{km}) + 32.45$, or $Loss_{dB} = 20\,log_{10}(f_{MHz}) + 20\,log_{10}(d_{feet}) - 37.892$, where f_{MHz} is the frequency in megahertz, d_{km} is the path length in kilometers, and d_{feet} is the path length in feet[2].
frequency	The number of times, typically per second, that a repetitive event happens. For an electromagnetic wave, frequency is the wave's rate of oscillation. Commonly measured or stated in units of hertz (Hz), which is the number of cycles per second.
frequency domain	A representation of a periodic signal (e.g., sinusoidal waveform) or aperiodic signal (e.g., rectangular pulse or random noise) as a function of frequency. When plotted on a graph or displayed on an instrument such as spectrum analyzer, the vertical axis is amplitude and the horizontal axis is frequency.
frequency modulation (FM)	A means of conveying information by varying the frequency of a carrier wave in proportion to a baseband modulating signal, such as audio, video, or digital data.
Fresnel zone	The locus of points above or below a direct path from a microwave transmitter to a microwave receiver where the distance from one end of the path to a locus point and then to the other end of the path is an integer number of half-wavelengths longer than the direct path. For example, the first Fresnel zone, F_1, has a total additional path length of ½ wavelength. The second Fresnel zone, F_2, has a total additional path length of 2 ∗ ½ wavelengths, and so on.
gain	1) An increase in the power of a signal or signals, usually measured in decibels. Expressed mathematically, $G_{dB} = 10log_{10}(P_{out}/P_{in})$, where G_{dB} is gain in decibels, P_{out} is output power in watts, P_{in} is input power in watts, and $P_{out} > P_{in}$. When signal power is stated in dBmV, G_{dB} =

[2] Real-world path loss seldom equals the calculated free space path loss, because of the constructive and/or destructive effects of signal reflection(s), refraction, and diffraction. In addition to free space path loss modeling, other models used to calculate path loss include, but are not limited to, Lee, Irregular Terrain Model (ITM; also known as the Longley-Rice model), Okumura-Hata, Walfish-Ikegami, Wireless World Initiative New Radio Phase II (WINNER II) and Young. This Operational Practice document uses free space path loss modeling in its examples.

	$P_{out(dBmV)} – P_{in(dBmV)}$. 2) An antenna characteristic related to directivity and efficiency. Mathematically, G = kD (dimensionless), where G is gain, k is the efficiency factor of the antenna ($0 \leq k \leq 1$), dimensionless, and D is directivity.
gigahertz (GHz)	One billion (10^9) hertz. See also *hertz*.
group delay distortion	The negative derivative of radian phase with respect to radian frequency. In a system, network, device, or component with no group delay distortion, all frequencies propagate through the system, network, device, or component in the same amount of time – that is, with equal time delay. If group delay distortion exists, signals (or parts of signals) at some frequencies propagate faster or slower than signals (or parts of signals) at other frequencies.
hertz (Hz)	A unit of frequency equivalent to one cycle per second.
hybrid fiber/coax (HFC)	A class of cable network architectures comprising a combination of optical fiber and coaxial cable for signal distribution.
impedance	The combined opposition to current in a component, circuit, device, or transmission line that contains both resistance and reactance. Represented by the symbol Z and expressed in ohms.
independent and identically distributed (IID)	A property of random variables (e.g., bit errors) that are independent and have the same probability distribution as other random variables – that is, with equal probability. For independent random variables, knowing the probability of one does not change the probability of the others.
index of refraction	A dimensionless number that describes the reduction in the speed of an electromagnetic signal in a given material, defined as the ratio of the speed of light in a vacuum to the speed of light in the material through which the electromagnetic signal is traveling. Expressed mathematically, $n = c_0/c$, where n is the index of refraction, c_0 is the speed of light in a vacuum, and c is the speed of light in the material.
inductive reactance	The opposition to alternating current by an inductor (or inductance). Represented by the symbol X_L and expressed in ohms.
isotropic source	A source that radiates energy uniformly in all directions. An analogy is a light bulb at the center of a sphere, illuminating the surface of that sphere uniformly.
loss	A decrease in the power of a signal or signals, usually measured in decibels. Expressed mathematically, $L_{dB} = 10\log_{10}(P_{in}/P_{out})$, where L_{dB} is loss in decibels, P_{in} is input power in watts, P_{out} is output power in watts, and $P_{out} < P_{in}$. When signal power is stated in dBmV, $L_{dB} = P_{in(dBmV)} – P_{out(dBmV)}$.
megahertz (MHz)	One million (10^6) hertz. See also *hertz*.
micro-reflection	An echo (reflection) with a relatively short time delay, typically from less than a symbol period to several symbol periods.
microvolt (µV)	One millionth (10^{-6}) of a volt.
microvolt per meter (µV/m)	A measure of the field strength of an RF signal, calculated by dividing the received intensity in microvolts by the receiving antenna maximum effective aperture.
millivolt (mV)	One thousandth (10^{-3}) of a volt.
minimum[n, …, n_n]	The smallest value in the set [n, …, n_n].
modulation error ratio (MER)	The ratio of average signal constellation power to average constellation error power, stated in decibels.

modulation rate	The signaling rate of the upstream modulator (for example, 1,280 kHz to 5,120 kHz). In S-CDMA, the chip rate. In TDMA, the channel symbol rate. (Note: This definition was first introduced in the DOCSIS 2.0 Radio Frequency Interface Specification.)
near-field	The space around an antenna comprising a reactive region and a radiating region. The radiating region is further subdivided into a near-field region and a far-field region. The radiating near-field is the propagation region where angular contributions from individual antenna elements vary significantly with distance from the antenna. See also *far-field*.
noise factor (F)	Degradation in signal-to-noise ratio as a signal passes through a device. Specifically, the noise factor of a system or device is a linear value defined as $F = SNR_i/SNR_o$ where SNR_i is system or device input signal-to-noise ratio and SNR_o is system or device output signal-to-noise ratio.
noise figure (NF)	Noise factor expressed in decibels, expressed mathematically as $NF = 10\log_{10}(F)$.
noise temperature	The noise temperature of an electrical device, circuit, or component is defined to be the temperature of a single passive resistance that contributes the same noise power spectral density as the device itself. The term applies to active devices as well as simple and complex passive circuits and components. Noise temperature is stated in kelvin, and while related to physical temperature, should not be confused with the physical temperature of the device that one would measure with a thermometer.
OFDM power	The average RF power of an OFDM signal, which is usually characterized in two ways: (1) OFDM power per CTA channel – that is, the average power per 6 MHz (which may not be uniform across the OFDM signal because of exclusion bands and other factors). (2) OFDM total power: The average power over the entire occupied bandwidth of the OFDM signal, defined mathematically as Total power = Power per CTA channel + $10\log_{10}$(Number of CTA channels occupied by the OFDM signal).
ohm (Ω)	1) A unit of resistance, where 1 ohm is defined as the resistance that allows 1 ampere of current to flow between two points that have a potential difference of 1 volt. 2) A unit of impedance. 3) A unit of reactance.
Ohm’s Law	The principle, named after Georg Ohm, in which the current through a conductor between two points is directly proportional to the voltage across the two points. Expressed mathematically, $I = E/R$, where I is current in amperes, E is electromotive force in volts, and R is resistance in ohms. Other variations of the formula are $E = IR$ and $R = E/I$.
optical modulation index (OMI)	A measure of the amount of optical modulation that is applied to a laser, commonly stated in terms of the extent of the range from a laser’s quiescence bias point to cutoff that is taken up by the modulating signal. Typically expressed in percent per channel (peak) or composite percent for all channels (root mean square). Note that optical intensity is proportional to input signal current.

peak (p)	The maximum value of an alternating quantity such as a sinusoidal AC waveform during one cycle, relative to a reference value such as 0 volts or 0 amperes (e.g., the horizontal line in Figure 10 labeled 0). Mathematically, the peak value of a sinusoidal AC waveform is $p = RMS * \sqrt{2}$.
peak envelope power (PEP)	The average power (watts) during one cycle of an RF signal at the crest of its modulation envelope.
peak-to-average power ratio (PAPR)	The ratio of the peak power of a signal to its average power.
peak-to-peak (p-p)	The sum of the maximum value of an alternating quantity such as a sinusoidal AC waveform during one cycle, above and below a reference such as 0 volts or 0 amperes (e.g., the horizontal line in Figure 10 labeled 0). Mathematically, the peak-to-peak value of a sinusoidal AC waveform is $p\text{-}p = p * 2$.
period	The duration in time of each cycle of a repetitive event; the reciprocal of frequency. In the case of symbol period, the reciprocal of symbol rate (or modulation rate).
phase noise	An undesired spreading of the signal spectrum in the frequency domain caused by phase fluctuations in the signal source; equivalent to jitter in the time domain.
power	The rate at which work is done, or energy per unit of time, expressed in watts. 1 watt of power is equal to 1 volt causing a current of 1 ampere. Mathematically, for sinusoidal waveforms $P_{AVG} = I_{RMS} * E_{RMS} * \cos\theta$, where P_{AVG} is average power, I_{RMS} is root mean square current, E_{RMS} is root mean square voltage, and $\cos\theta$ is the cosine of the phase angle difference in degrees between the current and voltage.
power factor	The ratio of real power to apparent power.
power per CTA channel	The average power in a 6 MHz bandwidth, typically expressed in dBmV.
power spectral density (PSD)	The average power present in a signal as a function of frequency, per unit frequency, typically expressed in units of power per hertz (e.g., W/Hz). Commonly used to describe how the power of a signal is distributed over frequency.
quadrature amplitude modulation (QAM)	A means of conveying information by varying the amplitude and phase of a carrier wave.
QAM-independent system efficiency	Spectral efficiency in b/s/Hz divided by the number of bits per symbol, stated in units of symbols per second per hertz (sym/s/Hz).
radian (rad)	A unit of angular measure, where an angle of 1 radian defines an arc on the circumference of a circle, and the arc has a length equal to the radius of that circle.
radio frequency (RF)	That portion of the electromagnetic spectrum from a few kilohertz to just below the frequency of infrared light.
real power	The actual power dissipated in an alternating current circuit. Expressed in units of watts.
receive modulation error ratio (RxMER)	The MER as measured in a digital receiver after demodulation, with or without adaptive equalization. See also *modulation error ratio*.
reflection coefficient	The ratio of reflected voltage to incident voltage, commonly represented by the Greek letter gamma (Γ), and sometimes by the Greek letter rho (ρ). The magnitude of reflection coefficient, |Γ|, can have values from 0 (indicating a reflectionless load – that is, all of the

	incident energy is absorbed by the load) to 1 (indicating that all of the incident energy is reflected by the load).
reflection loss	The loss, as the result of a reflection, in the power delivered to a load. Sometimes called transmission loss.
relative intensity noise (RIN)	Random amplitude fluctuation in the light from a laser, expressed in terms of noise power in a 1 Hz bandwidth compared with the laser's average optical power level.
reliability	The probability that a system or device will not fail during a specified period of time.
resistance	Opposition to the flow of current.
return loss	The ratio, in decibels, of the power incident upon an impedance discontinuity to the power reflected from the impedance discontinuity. Note: When $P_{reflected} < P_{incident}$ return loss is a positive number.
root mean square (RMS or rms)	RMS is based upon equating the values of AC and DC power to heat a resistive element to exactly the same degree. An RMS value is found by squaring the individual values of all the instantaneous values of voltage or current in a single AC cycle. Take the average of those squares and find the square root of the average. Mathematically, the root mean square value of a sinusoidal AC waveform is $RMS = p * \left(1/\sqrt{2}\right)$.
scattering parameters (S-parameters)	Voltage reflection coefficients and voltage transmission coefficients that can be used to characterize N-port networks (components, devices, etc.). S-parameters describe the electrical behavior of linear N-port networks when those networks undergo steady-state stimuli by electrical signals.
shot noise	Noise in an optical receiver caused by fluctuations in the number of photons detected due to the statistical variation in arriving photon distribution.
signal leakage	Unwanted emission of RF signals from a cable network into the surrounding over-the-air environment, typically caused by degraded shielding effectiveness of coaxial cable, connectors, and other network components, or by poorly shielded subscriber terminal equipment connected to the cable network.
signal-to-noise ratio (SNR)	1) A general measurement of the ratio of signal power to noise power. 2) In a specific context, a measurement of the ratio of signal power to noise power made at baseband before modulation or after detection or demodulation.
single carrier quadrature amplitude modulation (SC-QAM)	A term applied to legacy DOCSIS and digital video signals, in which information such as digital data is conveyed by varying the amplitude and phase of a single RF carrier.
skin depth	The depth at which the current density is 1/*e* of the current density at the surface of a conductor. Note: "*e*" is the mathematical constant that is the base of the natural logarithm. Skin depth, a measure of skin effect, is commonly denoted by the symbol δ.
skin effect	A phenomenon where the conduction of alternating current – including RF – is largely confined to a region at and near the surface of a conductor. The higher the frequency, the shallower the region (and closer to the surface) in which the current is conducted.
slant range distance	The line-of-sight distance along a slant direction between two points which are not at the same level relative to each other. As used in

	satellite-to-earth station free space path loss calculations, slant range distance is the distance between a satellite in geostationary orbit above the equator and an earth station (usually) north or south of the equator.
spectral efficiency	Describes the information bit rate that is supported in a given RF bandwidth. Mathematically, the net bit rate (that is, the bit rate excluding overhead) in bits per second, bps_{net}, divided by the channel bandwidth in hertz, stated in units of bits per second per hertz (b/s/Hz or bps/Hz).
standing wave ratio (SWR)	The ratio of maximum voltage to minimum voltage (or maximum current to minimum current) in the distribution of fields in a transmission line when a wave (or waves) reflected from an impedance discontinuity interacts with an incident wave (or incident waves).
symbol rate	The number of symbol, waveform, or signaling changes or events in a data transmission medium, typically expressed in units of symbols per second (e.g., kilosymbols per second or megasymbols per second). Also called baud. See also *modulation rate*.
thermal noise	Also called Johnson-Nyquist noise, the fluctuating voltage across a resistance caused by the random motion of free charge as a result of thermal agitation.
time domain	A representation of a periodic signal (e.g., sinusoidal waveform) or aperiodic signal (e.g., rectangular pulse or random noise) as a function of time. When plotted on a graph or displayed on an instrument such as an oscilloscope, the vertical axis is amplitude and the horizontal axis is time.
torque	The rotational equivalent of linear force, defined as the product of radius times force. Sometimes called moment of force or rotational force. Commonly described in units of pound·foot or pound·inch (the SI derived unit for moment of force is the newton·meter).
total power	The combined power of all signals and/or signal components in a defined bandwidth. Generally assumed to be average power, unless otherwise stated.
transmit modulation error ratio (TxMER)	The MER produced by a transmitter under test, as measured by an ideal test receiver. See also *modulation error ratio*.
velocity factor (VF)	The ratio, in decimal form, of the speed of an electromagnetic signal propagating through a medium (such as coaxial cable) to the speed of light in a vacuum.
velocity of propagation (VoP or VP)	Velocity factor expressed in percent.
volt (E or V)	A derived unit for electric potential (or electromotive force), where 1 volt is the potential difference between two points on a conductor (wire) carrying 1 ampere of current when the power dissipated between the points is 1 watt. Analogous to water pressure in a garden hose. Note: Electromotive force is the force of electrical attraction between two points of different charge potential.
watt (W)	The rate at which work is done, or energy per unit of time – that is, power can be described as the rate at which energy is consumed in a circuit. 1 watt of power is equal to 1 volt causing a current of 1 ampere. Watt is the power required to do work at a rate of 1 joule per second (J/s).

wavelength	An electromagnetic wave's speed of propagation divided by its frequency in cycles per second, and commonly represented by the symbol λ. If one could see an electromagnetic signal's waves, wavelength is the distance from a point on one cycle's wave to the same point on an adjacent cycle's wave. Note: By convention, when discussing the wavelength of an optical signal, that wavelength is the value in free space (vacuum).

6. Notes for the User of This Operational Practice

Where applicable, conventions used in this document conform with the International System of Units (SI). Appendix B includes tables summarizing SI prefixes, SI base units, examples of SI derived units, etc., that the reader may find helpful.[3]

Numbers used for quantities are printed with the decimal point represented by a period (.) and numbers from one thousand and higher are ordered in groups of three with a comma (,) separating each group. For example, the number one hundred sixty-three thousand two hundred thirty-four point seven eight is written as 163,234.78.

The reader is cautioned to pay attention to letters and symbols that are locally defined in the relevant sections used in various formulas and examples. In many instances the same letter or symbol can have more than one meaning, depending on application. Consider, for example, the letter "k". The following is a partial list of different uses of the letter "k" in this document.

Table 1. Example uses of the letter "k"

Letter or symbol	Definition
k	antenna efficiency
k	Boltzmann's Constant
k	SI prefix symbol for kilo (e.g., kHz)
k	number of observed failures
k_s	stranding factor (for coaxial cable shielding)
K	kelvin (thermodynamic temperature)
K factor	equivalent Earth radius factor
K factor	multiplying factor for antenna element length

The content provided in this Operational Practice has been compiled from various sources, many of which are summarized in Section 3.3. For more in-depth information, the reader is encouraged to consult the original source material and other references as noted.

Unless stated otherwise, most of the formulas and examples included in this Operational Practice pertain to radio frequency applications.

Terms in some formulas use letters from the Greek alphabet. Table 2 summarizes the 24 letters of the Greek alphabet, and includes both uppercase and lowercase forms. For example, the uppercase form of omega is Ω, and the lowercase form of omega is ω.

[3] For more information about SI, see https://www.nist.gov/pml/weights-and-measures/metric-si/si-units and https://www.nist.gov/pml/weights-and-measures/publications/metric-publications.

Table 2. Greek alphabet

Name	Letter	Name	Letter	Name	Letter	Name	Letter	Name	Letter	Name	Letter
alpha	Α, α	epsilon	Ε, ε	iota	Ι, ι	nu	Ν, ν	rho	Ρ, ρ	phi	Φ, φ
beta	Β, β	zeta	Ζ, ζ	kappa	Κ, κ	xi	Ξ, ξ	sigma	Σ, ς, σ	chi	Χ, χ
gamma	Γ, γ	eta	Η, η	lambda	Λ, λ	omicron	Ο, ο	tau	Τ, τ	psi	Ψ, ψ
delta	Δ, δ	theta	Θ, θ	mu	Μ, μ	pi	Π, π	upsilon	Υ, υ	omega	Ω, ω

Every effort has been made to ensure the accuracy of the content in this Operational Practice. Any errors or omissions are unintentional and should be brought to the attention of SCTE so that corrections can be made to subsequent versions of this document.

If the reader is aware of or has suggestions for other cable technology-related formulas and examples that could be considered for inclusion in future versions of this Operational Practice, please contact SCTE.

7. Scientific Notation and Engineering Notation

Cable math often requires calculations that use very large quantities and very small quantities. For example, the speed of light in a vacuum is approximately 300,000,000 meters per second (m/s) or approximately 186,000 miles per second (mi/s).[4] A typical wavelength of light used for fiber optics in cable systems is 0.000001550 meter (commonly stated as 1,550 nanometers). A typical bit error ratio might be 0.00000000153, that is, 153 bit errors per every 100 billion bits transmitted (153/100,000,000,000 = 0.00000000153).

It is difficult and awkward to carry along all those zeros when doing calculations. Scientific notation and engineering notation provide useful ways of writing very large and very small numbers.

7.1. A review of powers of 10

A helpful starting point for explaining scientific notation is a review of powers of 10. The number 10 raised to the first power is given by

$$10^1 = 10$$

10 to the second power is given by

$$10^2 = 100$$

10 to the third power is given by

$$10^3 = 100$$

and so forth. One million can be written as 10 to the sixth power or

$$10^6 = 1{,}000{,}000$$

10 can also be raised to a negative power. 10 to the power of negative one is given by

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

or one tenth. 10 to the power of negative two is given by

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

or one hundredth. 10 to power of negative three is given by

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

or one thousandth and so forth.

---

[4] The speed of light used in this document is the National Institute of Standards and Technology value of 299,792,458 meters per second, which equals 983,571,056.43 feet per second or 186,282.40 miles per second.

By definition 10 raised to the zero power is 1.

$$10^0 = 1$$

In fact, any number raised to the zero power is equal to 1.

7.2. Scientific notation

Scientific notation is a way of writing numbers in the following form:

$$A * 10^B$$

For example, we could write the approximate speed of light in meters per second as

$$300{,}000{,}000 = 3.0 * 100{,}000{,}000 = 3.0 * 10^8\ m/s$$

Thus, the approximate speed of light expressed in scientific notation is $3.0 * 10^8$ m/s. It would be read as "three point zero times ten to the eighth meters per second" or just "three times ten to the eighth meters per second". This is a much more convenient way to write this very large number.

Similarly, in the bit error ratio example given previously where there were 153 bit errors per every 100 billion bits transmitted, the BER could be written as

$$0.00000000153 = 1.53 * 0.000000001 = 1.53 * 10^{-9}$$

We would write the BER as $1.53 * 10^{-9}$ rather than 0.00000000153. It would be read as "one point five three times ten to the minus nine."

Many pieces of test equipment display results, such as BER, in scientific notation. The format used by test equipment often is of a slightly different form. For example, the BER result in the previous paragraph would be displayed as 1.53E–09. The "E" in this format refers to the exponent (or power) of 10. The display would still be read as "one point five three times ten to the minus nine."

Note that when dealing with BER, a result of 1.00E–09 is much better than a result of 9.99E–09. In the first case there is only one error per every billion bits transmitted whereas in the second case there would be almost 10 errors (9.99 errors) per every billion bits transmitted.

7.3. Engineering notation

Engineering notation is very similar to scientific notation. However, in engineering notation the power of 10 is always a multiple of three. For example, the approximate speed of light in meters per second, $3.0 * 10^8$ m/s in scientific notation, would be written in engineering notation as $300 * 10^6$ m/s. Note that the power of 10 is six which is a multiple of three.

As a second example, consider the number $3.14 * 10^{-8}$ as expressed in scientific notation. This same number would be $31.4 * 10^{-9}$ when expressed in engineering notation.

Note how engineering notation can be easily related to the prefixes used in Appendix B, Table 32.

8. Frequency, Wavelength, and Period

8.1. Units

Frequency is the number of times, typically per second, that a repetitive event happens. For an electromagnetic wave – for instance, a radio frequency (RF) signal carried in a cable network – frequency is the wave's rate of oscillation. Frequency is commonly measured or stated in units of hertz (Hz), which is the number of cycles per second; see Table 3.

Related to frequency is *wavelength*, an electromagnetic wave's speed of propagation divided by its frequency in cycles per second. Wavelength is commonly represented by the symbol λ. If you could see an electromagnetic signal's waves, wavelength would be the distance from a point on one cycle's wave to the same point on the adjacent cycle's wave. Yet another related term is *period*, which is the duration in time of each cycle of a repetitive event and is the reciprocal of frequency.

Electromagnetic waves can be thought of as analogous to the ripples that occur when one tosses a rock into a pond of water. For example, throw a rock in the water, then count the number of waves per second that pass by a wooden post sticking out of the water. That's the frequency. Next, measure the distance between adjacent water ripples' peaks or valleys to determine the wavelength. Finally, measure the time that it takes for each wave or ripple to pass the wooden post; that's the period.

Table 3. Cycles per second and frequency

Number of cycles per second	Factor	Unit	Abbreviation	Frequency (SI units)
1	10^0	hertz	Hz	1 hertz (1 Hz)
1,000	10^3	kilohertz	kHz	1 kilohertz (1 kHz)
1,000,000	10^6	megahertz	MHz	1 megahertz (1 MHz)
1,000,000,000	10^9	gigahertz	GHz	1 gigahertz (1 GHz)
1,000,000,000,000	10^{12}	terahertz	THz	1 terahertz (1 THz)
1,000,000,000,000,000	10^{15}	petahertz	PHz	1 petahertz (1 PHz)
1,000,000,000,000,000,000	10^{18}	exahertz	EHz	1 exahertz (1 EHz)

8.2. Frequency

The frequency of an electromagnetic signal is related to wavelength using the following formulas, which are simply the speed of light divided by wavelength:

$$f = \frac{983{,}571{,}056.43}{\lambda_{feet}}$$

where
f is frequency in hertz
λ_{feet} is wavelength in feet

$$f = \frac{299{,}792{,}458}{\lambda_{meters}}$$

where
f is frequency in hertz
λ_{meters} is wavelength in meters

Example:
What is the frequency of a signal whose wavelength in a vacuum is 30 meters?

Solution:

$$f = \frac{299{,}792{,}458}{\lambda_{meters}}$$

$$f = \frac{299{,}792{,}458}{30}$$

$$f = 9{,}993{,}081.933$$

Answer: The frequency is 9,993,081.933 Hz or 9.99 MHz.

8.3. Wavelength

The wavelength of an electromagnetic signal is related to frequency using the following formulas:

$$\lambda_{feet} = \frac{983{,}571{,}056.43}{f}$$

where
λ_{feet} is wavelength in feet
f is frequency in hertz

$$\lambda_{meters} = \frac{299{,}792{,}458}{f}$$

where
λ_{meters} is wavelength in meters
f is frequency in hertz

Example:
What is the wavelength in feet of a 100 MHz signal in a vacuum?

Solution:
First convert 100 MHz to Hz: 100 MHz = 100,000,000 Hz, then use the following formula:

$$\lambda_{feet} = \frac{983{,}571{,}056.43}{f}$$

$$\lambda_{feet} = \frac{983{,}571{,}056.43}{100{,}000{,}000}$$

$$\lambda_{feet} = 9.836$$

Answer: The free-space (that is, in a vacuum) wavelength in feet of a 100 MHz signal is 9.836 feet.

8.4. Period

The period of an electromagnetic signal is related to frequency with the following formulas:

$$T = 1/f$$

where
T is period in seconds
f is frequency in hertz

$$f = 1/T$$

where
f is frequency in hertz
T is period in seconds

Example 1:
What is the period of a 10 MHz RF signal?

Solution 1:
$T = 1/f$

$T = 1/10{,}000{,}000$

$T = 0.0000001$

Answer: The period of a 10 MHz signal is 0.0000001 second or 0.1 microsecond (µs).

Example 2:
What is the frequency of an RF signal whose period is 1 µs (0.000001 second)?

Solution 2:
$f = 1/T$

$f = 1/0.000001$

$f = 1{,}000{,}000$

Answer: The frequency is 1,000,000 Hz or 1 MHz.

8.5. Free-space wavelength formulas

The following table summarizes formulas to calculate the free-space wavelength in meters, feet, and inches, for a full wavelength, half-wavelength, and quarter-wavelength.

Table 4 - Wavelength formulas

λ_{meters}	=	$299.792458/f_{MHz}$
$\lambda/2_{meters}$	=	$149.896229/f_{MHz}$
$\lambda/4_{meters}$	=	$74.948115/f_{MHz}$
λ_{feet}	=	$983.571056/f_{MHz}$
$\lambda/2_{feet}$	=	$491.785528/f_{MHz}$
$\lambda/4_{feet}$	=	$245.892764/f_{MHz}$
λ_{inches}	=	$11{,}802.852677/f_{MHz}$
$\lambda/2_{inches}$	=	$5{,}901.426339/f_{MHz}$
$\lambda/4_{inches}$	=	$2{,}950.713169/f_{MHz}$

Assumptions for table:
Speed of light in a vacuum = 299,792,458 meters per second, or 983,571,056.43 feet per second
λ = wavelength
λ/2 = half wavelength
λ/4 = quarter wavelength
f_{MHz} = frequency in megahertz

Example:
What is the free space wavelength, in feet, of a 450 MHz RF signal?

Solution:
$$\lambda_{feet} = 983.571056/f_{MHz}$$

$$\lambda_{feet} = 983.571056/450$$

$$\lambda_{feet} = 2.19$$

Answer: The wavelength is 2.19 feet, or 2 feet, 2.28 inches.

8.6. Angular velocity (angular frequency)

Another measure of frequency is angular velocity (or angular frequency). The SI unit of angular velocity is expressed in radians per second (rad/s) and is commonly represented by the lowercase form of the Greek letter omega (ω). An angular frequency of ω = 1 rad/s corresponds to a frequency in hertz of f = (1 rad/s)/(2π) ≈ 0.159 Hz.

8.6.1. Radians

The concept of angular velocity (angular frequency) is based upon something known as the radian. A radian – abbreviated rad – is a unit of angular measure, where an angle of 1 radian defines an arc that has a length equal to the radius of a circle (see Figure 1). This results in the following relationships:

1 radian = 180/π degrees ≈ 57.3 degrees

From this, degrees can be expressed in radians and vice versa: 360° = 2π radians, 180° = π radians, 90° = π/2 radians, and so on. See Figure 2.

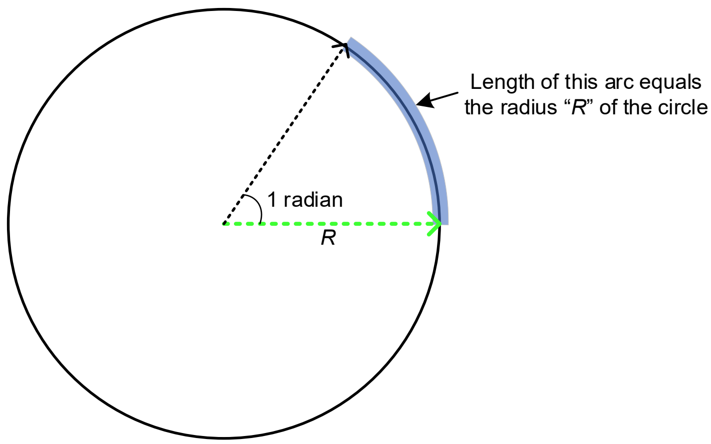


Figure 1 - The angle formed by an arc whose length is equal to the length of a circle's radius is 1 radian, which is approximately 57.3 degrees.

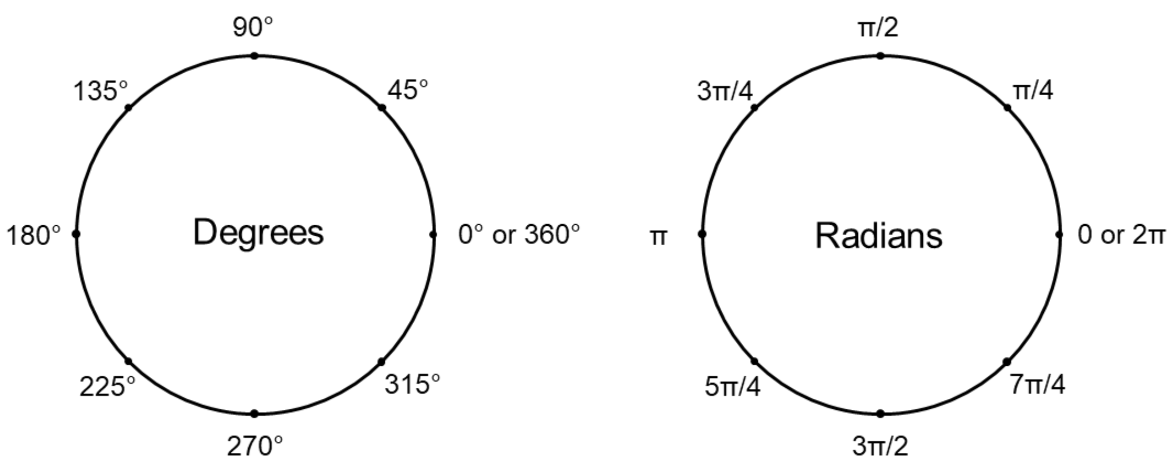


Figure 2. These two graphics illustrate the relationship between degrees and radians.

Here's one example of the application of radians to frequency: A sinusoidal signal such as an unmodulated RF carrier wave – more commonly known as a continuous wave (CW) carrier – can be expressed mathematically as

$$C_{CW} = A * sin(\omega_0 * t)$$

where
C_{CW} is an unmodulated carrier wave (that is, a CW carrier)
A is the peak amplitude of the carrier wave

t is time in seconds
ω_0 is the angular frequency of the carrier wave in radians per second

8.6.2. Convert hertz to radians per second

To convert frequency in hertz to radians per second use the following formula:

$$\omega = 2\pi * f$$

where
ω is angular velocity (angular frequency) in radians per second
f is frequency in hertz

Example:
What is a frequency of 1 MHz expressed in radians per second?

Solution:
First convert 1 MHz to hertz (1 MHz = 1,000,000 Hz) then use the following formula:

$\omega = 2\pi * f$

$\omega = 2\pi * 1{,}000{,}000$

$\omega = (2 * 3.1416) * 1{,}000{,}000$

$\omega = (6.2832) * 1{,}000{,}000$

$\omega = 6{,}283{,}185.31$

Answer: 1 MHz is 6,283,185.31 rad/s.

8.6.3. Convert radians per second to hertz

To convert angular velocity (angular frequency) in radians per second to frequency in hertz use the following formula:

$$f = \frac{\omega}{2\pi}$$

where
f is frequency in hertz
ω is angular velocity (angular frequency) in radians per second

Example:
What is an angular velocity (angular frequency) of 376.9912 rad/s expressed in hertz?

Solution:

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{376.9912}{2\pi}$$

$$f = \frac{376.9912}{2 * 3.1416}$$

$$f = \frac{376.9912}{6.2832}$$

$$f = 60$$

Answer: The frequency is 60 Hz.

9. Time Domain and Frequency Domain

Most electrical signals of interest in cable systems vary as a function of time.[5] One simple example is a sinusoidal voltage. Such a signal is a function with time as its domain and voltage as its range. We can write $V(t)$ indicating that the signal, V, is a function of time, t.

Mathematically the signal, $V(t)$, can be described using the following formula:[6]

$$V(t) = A * sin(2 * \pi * f * t)$$

where
$V(t)$ is the signal, V, as a function of time, t
A is the peak voltage of the signal in volts
sin is the sine function
f is frequency in hertz
t is time in seconds

Figure 3 illustrates a plot of $V(t)$ for a sinusoidal waveform (also called a sine wave), where A is the peak voltage. Note: The period of the sine wave in seconds is the reciprocal of its frequency, or $1/f$. Refer to Section 8.4 for more information on period.

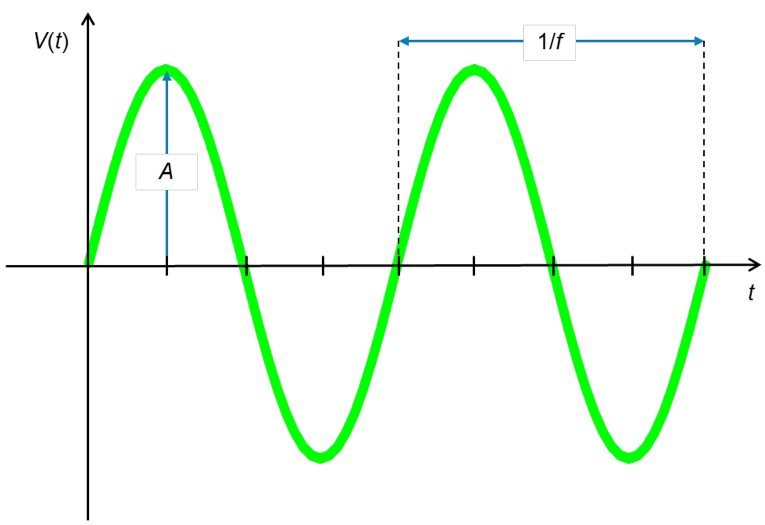


Figure 3 - V(t) for a sinusoidal waveform.

A good example is the 120 volts root mean square (RMS) alternating current "signal" from a North American household electric outlet. Its peak voltage is $120\ V_{RMS} * \sqrt{2} = 169.71$ volts and its frequency is 60 Hz. Here, $V(t) = 169.71 * sin(2 * \pi * 60 * t)$.

---

[5] Robert Gagliardi, *Introduction to Communications Engineering*, John Wiley and Sons, ©1978 pp 1-10
[6] The formula often includes a phase term, θ, for example, $V(t) = A * sin(2\pi f t + \theta)$. For this discussion, the phase term is omitted to simplify the concept.

Consider a sine wave at a frequency of 1 kHz (f = 1,000) and peak amplitude of 1 volt (A = 1), represented by the formula $V(t) = 1 * sin(2 * \pi * 1{,}000 * t)$.

If we were to connect this signal to an oscilloscope the resulting display of one cycle of that signal would appear as shown in Figure 4.

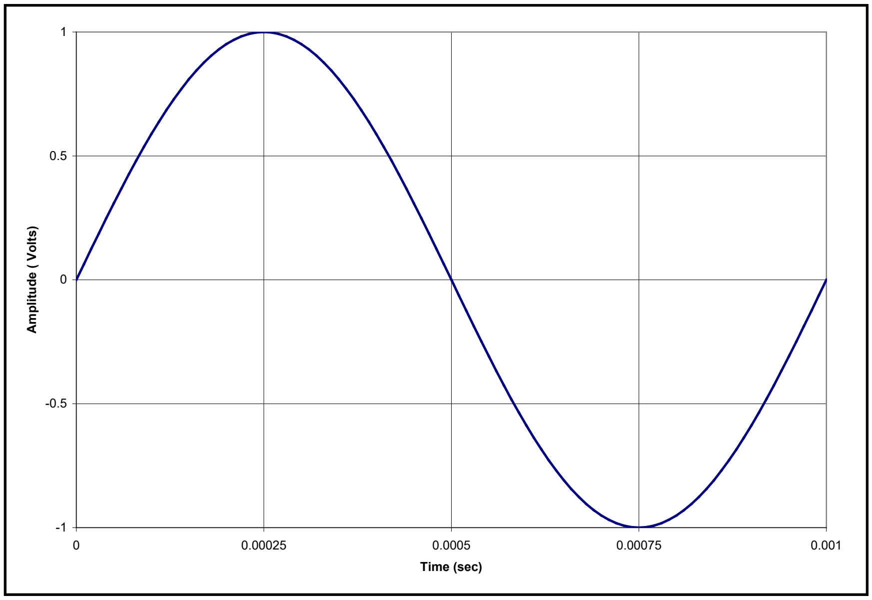


Figure 4 - One cycle of a 1 kHz sine wave as displayed on an oscilloscope.

The oscilloscope displays time (domain) on the horizontal axis and displays voltage (range) on the vertical axis. This is known as a time domain representation of the signal. Figure 5 shows examples of other types of signals as they would appear on an oscilloscope (i.e., in the time domain).

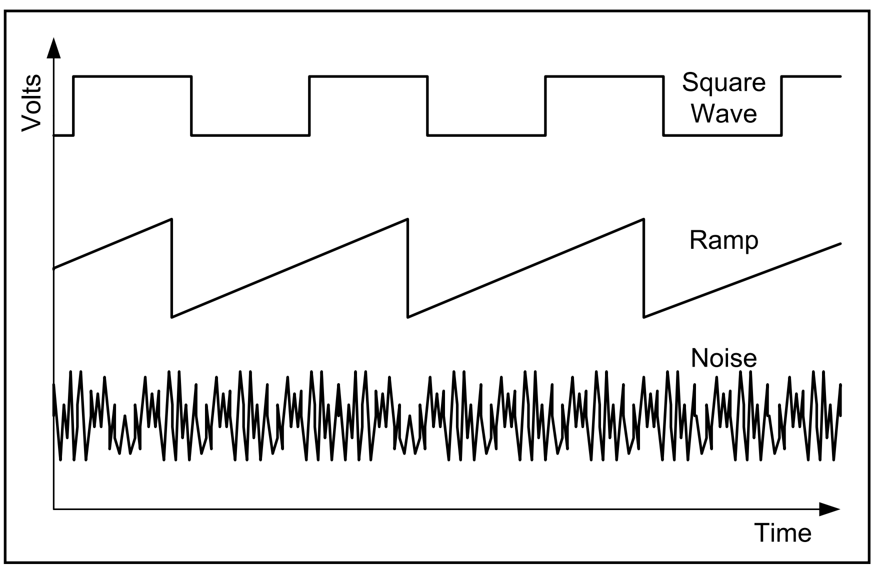


Figure 5 - Other examples of signals in the time domain.

Note: One must be cautious when using an oscilloscope to display and measure signals, particularly high frequency signals. Many analog oscilloscopes are intentionally designed with limited bandwidth. Signals *containing components at frequencies that exceed the bandwidth of the oscilloscope may not be displayed properly. Digital sampling oscilloscopes are of particular concern. The sampling rate must be significantly higher than the highest frequency component of the signal to be measured. Failure to follow this requirement can result in artifacts called aliases that can completely obscure the desired signal. It is important to confirm that the useful frequency range of an oscilloscope exceeds the range of frequencies in the signal to be measured.*

Equivalently it is possible to think of signals as a function of frequency. Consider again our original 1 kHz sinusoidal signal. It contains power at a single frequency, 1 kHz. If we were to plot power as a function of frequency that power would show up as a narrow vertical spike at 1 kHz on the frequency axis. This is illustrated in Figure 6.

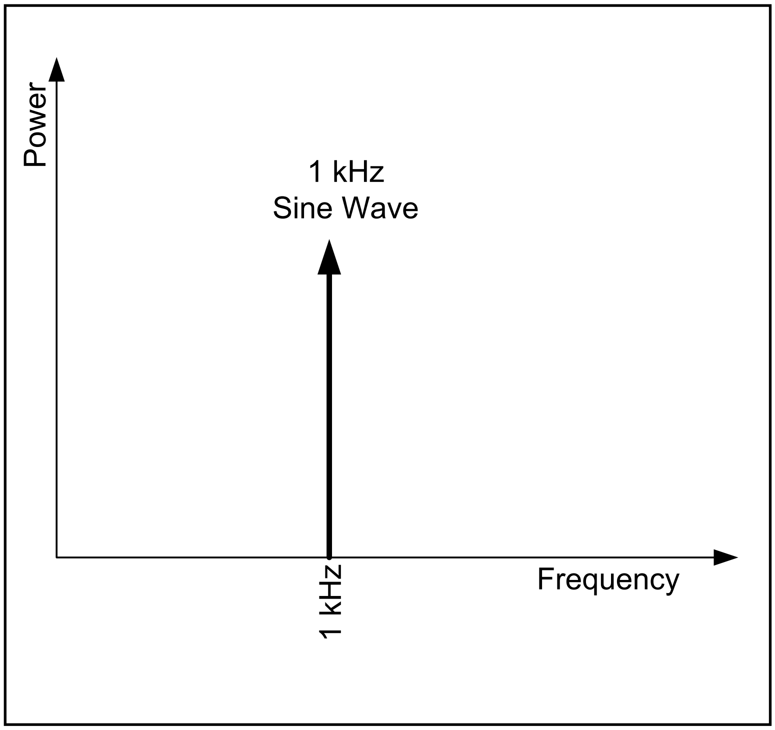


Figure 6 - A 1 kHz sine wave in the frequency domain.

In this case we consider frequency as the domain of the signal and power as the range. Thus, in Figure 6 the signal is shown in the frequency domain.[7]

The two ways of observing the signal, time domain and frequency domain, are both completely valid but emphasize different characteristics of the same signal. Techniques developed by Fourier[8] allow us to mathematically convert between the two domains (the details of this method are beyond the scope of what we wish to cover in this text).

[7] From a rigorous mathematical standpoint, the frequency domain representation of a signal must contain not only amplitude information but also phase information. Frequency spectra are, by definition, vector quantities. However, for the purposes of the discussions here we will focus on the amplitude of the frequency spectra. The inquisitive reader is encouraged to investigate the complete nature of frequency spectra by examining the Fourier transform.

[8] Athanasios Papoulis, *The Fourier Integral and its Applications*, McGraw-Hill Book Company, Inc., © 1962, pp 1-17

A conventional swept-tuned spectrum analyzer uses RF and analog circuitry to produce an approximate representation of a signal or signals in the frequency domain. If we were to apply our 1 kHz signal to the input of a spectrum analyzer the display would appear as shown in Figure 7.

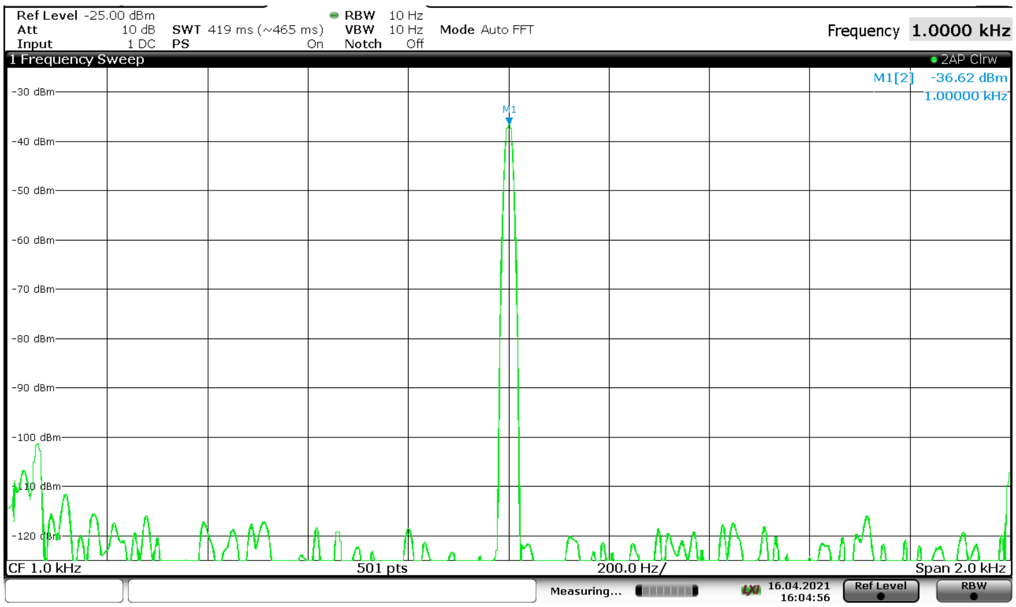


Figure 7 - A 1 kHz sine wave as displayed on a spectrum analyzer (courtesy of Comcast).

Spectrum analyzers display signals in the frequency domain just as oscilloscopes display signals in the time domain. Real world constraints prevent the spectrum analyzer from resolving the frequency components of the signal at its input with infinite resolution. Consequently, the signal as displayed on the spectrum analyzer has a width in the frequency domain that is a function of the width of the filters used inside the spectrum analyzer.

A sinusoidal signal such as that shown Figure 3, Figure 4, Figure 6, and Figure 7 is generally referred to as a *carrier*. This designation has to do with the use of sinusoidal signals in RF modulation. A carrier without modulation is often referred to as a continuous wave (CW) carrier.

Now consider a more complex signal consisting of the sum of three sine waves, each with an amplitude of 1 volt. The first sine wave will be at a frequency of 1 kHz, the second will be at a frequency of 2 kHz and the third at a frequency of 3 kHz. We can describe the composite voltage resulting from the sum of these three sine waves as

$$V(t) = [1 * sin(2 * \pi * 1{,}000 * t)] + [1 * sin(2 * \pi * 2{,}000 * t)] + [1 * sin(2 * \pi * 3{,}000 * t)]$$

If we were to connect this complex signal to an oscilloscope, the resulting time domain display would appear as shown in Figure 8.

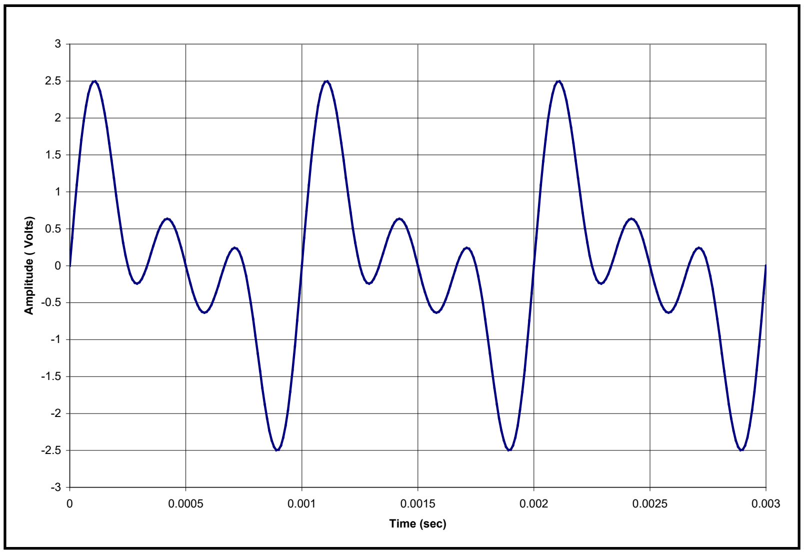


Figure 8 - The sum of a 1 kHz sine wave, 2 kHz sine wave, and 3 kHz sine wave as displayed on an oscilloscope.

It is not readily apparent by simple inspection in the time domain that this signal consists of the sum of three sine waves. Figure 9 shows the resulting display if this complex signal were connected to a spectrum analyzer.

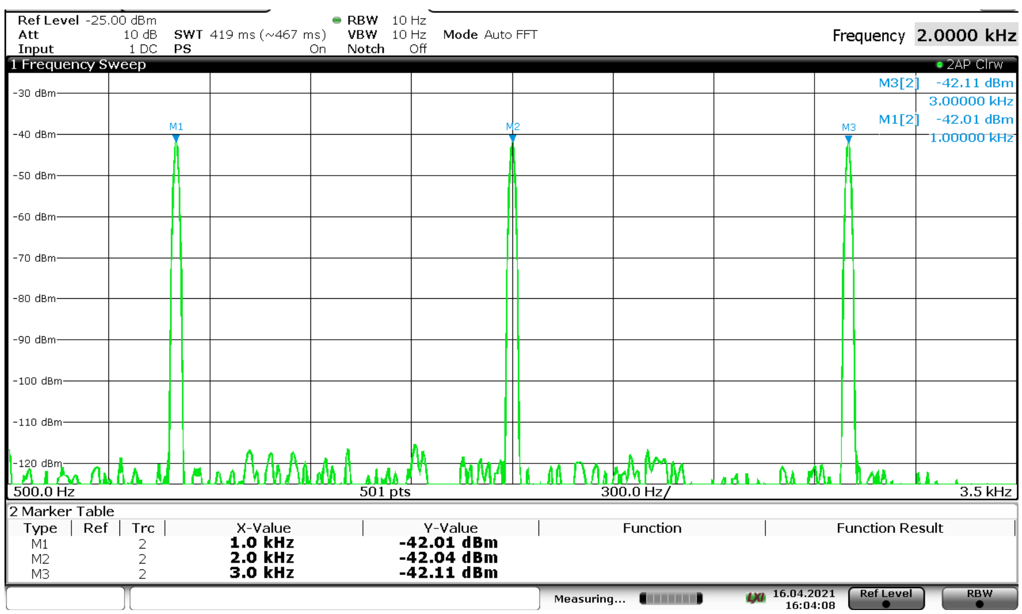


Figure 9 - The sum of three sine waves – 1 kHz, 2 kHz, and 3 kHz – in the frequency domain as displayed on a spectrum analyzer (courtesy of Comcast).

The spectrum analyzer makes it much easier to see that the signal contains power at three distinct frequencies.

When dealing with cable signals it is often useful to consider those signals in both the time domain and the frequency domain. The two ways of examining the signal emphasize different characteristics. Depending upon the task at hand one representation may be preferred to the other.

10. Conversion Factors for Sinusoidal AC Voltage and Current

An unmodulated RF carrier, also known as a continuous wave (CW) carrier, is a sinusoidal alternating current (AC) waveform. The 120 volts electricity from a North American household electrical outlet also is a sinusoidal waveform. Figure 10 shows an example of a sinusoidal waveform, or sine wave, in the time domain (that is, the horizontal axis is time and the vertical axis is amplitude). The figure's horizontal axis also shows the phase of the waveform in degrees, and the waveform's period (T) in units of seconds (see Section 8.4 for more information on period). The graphic in the figure is similar to what one would see on an oscilloscope display of a sine wave. The formulas in this section can be used to convert between the various ways commonly used to express the amplitude of sinusoidal AC voltage or current: average, peak, peak-to-peak, and root mean square.

average (avg): The average (or mean) value of a symmetrical alternating quantity such as a sine wave is defined here as the average value of the (full wave) rectified quantity measured over one complete cycle. Mathematically, the average value of a sinusoidal AC waveform is $avg = p * (2/\pi)$.

peak (p): The maximum value of an alternating quantity such as a sinusoidal AC waveform during one cycle, relative to a reference value such as 0 volts or 0 amperes (e.g., the horizontal line in Figure 10 labeled 0). Mathematically, the peak value of a sinusoidal AC waveform is $p = RMS * \sqrt{2}$.

peak-to-peak (p-p): The sum of the maximum value of an alternating quantity such as a sinusoidal AC waveform during one cycle, above and below a reference such as 0 volts or 0 amperes (e.g., the horizontal line in Figure 10 labeled 0). Mathematically, the peak-to-peak value of a sinusoidal AC waveform is $p\text{-}p = p * 2$.

root mean square RMS): In the context as used here, RMS is based upon equating the values of AC and DC power to heat a resistive element to exactly the same degree. An RMS value is found by squaring the individual values of all the instantaneous values of voltage or current in a single AC cycle. Take the average of those squares and find the square root of the average. Mathematically, the root mean square value of a sinusoidal AC waveform is $RMS = p * \left(1/\sqrt{2}\right)$.

The formulas and examples for a sinusoidal waveform are NOT applicable to the quasi-square wave alternating current waveform provided by ferroresonant transformer-based cable network line power supplies. (Note: The peak-to-peak value of a quasi-square wave waveform is $p\text{-}p = p * 2$; the peak value of that type of waveform is $p = p\text{-}p * 0.5$.)

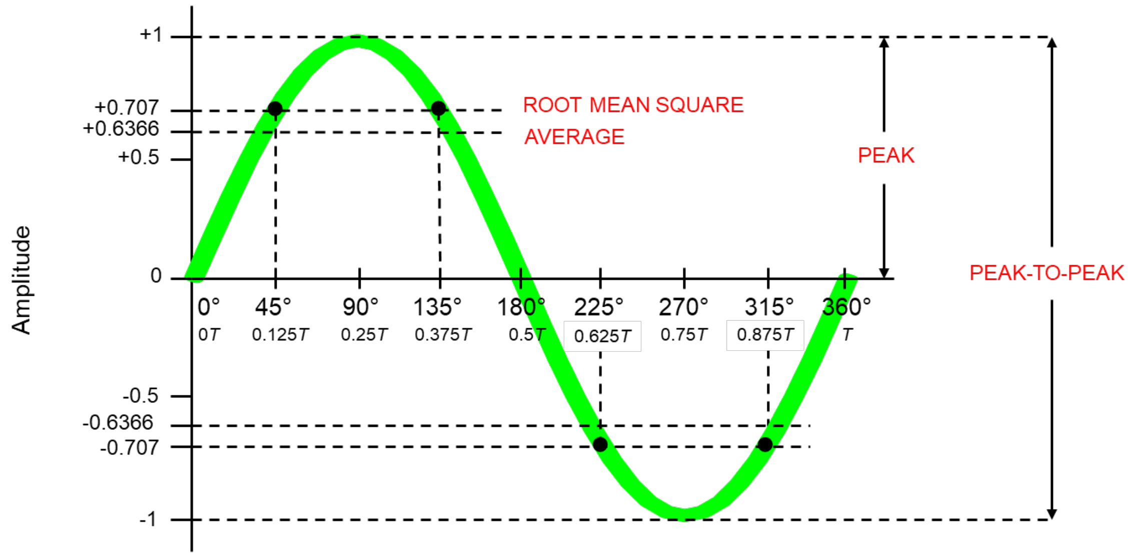


Figure 10 - Relationships between root mean square, average, peak, and peak-to-peak values of AC voltage (or AC current) in a sinusoidal waveform. Adapted from [1].

10.1. Convert peak voltage to peak-to-peak voltage (or peak current to peak-to-peak current)

The following formula can be used to convert peak voltage to peak-to-peak voltage (or peak current to peak-to-peak current):

$$p\text{-}p = 2 * p$$

where
$p\text{-}p$ is peak-to-peak voltage or current
p is peak voltage or current

Example:
What is the peak-to-peak voltage of a 30 volts peak sine wave?

Solution:
$p\text{-}p = 2 * p$

$p\text{-}p = 2 * 30$

$p\text{-}p = 60$

Answer: The peak-to-peak voltage is 60 volts.

10.2. Convert peak-to-peak voltage to peak voltage (or peak-to-peak current to peak current)

The following formulas can be used to convert peak-to-peak voltage to peak voltage (or peak-to-peak current to peak current):

$$p = \frac{p\text{-}p}{2}$$

$$p = p\text{-}p * 0.5$$

where
p is peak voltage or current
$p\text{-}p$ is peak-to-peak voltage or current

Example:
What is the peak voltage of a 60 volts peak-to-peak sine wave?

Solution:

$$p = \frac{p\text{-}p}{2}$$

$$p = \frac{60}{2}$$

$$p = 30$$

Answer: The peak voltage is 30 volts.

10.3. Convert root mean square voltage to peak voltage (or root mean square current to peak current)

The following formulas can be used to convert root mean square voltage to peak voltage (or root mean square current to peak current):

$$p = RMS * \sqrt{2}$$

$$p = RMS * 1.414214$$

where
p is peak voltage or current
RMS is root mean square voltage or current

Example:
What is the peak current of a 12 amperes RMS sinusoidal current?

Solution:
$p = RMS * 1.414214$

$p = 12 * 1.414214$

$p = 16.97$

Answer: The peak current is 16.97 amperes.

10.4. Convert root mean square voltage to peak-to-peak voltage (or root mean square current to peak-to-peak current)

The following formulas can be used to convert root mean square voltage to peak-to-peak voltage (or root mean square current to peak-to-peak current):

$$p\text{-}p = RMS * \left(2 * \sqrt{2}\right)$$

$$p\text{-}p = RMS * 2.828427$$

where
$p\text{-}p$ is peak-to-peak voltage or current
RMS is root mean square voltage or current

Example:
What is the peak-to-peak voltage of a 120 volts RMS electrical outlet?

Solution:
$p\text{-}p = RMS * \left(2 * \sqrt{2}\right)$

$p\text{-}p = 120 * \left(2 * \sqrt{2}\right)$

$p\text{-}p = 120 * (2 * 1.4142)$

$p\text{-}p = 120 * (2.8284)$

$p\text{-}p = 339.41$

Answer: The peak-to-peak voltage is 339.41 volts.

10.5. Convert average voltage to peak voltage (or average current to peak current)

The following formulas can be used to convert average voltage to peak voltage (or average current to peak current):

$$p = avg * \frac{\pi}{2}$$

$$p = avg * 1.570796$$

where
p is peak voltage or current
avg is average voltage or current

Example:
Assume that the average current of a sinusoidal waveform is 10 amperes. What is the peak current?

Solution:
$p = avg * \frac{\pi}{2}$

$p = 10 * \frac{3.141593}{2}$

$p = 10 * 1.570796$

$p = 15.71$

Answer: The peak current is 15.71 amperes.

10.6. Convert peak voltage to average voltage (or peak current to average current)

The following formulas can be used to convert peak voltage to average voltage (or peak current to average current):

$$avg = p * \frac{2}{\pi}$$

$$avg = p * 0.636620$$

where
avg is average voltage or current
p is peak voltage or current

Example:
Assume the peak current of a sinusoidal waveform is 15.71 amperes. What is the average current?

Solution:

$$avg = p * \frac{2}{\pi}$$

$$avg = 15.71 * \frac{2}{3.141593}$$

$$avg = 15.71 * 0.636620$$

$$avg = 10.00$$

Answer: The average current is 10 amperes.

10.7. Convert root mean square voltage to average voltage (or root mean square current to average current)

The following formulas can be used to convert root mean square voltage to average voltage (or root mean square current to average current):

$$avg = RMS * \frac{(2 * \sqrt{2})}{\pi}$$

$$avg = RMS * 0.900316$$

where
avg is average voltage or current
RMS is root mean square voltage or current

Example:
What is the average voltage of a 120 volts RMS electrical outlet?

Solution:

$$avg = RMS * 0.900316$$

$$avg = 120 * 0.900316$$

$$avg = 108.04$$

Answer: The average voltage is 108.04 volts.

10.8. Convert peak voltage to root mean square voltage (or peak current to root mean square current)

The following formulas can be used to convert peak voltage to root mean square voltage (or peak current to root mean square current):

$$RMS = p * \frac{1}{\sqrt{2}}$$

$$RMS = p * 0.707107$$

where
$p\text{-}p$ is peak-to-peak voltage or current
p is peak voltage or current
RMS is root mean square voltage or current

Example:
What is the root mean square voltage of a North American electrical outlet that provides a peak voltage of 169.7057 volts?

Solution:
$RMS = p * 0.707107$

$RMS = 169.7057 * 0.707107$

$RMS = 120.00$

Answer: The RMS voltage is 120 volts.

10.9. Convert peak-to-peak voltage to root mean square voltage (or peak-to-peak current to root mean square current)

The following formulas can be used to convert peak-to-peak voltage to root mean square voltage (or peak-to-peak current to root mean square current):

$$RMS = p\text{-}p * \frac{1}{\left(2 * \sqrt{2}\right)}$$

$$RMS = p\text{-}p * 0.353553$$

where
RMS is root mean square voltage or current
$p\text{-}p$ is peak-to-peak voltage or current

Example:
What is the root mean square voltage of a 339.41 volts peak-to-peak sinusoidal waveform?

Solution:
$RMS = p\text{-}p * 0.353553$

$RMS = 339.41 * 0.353553$

$RMS = 119.99$

Answer: The RMS voltage is about 120 volts.

10.10. Convert average voltage to root mean square voltage (or average current to root mean square current)

The following formulas can be used to convert average voltage to root mean square voltage (or average current to root mean square current):

$$RMS = avg * \frac{\pi}{\left(2 * \sqrt{2}\right)}$$

$$RMS = avg * 1.110721$$

where
RMS is root mean square voltage or current
avg is average voltage or current

Example:
What is the root mean square current of a sinusoidal waveform when the average current is 10.8 amperes?

Solution:
$RMS = avg * 1.110721$

$RMS = 10.8 * 1.110721$

$RMS = 11.9958$

Answer: The RMS current is about 12 amperes.

11. Common Decibel-Based Calculations

11.1. The decibel

The foundation of much of the mathematics of cable – and many of the formulas in this document – is the decibel, which is a logarithmic-based expression of the ratio between two values of a physical quantity, typically power or intensity. The decibel provides an efficient way to express ratios which span one or more powers of the logarithmic base, most commonly 10 (see sidebar immediately prior to Section 11.4).

11.1.1. Why use logarithms and decibels?

RF signal power levels used in cable networks can exist over a very large dynamic range. Radio receivers recover signals that are about 10 times bigger than thermal noise power. This is a very small amount of energy at room temperature. Coaxial cable and taps attenuate the RF signals launched by optical nodes and amplifiers by a factor of about 10,000. Tracking signal levels as they travel from node or amplifier to the subscriber drop and coax outlet inside the customer premises is easiest if referenced back to some convenient level. Let's say an amplifier output level is 1 volt of RF signal. The set-top box or cable modem in a subscriber's home can recover a useable signal that is as little as 0.0001 volt. That is a dynamic range of 10,000-fold compared to the amplifier output.

We could use volts, or millivolts (or even units of watts), to describe RF signal power levels in cable networks, but it is cumbersome with such a large range of values to accommodate. It is easier to convert these signal levels into decibels by employing logarithms.

Another neat feature of decibels is that they make it much easier to determine the impact of an amplifier or length of coaxial cable on signal levels. Amplifiers have gain which can be represented in decibels – the ratio of output signal level to input signal level. Similarly, a distribution feeder will attenuate the signal level by the ratio of tap port signal level divided by amplifier launch level. When expressed in decibels, the impact of amplifiers or passive components is easy to determine by simply adding or subtracting the applicable number of decibels. In essence, logarithms translate multiplication and division into addition and subtraction – a much easier process to do in one's head.

11.2. Convert the ratio of two power levels to decibels

The ratio in decibels of two power levels P_1 and P_2 can be calculated as follows:

$$dB = 10log_{10}\left(\frac{P_1}{P_2}\right)$$

where
dB is the value in decibels
log_{10} is base 10 logarithm
P_1 is power in watts (or units of watts, e.g., milliwatts)
P_2 is power in watts (the same units of watts as P_1)

Example 1:
Let's say your company truck has a 50 watts two-way radio, and the dispatch office uses a 100 watts base station. How much more power, in decibels, does dispatch's base station radio have than the one in your truck?

Solution 1:

$$dB = 10log_{10}\left(\frac{P_1}{P_2}\right)$$

$$dB = 10 * log_{10}\left(\frac{100}{50}\right)$$

$$dB = 10 * log_{10}(2)$$

$$dB = 10 * (0.301)$$

$$dB = 3.01$$

Answer: The 100 watts dispatch office radio has 3.01 dB greater power than the 50 watts radio in the truck.

Example 2:
Assume a local FM radio station installs a new 40,000 watts transmitter to replace an old 20,000 watts transmitter. How much more powerful, in decibels, is the new transmitter than the old one?

Solution 2:

$$dB = 10log_{10}\left(\frac{P_1}{P_2}\right)$$

$$dB = 10 * log_{10}\left(\frac{40{,}000}{20{,}000}\right)$$

$$dB = 10 * log_{10}(2)$$

$$dB = 10 * (0.301)$$

$$dB = 3.01$$

Answer: The 40,000 watts transmitter is 3.01 dB more powerful than the 20,000 watts transmitter.

How can that be? In the first example, the power difference is 50 watts, and in the second example the power difference is 20,000 watts! Why does 3.01 dB apply to both examples? The absolute difference between the two power levels isn't what matters, it's the *ratio* of the two power levels. In both examples the ratio is 2:1. That is, one power level is twice as much as the other. (Note: Reducing the power by half, say, from 20,000 watts to 10,000 watts, also is a 3.01 dB change.)

Using the decibel for voltage ratios
The decibel, while technically a ratio of two power levels, also can be used to represent the ratio of two voltage levels, assuming the two voltages are across (or in) the same impedance.

Here is how that relationship is derived: The unit of electrical power, the watt, equals 1 volt multiplied by 1 ampere. Equation-wise, $P = EI$, where P is power in watts, E is voltage in volts, and I is current in amperes. Substituting the Ohm's Law equivalent for E and I gives additional formulas for power: $P = E^2/R$ and $P = I^2R$. If the right-hand side of the power equation $P = E^2/R$ is substituted for both P_1 and P_2 in the formula $dB = 10log_{10}(P_1/P_2)$, the equation becomes $dB = 10log_{10}[(E^2/R)/(E^2/R)]$ which is the same as $dB = 10log_{10}[(E_1^2)/R_1)/(E_2^2/R_2)]$. In this example, R represents the 75 ohms impedance of a cable network. Since R_1 and R_2 are both equal to 75 ohms, those equation terms cancel, leaving the equation $dB = 10log_{10}(E_1^2/E_2^2)$. This can be simplified somewhat and written as $dB = 10log_{10}(E_1/E_2)^2$ which is the same as $dB = 2 * 10log_{10}(E_1/E_2)$ or $dB = 20log_{10}(E_1/E_2)$.

11.3. Convert the ratio of two voltage levels to decibels

The ratio in decibels of two voltage levels E_1 and E_2 can be calculated as follows:

$$dB = 20log_{10}\left(\frac{E_1}{E_2}\right)$$

where
dB is the value in decibels
log_{10} is base 10 logarithm
E_1 is voltage in volts (or units of volts, e.g., millivolts)
E_2 is voltage in volts (the same units of volts as E_1)

Note: Voltages E_1 and E_2 must be across the same impedance.

Example:
What is the difference, in decibels, between 100 millivolts (mV) and 50 mV, in the same 75 ohms impedance network?

Solution:

$$dB = 20log_{10}\left(\frac{E_1}{E_2}\right)$$

$$dB = 20 * log_{10}\left(\frac{100}{50}\right)$$

$$dB = 20 * log_{10}(2)$$

$$dB = 20 * (0.301)$$

$$dB = 6.02$$

Answer: The difference, in decibels, between 100 mV and 50 mV is 6.02 dB.

An important point: By itself, the decibel cannot be used to express absolute values.[9] For instance, one can correctly say that an amplifier has 20 dB of gain, or a splitter has 4 dB of loss. It is incorrect to say that the RF signal level at the input to a set-top box is –2 dB, or the RF signal level at a line extender output is 48 dB. For that, "dB" must be appended with a reference suffix, such as dBmV, discussed later. The two examples here, when correctly stated, are –2 dBmV and +48 dBmV respectively.

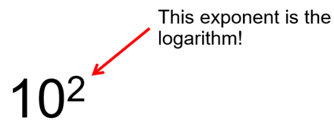
[9] The decibel (dB) is used to express gain, loss or attenuation, return loss, structural return loss (SRL), isolation, carrier-to-noise ratio (CNR), composite intermodulation noise (CIN) ratio, signal-to-noise ratio (SNR), receive modulation error ratio (RxMER), various carrier-to-distortion ratios, and similar parameters.

Logarithms

The formulas in the previous section include "$\log_{10}$," which is an abbreviation for "base 10 logarithm." Consider the numbers 100, 1,000, 10,000 and 0.001, and the different ways in which they can be represented:

$100 = 10 * 10 = 10^2$
$1{,}000 = 10 * 10 * 10 = 10^3$
$10{,}000 = 10 * 10 * 10 * 10 = 10^4$
$0.001 = 1/(10 * 10 * 10) = 10^{-3}$

What do the numbers 100, 1,000, 10,000, and 0.001 have in common? Each can be represented as 10 raised to some power – for instance, 10 raised to the power of 2 equals 100, written as $10^2 = 100$. The exponent – the power to which 10 is raised – is the base 10 logarithm!



From this, the logarithms of the previous example numbers can be shown.

$\log_{10}(100) = 2$
That is, $10^2 = 100$

$\log_{10}(1{,}000) = 3$
That is, $10^3 = 1{,}000$

$\log_{10}(10{,}000) = 4$
That is, $10^4 = 10{,}000$

$\log_{10}(0.001) = –3$
That is, $10^{–3} = 0.001$

What about a number such as 523?
$\log_{10}(523) = 2.7185$
That is, $10^{2.7185} = 523$

Note: The subscript "10" is sometimes dropped from "$\log_{10}$" when dealing with base 10 logarithms, so the formula $dB = 10\log_{10}(P_2/P_1)$ becomes $dB = 10\log(P_2/P_1)$. In this document, the subscript denoting base is retained in formulas using logarithms.

Logarithms are not limited to base 10. Indeed, they can be any base. One common logarithmic base used in data communications is 2. Here, the base 2 logarithm of a number is the power to which 2 is raised to obtain that number. For example, $\log_2(256) = 8$; that is, $2^8 = 256$. Refer to Appendix A for information about how to calculate base 2 logarithms.

For more on logarithms, an on-line search will return a variety of helpful resources and references.

11.4. Gain

Gain is an increase in the power of a signal or signals, usually measured in decibels. Expressed mathematically:

$$G_{dB} = 10log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

where
G_{dB} is gain in decibels
log_{10} is base 10 logarithm
P_{out} is output power in watts
P_{in} is input power in watts (the same units of watts as P_{out})
and $P_{out} > P_{in}$

When signal power is stated in dBmV

$$G_{dB} = P_{out(dBmV)} - P_{in(dBmV)}$$

where
G_{dB} is gain in decibels
$P_{out(dBmV)}$ is output power in dBmV
$P_{in(dBmV)}$ is input power in dBmV

Example 1:
What is the nominal gain of an amplifier whose input power is 1 watt and output power is 10 watts?

Solution 1:

$$G_{dB} = 10log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$G_{dB} = 10log_{10}\left(\frac{10\ watts}{1\ watt}\right)$$

$$G_{dB} = 10 * log_{10}(10)$$

$$G_{dB} = 10 * (1.00)$$

$$G_{dB} = 10$$

Answer: The gain of the amplifier is 10 dB.

Example 2:
What is the nominal gain of a line extender amplifier whose per-channel RF input signal level is +20 dBmV and output per-channel signal level is +48 dBmV?

Solution 2:

$$G_{dB} = P_{out(dBmV)} - P_{in(dBmV)}$$

$$G_{dB} = 48\ dBmV - 20\ dBmV$$

$G_{dB} = 28$

Answer: The gain of the line extender amplifier is 28 dB.

Note: When adding or subtracting one value in dBmV to or from another value in dBmV, the difference is in dB, not dBmV.

11.4.1. Convert linear power gain to gain in decibels

The following formula can be used to convert a linear power gain value (sometimes called a linear power ratio) to a value in decibels. Note: The formula is essentially the same as formulas used in Section 11.2 and Section 11.4, but with the ratio (fraction) in parentheses already solved to yield the linear power gain value.

$$G_{dB} = 10log_{10}(G)$$

where
G_{dB} is gain in decibels
log_{10} is base 10 logarithm
G is the linear gain or linear power ratio

Example:
Assume an amplifier has a linear power gain of 100, that is, the amplifier's gain increases the input power to the amplifier by a factor of 100. What is the amplifier's gain in decibels?

Solution:
$G_{dB} = 10log_{10}(G)$

$G_{dB} = 10 * log_{10}(100)$

$G_{dB} = 10 * (2.00)$

$G_{dB} = 20$

Answer: The gain in decibels is 20 dB.

11.4.2. Convert gain in decibels to a linear power gain value

The following formula can be used to convert gain in decibels to a linear power gain value (or linear power ratio).

$$G = 10^{G_{dB}/10}$$

where
G is the linear gain or linear power ratio
G_{dB} is gain in decibels

Example:
Assume an amplifier has 20 dB of gain. What is that power gain expressed as a linear value?

Solution:
$G = 10^{G_{dB}/10}$

$G = 10^{20/10}$

$G = 10^{2.00}$

$G = 100$

Answer: The linear gain (or linear power ratio) is 100.

11.5. Loss[10]

Loss (or attenuation) is a decrease in the power of a signal or signals, usually measured in decibels. Expressed mathematically:

$$L_{dB} = 10log_{10}\left(\frac{P_{in}}{P_{out}}\right)$$

where
L_{dB} is loss in decibels
log_{10} is base 10 logarithm
P_{in} is input power in watts
P_{out} is output power in watts (the same units of watts as P_{in})
and $P_{out} < P_{in}$

When signal power is stated in dBmV

$$L_{dB} = P_{in(dBmV)} - P_{out(dBmV)}$$

where
L_{dB} is loss in decibels
$P_{in(dBmV)}$ is input power in dBmV
$P_{out(dBmV)}$ is output power in dBmV

[10] Coaxial cable loss (attenuation) is discussed later in Section 20.3.

Example 1:
What is the loss of a high-power attenuator when the input power is 100 watts and the output power is 1 watt?

Solution 1:
$L_{dB} = 10log_{10}\left(\frac{P_{in}}{P_{out}}\right)$

$L_{dB} = 10log_{10}\left(\frac{100\ watts}{1\ watt}\right)$

$L_{dB} = 10 * log_{10}(100)$

$L_{dB} = 10 * (2)$

$L_{dB} = 20$

Answer: The attenuator's loss (or attenuation) is 20 dB.

Example 2:
What is the tap loss of an unmarked directional coupler when the per-channel RF input signal level is +10 dBmV and the per-channel output signal level is +4 dBmV?

Solution 2:
$L_{dB} = P_{in(dBmV)} - P_{out(dBmV)}$

$L_{dB} = 10\ dBmV - 4\ dBmV$

$L_{dB} = 6$

Answer: The directional coupler's tap loss is 6 dB.

Note: When adding or subtracting one value in dBmV to or from another value in dBmV, the difference is in dB, not dBmV.

11.6. Ideal splitter insertion loss

The insertion loss through an ideal multiple-output splitter that has equal insertion loss or attenuation between the input port and each of the output ports can be calculated using the following formula:

$$L_{dB} = 10log_{10}(N)$$

where
L_{dB} is loss in decibels
log_{10} is base 10 logarithm
N is the number of output ports

Example:
What is the insertion loss of an ideal two-way splitter (see Figure 11)?

Solution:
$L_{dB} = 10log_{10}(N)$

$L_{dB} = 10 * log_{10}(2)$

$L_{dB} = 10 * 0.301$

$L_{dB} = 3.01$

Answer: The ideal two-way splitter's insertion loss is 3.01 dB.

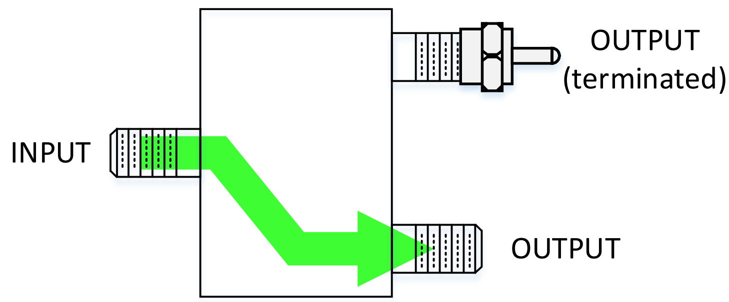


Figure 11 - Insertion loss of a splitter is measured between the input port and each output port. (Note: During the measurement, all unused ports are terminated.)

Real-world splitters have somewhat higher insertion loss than the ideal value calculated here. That additional or excess insertion loss is on the order of 0.5 dB to 1 dB (for a total insertion loss of about 3.5 dB to 4 dB in a two-way splitter), and is caused by losses in the splitter's internal toroidal transformers' ferrite-core material and their very small gauge wire windings. The next section provides a formula to calculate the insertion loss of a real-world splitter.

11.6.1. Real-world splitter insertion loss

The insertion loss through a real-world multiple-output splitter that has equal insertion loss or attenuation between the input port and each of the output ports, and which uses combinations of internal two-way splitters to achieve the desired number of outputs can be calculated using the following formula:

$$L_{dB} = \left[\frac{L_1}{0.301}\right] * log_{10}(N)$$

where
L_{dB} is loss in decibels
L_1 is the actual insertion loss of a single two-way splitter (typ. 3.5 dB to 4.0 dB)
log_{10} is base 10 logarithm
N is the number of output ports

Example:
What is the insertion loss of a real-world four-way splitter, assuming a two-way splitter has 3.5 dB of insertion loss?

Solution:

$$L_{dB} = \left[\frac{L_1}{0.301}\right] * log_{10}(N)$$

$$L_{dB} = \left[\frac{3.5}{0.301}\right] * log_{10}(4)$$

$$L_{dB} = [11.6279] * log_{10}(4)$$

$$L_{dB} = [11.6279] * (0.6021)$$

$$L_{dB} = 7.00$$

Answer: The insertion loss is 7 dB.

11.6.2. Splitter used as a combiner

A splitter installed "backwards" can be used as a combiner. The splitter's original output ports become the input ports for the sources being combined, and the splitter's original input port becomes the combined output port.

The following formula can be used to calculate the combined output power when the signals being combined have the same frequency, amplitude, and phase. The splitter is assumed to have no internal phase shift.[11]

$$P_{combined} = 10log_{10}\left(10^{P_1/10} + 10^{P_2/10} + \cdots 10^{P_n/10}\right)$$

where
$P_{combined}$ is the combined output power in decibel millivolt
log_{10} is base 10 logarithm
P_1 is the first signal's input power in decibel millivolt
P_2 is the second signal's input power in decibel millivolt
P_n is the n^{th} signal's input power in decibel millivolt

Example:
Assume that a two-way splitter is being used to combine signals from two identical antennas receiving the same over-the-air TV channel. If the signal level at the input to each of the backwards splitter's two output ports is 5 dBmV, what is the combined output signal level at the splitter's input port (refer to Figure 12)?

[11] If the signals being combined are on different frequencies – for example, two different channels – then the combined output will be *lower* in amplitude than the individual input signal levels by an amount equal to the splitter's insertion loss. See Section 11.6.

Solution:

$P_{combined} = 10log_{10}\left(10^{P_1/10} + 10^{P_2/10} + \cdots 10^{P_n/10}\right)$

$P_{combined} = 10 * log_{10}\left(10^{5/10} + 10^{5/10}\right)$

$P_{combined} = 10 * log_{10}(10^{0.50} + 10^{0.50})$

$P_{combined} = 10 * log_{10}(3.1623 + 3.1623)$

$P_{combined} = 10 * log_{10}(6.3246)$

$P_{combined} = 10 * (0.8010)$

$P_{combined} = 8.0103$

Answer: The combined output is about 8.01 dBmV. (Note: The combined output will be somewhat less than the ideal value calculated here, because of losses in the splitter's internal toroidal transformers' ferrite-core material and their very small gauge wire windings. Also, if there are any phase differences between the two inputs, that will affect the combined output signal level.)

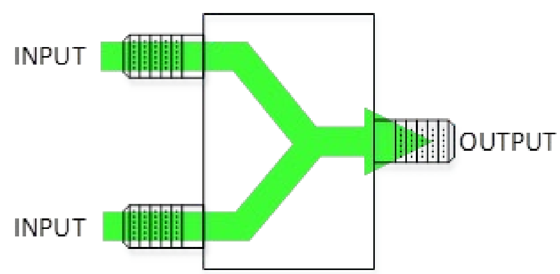


Figure 12 - Backwards two-way splitter used as a combiner.

11.7. Converting decibels and percentage

It is often desirable to convert between percentage and decibel values. The following formulas can be used to make those conversions.

To calculate the decibel equivalent of a percentage of power or voltage:

$$dB = 10log_{10}\left[\frac{percentage\ power}{100}\right]$$

$$dB = 20log_{10}\left[\frac{percentage\ voltage}{100}\right]$$

To convert from percentages to decibels:

$$percentage\ power = 100 * 10^{(dB/10)}$$

$$percentage\ voltage = 100 * 10^{(dB/20)}$$

where
dB is the value in decibels
log_{10} is base 10 logarithm
$percentage\ power$ is a ratio of power expressed as a fraction of 100, denoted using the percent sign (%)
$percentage\ voltage$ is a ratio of voltage expressed as a fraction of 100, denoted using the percent sign (%)

Example 1:
You are using an antenna feed line that has a signal loss of 0.68 dB at the frequency of interest. Calculate the percentage of transmitter power that is reaching your antenna.

Solution 1:
$percentage\ power = 100 * 10^{(dB/10)}$

$percentage\ power = 100 * 10^{(-0.68/10)}$

$percentage\ power = 100 * 10^{(-0.068)}$

$percentage\ power = 100 * 0.855067$

$percentage\ power = 85.51$

Answer: The percentage of transmitter power that is reaching the antenna is 85.51%.

Note: Negative decibel values represent signal loss.

Example 2:
You are using an antenna feed line to connect the output of the transmitter to your antenna. Assume the input power at the antenna is equal to 85.51% of the transmitter output power. What is the signal loss of the antenna feed line dB?

Solution 2:

$$dB = 10log_{10}\left[\frac{percentage\ power}{100}\right]$$

$$dB = 10 * log_{10}\left[\frac{85.51\%}{100}\right]$$

$$dB = 10 * log_{10}(0.8551)$$

$$dB = 10 * \ -0.067983$$

$$dB = -0.67983$$

Answer: The signal loss of the antenna feed line is 0.68 dB.

Example 3:
Calculate the output voltage of a 6 dB attenuator as a percentage of the input voltage.

Solution 3:

$$percentage\ voltage = 100 * 10^{(dB/20)}$$

$$percentage\ voltage = 100 * 10^{(-6/20)}$$

$$percentage\ voltage = 100 * 10^{(-0.3)}$$

$$percentage\ voltage = 100 * 0.501187$$

$$percentage\ voltage = 50.12$$

Answer: The output voltage of a 6 dB attenuator is equal to 50.12% of the input voltage.

Example 4:
Assume the output voltage of a 6 dB attenuator is equal to 50.12% of the input voltage. What is the attenuation in dB?

Solution 4:

$$dB = 20log_{10}\left[\frac{percentage\ voltage}{100}\right]$$

$$dB = 20 * log_{10}\left[\frac{50.12\%}{100}\right]$$

$$dB = 20 * log_{10}\left[\frac{50.12\%}{100}\right]$$

$dB = 20 * log_{10}(0.5012)$

$dB = 20 * \ -0.2999989$

$dB = 20 * \ -0.2999989$

$dB = -5.999978$

Answer: The attenuation is equal to about 6 dB.

11.8. Decibel millivolt (dBmV)

Decibel millivolt (dBmV) is a unit of power expressed in terms of voltage. RF signal levels in cable networks are commonly stated in dBmV, although some areas of the world use decibel microvolt (dBµV) instead of dBmV, which is discussed later. While dBmV is commonly used as an expression of absolute values, technically speaking dBmV is a logarithmic ratio of some value to a specified 0 dB reference, in this case 0 dBmV.[12] For instance, a signal level of +20 dBmV (10 millivolts) is another way of saying that 10 millivolts is 20 dB greater than 1 millivolt (0 dBmV).

11.8.1. Converting millivolts to dBmV

A value in millivolts can be converted to decibel millivolt using the following formulas:

$$dBmV = 20log_{10}\left(\frac{value\ in\ mV}{1\ mV}\right)$$

where
$dBmV$ is the value in decibel millivolt
log_{10} is base 10 logarithm
$value\ in\ mV$ is voltage in millivolts
$1\ mV$ is the 0 dB reference for dBmV

Note: The "1 mV" in the denominator is sometimes not included, making the formula

$$dBmV = 20log_{10}(value\ in\ mV)$$

Example:
What is the value in dBmV for a voltage of 40 mV?

Solution:
$dBmV = 20log_{10}(value\ in\ mV)$

$dBmV = 20 * log_{10}(40)$

$dBmV = 20 * (1.60)$

[12] In the context used here, the 0 dB reference for decibel millivolt, 0 dBmV, equals 13.33 nanowatts (nW) of power, defined as 1 millivolt (RMS) across an impedance of 75 ohms.

$dBmV = 32.04$

Answer: 40 mV is equal to 32.04 dBmV.

11.8.2. Converting dBmV to millivolts

A value in decibel millivolt can be converted to millivolts using the following formula:

$$mV = 10^{(dBmV/20)}$$

where
mV is the value in millivolt
$dBmV$ is the value in decibel millivolt

Example:
What is the value in millivolts for a signal level of +20 dBmV?

Solution:
$mV = 10^{(dBmV/20)}$

$mV = 10^{(20/20)}$

$mV = 10^1$

$mV = 10$

Answer: A signal level of +20 dBmV equals 10 millivolts.

11.8.3. Converting microvolts to dBmV

A value in microvolts can be converted to decibel millivolt using the following formula:

$$dBmV = 20log_{10}\left(\frac{value\ in\ \mu V}{1000}\right)$$

where
$dBmV$ is the value in decibel millivolt
log_{10} is base 10 logarithm
$value\ in\ \mu V$ is voltage in microvolts

Example:
What is the value in dBmV for a voltage of 500 µV?

Solution:

$dBmV = 20log_{10}\left(\frac{value\ in\ \mu V}{1000}\right)$

$dBmV = 20 * log_{10}\left(\frac{500}{1000}\right)$

$dBmV = 20 * log_{10}(0.5)$

$dBmV = 20 * (-0.301)$

$dBmV = -6.02$

Answer: 500 microvolts equals –6.02 dBmV.

11.8.4. Converting dBmV to microvolts

A value in decibel millivolt can be converted to microvolts using the following formula:

$$\mu V = 1{,}000 * 10^{(dBmV/20)}$$

where
μV is the value in microvolts
$dBmV$ is the value in decibel millivolt

Example:
What is the value in microvolts for a signal level of +10 dBmV?

Solution:

$\mu V = 1000 * 10^{(dBmV/20)}$

$\mu V = 1000 * 10^{(10/20)}$

$\mu V = 1000 * 10^{0.500}$

$\mu V = 1000 * 3.162$

$\mu V = 3{,}162$

Answer: A signal level of +10 dBmV equals 3,162 microvolts.

11.8.5. Converting dBmV to watts

Part 76 of the FCC Rules includes aeronautical band power thresholds stated in watts, which can cause some confusion for cable operators because of the widespread use of dBmV for RF signal level (power). The following is one way to convert RF signal level in dBmV to a value in watts.

First convert the power in dBmV to millivolts using the formula:

$$mV = 10^{(dBmV/20)}$$

where
mV is the value in millivolt
$dBmV$ is the value in decibel millivolt

Next, convert the value in millivolts to volts:

$$E = \frac{mV}{1{,}000}$$

where
E is voltage in volts
mV is voltage in millivolts

Finally, convert the value in volts to watts:

$$P = \frac{E^2}{R}$$

where
P is power in watts
E is voltage in volts
R is resistance or impedance in ohms (75 ohms for cable networks)

Example:
What is +38.75 dBmV in watts?

Solution:
First, convert +38.75 dBmV to millivolts:

$mV = 10^{(dBmV/20)}$

$mV = 10^{(38.75/20)}$

$mV = 10^{1.94}$

$mV = 86.596$

Next, convert 86.596 millivolts to volts:

$$V = \frac{mV}{1{,}000}$$

$$V = \frac{86.596}{1{,}000}$$

$$V = 0.0866$$

Finally, convert 0.0866 volt to watts:

$$P = \frac{E^2}{R}$$

$$P = \frac{0.0866^2}{75}$$

$$P = \frac{0.007}{75}$$

$$P = 0.0001$$

Answer: +38.75 dBmV is 0.0001 watt (10^{-4} watt), or 100 microwatts.

11.8.6. Converting watts to dBmV

The following is one way to convert power in watts to dBmV.

First convert the power in watts to volts using the formula:

$$E = \sqrt{P * R}$$

where
E is voltage in volts
P is power in watts
R is resistance or impedance in ohms (75 ohms for cable networks)

Next, convert the value in volts to millivolts:

$$mV = E * 1{,}000$$

where
mV is voltage in millivolts
E is voltage in volts

Finally, convert the value in millivolts to dBmV:

$$dBmV = 20log_{10}\left(\frac{value\ in\ mV}{1\ mV}\right)$$

where
$dBmV$ is the value in decibel millivolt
log_{10} is base 10 logarithm
$value\ in\ mV$ is voltage in millivolts
$1\ mV$ is the 0 dB reference for dBmV

Note: The “1 mV” in the denominator is sometimes not included, making the formula

$$dBmV = 20log_{10}(value\ in\ mV)$$

Example:
What is 75.85 microwatts expressed in dBmV?

Solution:
First, convert 75.85 microwatts (0.00007585 watt) to volts:

$E = \sqrt{P * R}$

$E = \sqrt{0.00007585 * 75}$

$E = \sqrt{0.00568875}$

$E = 0.075424$

Next, convert 0.075424 volt to millivolts:

$mV = 0.075424 * 1{,}000$

$mV = 75.4238$

Finally, convert 75.42 millivolts to dBmV:

$dBmV = 20log_{10}(value\ in\ mV)$

$dBmV = 20log_{10}(75.42)$

$dBmV = 20 * log_{10}(75.42)$

$dBmV = 20 * (1.88)$

$dBmV = 37.55$

Answer: 75.85 microwatts is +37.55 dBmV.

11.8.7. dBmV in different impedances

The following formula can be used to convert a dBmV value in one impedance into a dBmV value in another impedance, assuming uniform power.[13] The conversion does not take into account the insertion loss of a minimum loss pad or impedance matching transformer.

$$dBmV(z_2) = dBmV(z_1) + 10log_{10}\left(\frac{z_2}{z_1}\right)$$

where
$dBmV(z_1)$ is the dBmV value in the first impedance
$dBmV(z_2)$ is the dBmV value in the second impedance
z_1 is the first impedance in ohms
z_2 is the second impedance in ohms

Example 1:
If a 75 ohms impedance circuit with a signal level of 3 dBmV is connected to a 50 ohms impedance circuit, what is the signal level in dBmV in the 50 ohms impedance circuit?

Solution 1:
$dBmV(z_2) = dBmV(z_1) + 10log_{10}\left(\frac{z_2}{z_1}\right)$

[13] Units of decibel millivolt are commonly used in 75 ohms impedance applications, but dBmV can be used in impedances other than 75 ohms (the same is true of units of decibel microvolt, or dBµV). The formula and examples in this section assume uniform power. It is recommended that the impedance be specified when dBmV is used in other impedances.

$$dBmV(z_2) = 3 + 10log_{10}\left(\frac{50}{75}\right)$$

$$dBmV(z_2) = 3 + \left[10 * log_{10}\left(\frac{50}{75}\right)\right]$$

$$dBmV(z_2) = 3 + [10 * log_{10}(0.6667)]$$

$$dBmV(z_2) = 3 + [10 * (-0.1761)]$$

$$dBmV(z_2) = 3 + [-1.7609]$$

$$dBmV(z_2) = 1.2391$$

Answer: The signal level in the 50 ohms impedance circuit is 1.24 dBmV.

Example 2:
If a 75 ohms impedance circuit with a signal level of 3 dBmV is connected to a 300 ohms impedance circuit, what is the signal level in dBmV in the 300 ohms impedance circuit?

Solution 2:

$$dBmV(z_2) = dBmV(z_1) + 10log_{10}\left(\frac{z_2}{z_1}\right)$$

$$dBmV(z_2) = 3 + 10log_{10}\left(\frac{300}{75}\right)$$

$$dBmV(z_2) = 3 + \left[10 * log_{10}\left(\frac{300}{75}\right)\right]$$

$$dBmV(z_2) = 3 + [10 * log_{10}(4.00)]$$

$$dBmV(z_2) = 3 + [10 * (0.6021)]$$

$$dBmV(z_2) = 3 + [6.0206]$$

$$dBmV(z_2) = 9.0206$$

Answer: The signal level in the 300 ohms impedance circuit is 9.02 dBmV.

The following table summarizes some common conversions from dBmV in one impedance (z_1) to dBmV in another impedance (z_2). For example, to convert 5 dBmV in 300 ohms to dBmV in 75 ohms, subtract 6.02 dB from 5 dBmV, giving –1.02 dBmV in 75 ohms.

Table 5 - Add or subtract the value in dB to convert from dBmV in one impedance to dBmV in another impedance. See text.

		… to dBmV in impedance z_2, add or subtract the value (in dB) shown			
		50 Ω	**75 Ω**	**300 Ω**	**600 Ω**
To convert from dBmV in impedance z_1 …	**50 Ω**	0	+1.76 dB	+7.78 dB	+10.79 dB
	75 Ω	–1.76 dB	0	+6.02 dB	+9.03 dB
	300 Ω	–7.78 dB	–6.02 dB	0	+3.01 dB
	600 Ω	–10.79 dB	–9.03 dB	–3.01 dB	0

11.9. Decibel microvolt (dBµV)

Decibel microvolt (dBµV) is a unit of power expressed in terms of voltage. RF signal levels in cable networks are commonly stated in dBmV, although some areas of the world use dBµV instead of dBmV.

While dBµV is commonly used as an expression of absolute values, technically speaking dBµV is a logarithmic ratio of some value to a specified 0 dB reference, in this case 0 dBµV.[14] For instance, a signal level of +70 dBµV (3,162 microvolts) is another way of saying that 3,162 microvolts is 70 dB greater than 1 microvolt (0 dBµV).

11.9.1. Converting microvolts to dBµV

The following formulas can be used to convert a value in microvolts to decibel microvolt:

$$dB\mu V = 20log_{10}\left(\frac{value\ in\ \mu V}{1\ \mu V}\right)$$

where
$dB\mu V$ is the value in decibel microvolt
log_{10} is base 10 logarithm
$value\ in\ \mu V$ is voltage in microvolts
$1\ \mu V$ is the 0 dB reference for dBµV

Note: The "$1\ \mu V$" in the denominator is sometimes not included, making the formula

$$dB\mu V = 20log_{10}(value\ in\ \mu V)$$

Example:
What is the value in dBµV for a voltage of 100 µV?

Solution:
$dB\mu V = 20log_{10}(value\ in\ \mu V)$

$dB\mu V = 20 * log_{10}(100)$

$dB\mu V = 20 * (2.00)$

[14] In the context used here, the 0 dB reference for decibel microvolt, 0 dBµV, equals 13.33 femtowatts (fW) of power, defined as 1 microvolt (RMS) across an impedance of 75 ohms.

$dB\mu V = 40$

Answer: 100 microvolts us 40 dBµV.

11.9.2. Converting dBµV to microvolts

A value in decibel microvolt can be converted to microvolts using the following formula:

$$\mu V = 10^{(dB\mu V/20)}$$

where
μV is the value in microvolts
$dB\mu V$ is the value in decibel microvolt

Example:
What is the value in microvolts for a signal level of +80 dBµV?

Solution:
$\mu V = 10^{(dB\mu V/20)}$

$\mu V = 10^{(80/20)}$

$\mu V = 10^{4}$

$\mu V = 10{,}000$

Answer: A signal level of +80 dBµV is 10,000 microvolts.

11.10. Decibel milliwatt (dBm)

Decibel milliwatt (dBm) is a unit of power, in particular a logarithmic-based expression of the ratio of a value in milliwatts to 1 milliwatt (mW), and is usually referenced to a specified impedance – for example, 50 ohms or 75 ohms in RF applications, and 600 ohms in baseband audio and telephony applications. Optical power in fiber optic links is commonly expressed in dBm. The 0 dB reference for decibel milliwatt is 0 dBm, which equals 1 mW.

11.10.1. Converting milliwatts to dBm

A value in milliwatts can be converted to decibel milliwatt using the following formulas:

$$dBm = 10log_{10}\left(\frac{value\ in\ mW}{1\ mW}\right)$$

where
dBm is the value in decibel millivolt
log_{10} is base 10 logarithm
$value\ in\ mW$ is power in milliwatts
$1\ mW$ is the 0 dB reference for dBm

Note: The "1 mW" in the denominator is sometimes not included, making the formula

$$dBm = 10log_{10}(value\ in\ mW)$$

Example:
What is the value in dBm for a power of 20 mW?

Solution:
$dBm = 10log_{10}(value\ in\ mW)$

$dBm = 10 * log_{10}(20)$

$dBm = 10 * (1.301)$

$dBm = 13.01$

Answer: A power of 20 mW is equal to 13.01 dBm.

11.10.2. Converting dBm to milliwatts

A value in decibel milliwatt can be converted to milliwatts using the following formula:

$$mW = 10^{(dBm/10)}$$

where
mW is the power in milliwatts
dBm is the value in decibel milliwatt

Example:
What is the power in milliwatts for a value of –10 dBm?

Solution:
$mW = 10^{(dBm/10)}$

$mW = 10^{(-10/10)}$

$mW = 10^{-1.00}$

$mW = 0.10$

Answer: A value of –10 dBm is 0.10 mW.

11.11. Decibel watt (dBW)

Decibel watt (dBW) is a unit of power, in particular a logarithmic-based expression of the ratio of a value in watts to 1 watt (W). The 0 dB reference for decibel watt is 0 dBW, which equals 1 watt.

11.11.1. Converting watts to dBW

A value in watts can be converted to decibel watt using the following formulas:

$$dBW = 10log_{10}\left(\frac{value\ in\ W}{1\ W}\right)$$

where
dBW is the value in decibel watt
log_{10} is base 10 logarithm
$value\ in\ W$ is power in watts
$1\ W$ is the 0 dB reference for dBW

Note: The "1 W" in the denominator is sometimes not included, making the formula

$$dBW = 10log_{10}(value\ in\ W)$$

Example:
What is the value in dBW for a power of 4 watts?

Solution:
$dBW = 10log_{10}(value\ in\ W)$

$dBW = 10 * log_{10}(4)$

$dBW = 10 * (0.602)$

$dBW = 6.02$

Answer: A power of 4 watts equals 6.02 dBW.

11.11.2. Converting dBW to watts

A value in decibel watt can be converted to watts using the following formula:

$$W = 10^{(dBW/10)}$$

where
W is the power in watts
dBW is the value in decibel watt

Example:
What is the power in watts for a value of 20 dBW?

Solution:
$W = 10^{(dBW/10)}$

$W = 10^{(20/10)}$

$W = 10^{2.00}$

$W = 100$

Answer: A value of 20 dBW is 100 watts.

11.12. Decibel volt (dBV)

Decibel volt (dBV) is a unit of power in terms of voltage, in particular a logarithmic-based expression of the ratio of a value in volts to 1 volt. The 0 dB reference for decibel volt is 0 dBV, which equals 1 volt.

11.12.1. Converting volts to dBV

A value in volts can be converted to decibel volt using the following formulas:

$$dBV = 20log_{10}\left(\frac{E}{1\,V}\right)$$

where
dBV is the value in decibel volt
log_{10} is base 10 logarithm
E is voltage in volts
$1\,V$ is the 0 dB reference for dBV

Note: The "1 V" in the denominator is sometimes not included, making the formula

$$dBV = 20log_{10}(E)$$

Example:
What is the value in dBV for a voltage of 10 V?

Solution:
$dBV = 20log_{10}(E)$

$dBV = 20 * log_{10}(10)$

$dBV = 20 * (1.00)$

$dBV = 20$

Answer: A voltage of 10 volts is 20 dBV.

11.12.2. Converting dBV to volts

A value in decibel volt can be converted to volts using the following formula:

$$E = 10^{(dBV/20)}$$

where
E is the value in volts
dBV is the value in decibel volt

Example:
What is the value in volts for –10 dBV?

Solution:
$E = 10^{(dBV/20)}$

$E = 10^{(-10/20)}$

$E = 10^{-0.50}$

$E = 0.316$

Answer: –10 dBV is 0.316 volt.

11.13. Decibel conversions

The following table includes formulas that can be used to convert between decibel microvolt, decibel millivolt, decibel volt, decibel milliwatt, and decibel watt. For these formulas, the impedance is assumed to be 75 ohms.

Table 6 - Decibel conversion formulas (75 ohms impedance)

dBμV – 138.75	=	dBW
dBμV – 60	=	dBmV
dBμV – 108.75	=	dBm
dBμV – 120	=	dBV
dBmV – 78.75	=	dBW
dBmV + 60	=	dBμV
dBmV – 48.75	=	dBm
dBmV – 60	=	dBV
dBV – 18.75	=	dBW
dBV + 120	=	dBμV
dBV + 60	=	dBmV
dBV + 11.25	=	dBm
dBm – 30	=	dBW
dBm + 108.75	=	dBμV
dBm + 48.75	=	dBmV
dBm – 11.25	=	dBV
dBW + 138.75	=	dBμV
dBW + 78.75	=	dBmV
dBW + 30	=	dBm
dBW + 18.75	=	dBV

The following four sections include examples of some of the conversions from the table.

11.13.1. Convert decibel millivolt (dBmV) to decibel microvolt (dBμV)

A value in decibel millivolt can be converted to decibel microvolt using the following formula:

$$dB\mu V = dBmV + 60$$

where
$dB\mu V$ is RF signal level in decibel microvolt
$dBmV$ is RF signal level in decibel millivolt

Example:
What is the dBμV equivalent of –48 dBmV?

Solution:
$dB\mu V = dBmV + 60$

$dB\mu V = -48 + 60$

$dB\mu V = 12$

Answer: –48 dBmV is 12 dBµV.

11.13.2. Convert decibel microvolt (dBµV) to decibel millivolt (dBmV)

A value in decibel microvolt can be converted to decibel millivolt using the following formula:

$$dBmV = dB\mu V - 60$$

where
$dBmV$ is RF signal level in decibel millivolt
$dB\mu V$ is RF signal level in decibel microvolt

Example:
What is the dBmV equivalent of 12 dBµV?

Solution:
$dBmV = dB\mu V - 60$

$dBmV = 12 - 60$

$dBmV = -48$

Answer: 12 dBµV is –48 dBmV.

11.13.3. Convert decibel millivolt (dBmV) to decibel milliwatt (dBm)

A value in decibel millivolt can be converted to decibel milliwatt using the following formula:

$$dBm = dBmV - 48.75$$

where
dBm is RF signal level in decibel milliwatt
$dBmV$ is RF signal level in decibel millivolt

Example:
What is the 75 ohms power equivalent (in dBm) of –48 dBmV?

Solution:
$dBm = dBmV - 48.75$

$dBm = -48 - 48.75$

$dBm = -96.75$

Answer: –48 dBmV equals –96.75 dBm.

11.13.4. Convert decibel milliwatt (dBm) to decibel millivolt (dBmV)

A value in decibel milliwatt can be converted to decibel millivolt using the following formula:

$$dBmV = dBm + 48.75$$

where
$dBmV$ is RF signal level in decibel millivolt
dBm is RF signal level in decibel milliwatt

Example:
What is the dBmV equivalent of –96.75 dBm (assume 75 ohms impedance)?

Solution:
$dBmV = dBm + 48.75$

$dBmV = -96.75 + 48.75$

$dBmV = -48$

Answer: –96.75 dBm equals –48 dBmV.

12. Noise Factor, Noise Figure, and Noise Temperature

Among the important metrics for terrestrial and satellite communications; amplifier, radio, and other receiver performance; and certain cable network performance calculations are noise factor, noise figure, and noise temperature.

The noise factor (F) of a system or device is a linear value defined as F = SNR_i/SNR_o where SNR_i is system or device input signal-to-noise ratio when the input noise is due only to thermal noise from a passive, resistive input termination of value equal to the system impedance (75 ohms for cable) and at a standard noise temperature T_0 (usually 290 K), and SNR_o is system or device output signal-to-noise ratio.

Noise figure (NF), defined in [21] as the "…degradation in signal-to-noise ratio as the signal passes through the [device under test]," is noise factor expressed in decibels. Amplifier specifications usually include noise figure among the listed parameters, in part because of its application to carrier-to-noise ratio calculations.

Noise temperature is more commonly used in satellite communications – for instance, to characterize a satellite antenna's low noise amplifier (LNA) or low noise block converter (LNB). The noise temperature of an electrical device, circuit, or component is defined to be the temperature of a single passive resistance that contributes the same noise power spectral density as the device itself. The term applies to active devices as well as simple and complex passive circuits and components. Noise temperature is stated in kelvin, and while related to physical temperature, should not be confused with the physical temperature of the device that one would measure with a thermometer.

The previous definitions assume the input termination is at a standard noise temperature of T_0 = 290 K.

12.1. Convert noise factor to noise figure

The following formula can be used to convert noise factor to noise figure in decibels:

$$NF = 10log_{10}(F)$$

where
NF is noise figure in decibels
log_{10} is base 10 logarithm
F is noise factor

Example:
What is the noise figure in decibels for a noise factor of 6.31?

Solution:
$NF = 10log_{10}(F)$

$NF = 10 * log_{10}(6.31)$

$NF = 10 * (0.80)$

$NF = 8.0$

Answer: The noise figure is 8 dB.

12.2. Convert noise figure to noise factor

The following formula can be used to convert noise figure to noise factor:

$$F = 10^{NF/10}$$

where
F is noise factor
NF is noise figure in decibels

Example:
What is the noise factor for a noise figure of 12 dB?

Solution:
$F = 10^{NF/10}$

$F = 10^{12/10}$

$F = 10^{1.20}$

$F = 15.85$

Answer: The noise factor is 15.85.

12.3. Convert noise temperature to noise figure

The following formula can be used to convert noise temperature in kelvin to noise figure in decibels:

$$NF = 10log_{10}\left[\left(\frac{T_e}{290}\right) + 1\right]$$

where
NF is noise figure in decibels
log_{10} is base 10 logarithm
T_e is noise temperature in kelvin (K)

Example:
What is the noise figure of a satellite antenna LNA that has a noise temperature of 85 K?

Solution:

$NF = 10log_{10}\left[\left(\frac{T_e}{290}\right) + 1\right]$

$NF = 10 * log_{10}\left[\left(\frac{85}{290}\right) + 1\right]$

$NF = 10 * log_{10}[(0.29) + 1]$

$NF = 10 * log_{10}[1.29]$

$NF = 10 * [0.112]$

$NF = 1.12$

Answer: The noise figure is 1.12 dB.

12.4. Convert noise figure to noise temperature

The following formula can be used to convert noise figure in decibels to noise temperature in kelvin:

$$T_e = 290 * \left[10^{NF/10} - 1\right]$$

where
T_e is noise temperature in kelvin (K)
NF is noise figure in decibels

Example:
What is the noise temperature in kelvin of a satellite antenna LNA that has a noise figure of 2 dB?

Solution:

$T_e = 290 * \left[10^{NF/10} - 1\right]$

$T_e = 290 * \left[10^{2/10} - 1\right]$

$T_e = 290 * [10^{0.20} - 1]$

$T_e = 290 * [1.58 - 1]$

$T_e = 290 * [0.58]$

$T_e = 169.62$

Answer: The noise temperature is 169.62 K.

13. Thermal Noise in a 75 Ω Network

When calculating carrier-to-noise ratio (CNR) in a cable network, one needs to first know the power of the thermal noise. Thermal noise power in a CNR calculation is typically specified in a given bandwidth, such as 4 MHz for analog NTSC visual carrier CNR measurements, or the symbol rate bandwidth for single carrier quadrature amplitude modulation (SC-QAM) CNR measurements. Two calculation methods are included here.

13.1. Noise calculation method #1

The first method involves starting with calculation of thermal noise power in watts from a thermal noise source, then converting the value in watts to decibel millivolt.

The power delivered by a thermal source into an impedance matched load can be calculated using the following formula:

$$P = kTB$$

where
P is the thermal noise power in watts
k is Boltzmann's Constant ($1.38 * 10^{-23}$ joules/kelvin)
T is the temperature in kelvin (K)
B is bandwidth in hertz

Example 1:
What is the noise power in a 4 MHz bandwidth delivered by a thermal source at room temperature (68 °F or 293.15 K)?

Solution 1:
$P = kTB$

$P = (1.38 * 10^{-23}) * 293.15 * 4{,}000{,}000$

$P = 1.62 * 10^{-14}$

Answer: The power is $1.62 * 10^{-14}$ watt.

Example 2:
What is the thermal noise power from Example 1 expressed in dBmV, assuming 75 ohms impedance?

Solution 2:
Use the formulas in Section 11.8.6 (Converting watts to dBmV)

First convert the power in watts to volts.

$E = \sqrt{P * R}$

$E = \sqrt{(1.62 * 10^{-14}) * 75}$

$$E = \sqrt{1.21 * 10^{-12}}$$

$$E = 1.10 * 10^{-6}$$

Answer: The voltage is $1.10 * 10^{-6}$ volt.

Next, convert the value in volts to millivolts:

$$mV = E * 1{,}000$$

$$mV = (1.10 * 10^{-6}) * 1{,}000$$

$$mV = 1.10 * 10^{-3}$$

Answer: The value in millivolts is $1.10 * 10^{-3}$ mV, or 0.0011 mV.

Finally, convert the value in millivolts to dBmV:

$$dBmV = 20log_{10}(mV)$$

$$dBmV = 20 * log_{10}(1.10 * 10^{-3})$$

$$dBmV = 20 * (-2.96)$$

$$dBmV = -59.16$$

Answer: The noise power in 4 MHz delivered by a thermal source into an impedance matched load (75 ohms) at room temperature is –59.16 dBmV.

13.2. Noise calculation method #2

The second method involves starting with calculation of open-circuit noise voltage from a resistance or impedance (e.g., 75 ohms), followed by calculating the voltage when the source is connected to a matched resistance or impedance, then converting to decibel millivolt.

13.2.1. Open-circuit noise voltage

To calculate the open-circuit noise voltage from a resistance or impedance, use the formula:

$$e_n = \sqrt{4kTBR}$$

where
e_n is the open-circuit noise voltage
k is Boltzmann's Constant ($1.38 * 10^{-23}$ joules/kelvin)
T is the temperature in kelvin (K)
B is bandwidth in hertz
R is resistance (or impedance) in ohms

Example:
What is the open-circuit noise voltage over a 4 MHz bandwidth (the noise power bandwidth used for analog NTSC television channel CNR measurements) generated by a 75-ohms resistor at room temperature (68 °F, or 293.15 K)?

Solution:
$e_n = \sqrt{4kTBR}$

$e_n = \sqrt{4 * (1.38 * 10^{-23}) * 293.15 * 4{,}000{,}000 * 75}$

$e_n = \sqrt{4.8546 * 10^{-12}}$

$e_n = 2.2033075 * 10^{-6}$

Answer: The open-circuit noise voltage is $2.2033075 * 10^{-6}$ volt, or about 2.2 microvolts (see Figure 13).

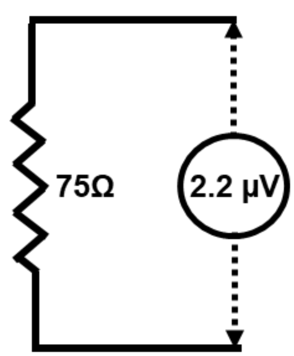


Figure 13 - Open-circuit noise voltage in a 4 MHz bandwidth for a 75 ohms resistance (or impedance) at room temperature.

13.2.2. Terminated noise voltage

When the 75 ohms impedance noise source is terminated by an equal value resistance or impedance – say, connected to the input of a 75 ohms impedance amplifier – the thermal noise is $e_n/2$ or 1.10165375 µV. See Figure 14.

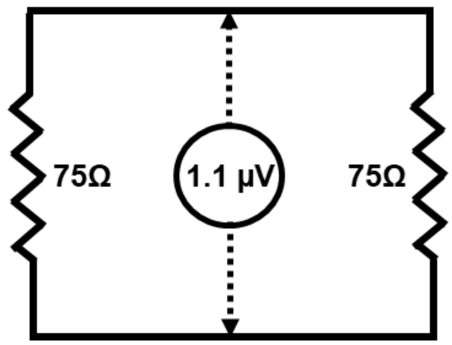


Figure 14 - When the open-circuit noise source is terminated by an equal-impedance load, the noise voltage is $e_n/2$, or about 1.1 µV in this example.

13.2.3. Convert noise voltage to decibel millivolt

To convert the terminated noise voltage to dBmV for use in carrier-to-noise ratio calculations, use the following formula:

$$dBmV = 20log_{10}\left(\frac{\mu V}{1{,}000}\right)$$

where
$dBmV$ is decibel millivolt
log_{10} is base 10 logarithm
μV is the value in microvolts

Example:
What is the dBmV value of 1.10165375 microvolt?

Solution:

$$dBmV = 20log_{10}\left(\frac{\mu V}{1{,}000}\right)$$

$$dBmV = 20 * log_{10}\left(\frac{1.10165375}{1{,}000}\right)$$

$$dBmV = 20 * log_{10}(0.00110165)$$

$$dBmV = 20 * (-2.95795488)$$

$$dBmV = -59.16$$

Answer: 1.10165375 microvolt equals –59.16 dBmV.

For those who prefer to use the standard noise temperature T_0 of 290 K rather than room temperature (293.15 K), the answer is –59.21 dBmV for a 4 MHz noise power bandwidth, although many cable network CNR calculations assume room temperature. (Note: Some use –59.2, which accommodates either reference source temperature after rounding.)

When calculating CNR for SC-QAM signals, the preferred noise power bandwidth for thermal noise is usually equal to the symbol rate bandwidth, although some prefer to use occupied bandwidth. The following table summarizes noise power bandwidths for downstream and upstream SC-QAM signals, based upon symbol rate bandwidths.

Table 7 - Noise power bandwidths (symbol rate bandwidths) for downstream and upstream SC-QAM signals.

Channel RF bandwidth[15]	Symbol rate[16]	Noise power bandwidth	Thermal noise level at 68 °F (75 ohms impedance)	
6 MHz	5.056941 Msym/s	5,056,941 Hz	1.24 µV	–58.14 dBmV
6 MHz	5.360537 Msym/s	5,360,537 Hz	1.28 µV	–57.89 dBmV
8 MHz	6.952 Msym/s	6,952,000 Hz	1.45 µV	–56.76 dBmV
200 kHz	160 ksym/s	160,000 Hz	0.22 µV	–73.14 dBmV
400 kHz	320 ksym/s	320,000 Hz	0.31 µV	–70.13 dBmV
800 kHz	640 ksym/s	640,000 Hz	0.44 µV	–67.12 dBmV
1.6 MHz	1,280 ksym/s	1,280,000 Hz	0.62 µV	–64.11 dBmV
3.2 MHz	2,560 ksym/s	2,560,000 Hz	0.88 µV	–61.10 dBmV
6.4 MHz	5,120 ksym/s	5,120,000 Hz	1.25 µV	–58.09 dBmV

Table 8 summarizes noise power bandwidths for downstream and upstream SC-QAM signals, based upon occupied bandwidths.

Table 8 - Noise power bandwidths (occupied bandwidths) for downstream and upstream SC-QAM signals.

Channel RF bandwidth	Symbol rate	Noise power bandwidth	Thermal noise level at 68 °F (75 ohms impedance)	
6 MHz	5.056941 and 5.360537 Msym/s	6 MHz	1.35 µV	–57.40 dBmV
8 MHz	6.952 Msym/s	8 MHz	1.56 µV	–56.15 dBmV
200 kHz	160 ksym/s	200,000 Hz	0.25 µV	–72.17 dBmV
400 kHz	320 ksym/s	400,000 Hz	0.35 µV	–69.16 dBmV
800 kHz	640 ksym/s	800,000 Hz	0.49 µV	–66.15 dBmV
1.6 MHz	1,280 ksym/s	1,600,000 Hz	0.70 µV	–63.14 dBmV
3.2 MHz	2,560 ksym/s	3,200,000 Hz	0.99 µV	–60.13 dBmV
6.4 MHz	5,160 ksym/s	6,400,000 Hz	1.39 µV	–57.12 dBmV

[15] Occupied bandwidth
[16] DOCSIS 2.0 and later use modulation rate in kHz rather than symbol rate for upstream SC-QAM carriers.

14. Carrier-to-Noise Ratio

Carrier-to-noise ratio, abbreviated C/N ratio or CNR, is in cable industry vernacular a pre-detection measurement – that is, a measurement performed in the frequency domain. CNR is the difference, in decibels, between the amplitude of an RF signal and the amplitude of noise present in the RF signal's transmission path. The RF signal may be unmodulated (also called continuous wave, or CW) or modulated. The noise may be one or a combination of several types: thermal noise; shot noise and relative intensity noise (RIN) in optical fiber links; and in cable systems carrying a mix of analog TV signals and digitally modulated signals, non-thermal noise such as composite and intermodulation noise. This section focuses on thermal noise generated by passive and active devices through which the RF signal is transmitted. The amplitude of thermal noise is usually specified over a certain bandwidth, called noise power bandwidth, discussed in the previous section.

14.1. CNR of an individual amplifier

The carrier-to-noise ratio of an individual amplifier can be calculated using the following formula:

$$CNR_i = N_t - NF + I$$

where
CNR_i is the carrier-to-noise ratio of an individual amplifier
N_t is the thermal noise level of a 75 ohms impedance in dBmV (expressed as a positive number so the formula's answer will come out positive, e.g., 59.16 for NTSC TV signals with a 4 MHz noise power bandwidth at 68 °F).
NF is the amplifier's noise figure in dB[17]
I is the amplifier's per-channel RF input level in dBmV

Example 1:
What is the standalone carrier-to-noise ratio of an amplifier with the following operating conditions? Assume the RF signals are analog NTSC TV signals (4 MHz noise power bandwidth).

Thermal noise level N_t = –59.16 dBmV
Noise figure NF = 8 dB
Per-channel RF input I = +15 dBmV

Solution 1:
Remember to change the thermal noise level to a positive number.

$$CNR_i = 59.16 - 8 + 15$$

$$CNR_i = 66.16$$

Answer: The CNR is 66.16 dB.

[17] The formula assumes that the amplifier's plug-in attenuator and equalizer are 0 dB values. If other than 0 dB, the additional insertion loss must be added to the noise figure number. For example, if the amplifier's actual noise figure is 10 dB, the attenuator is 0 dB, and the equalizer has 1 dB of insertion loss, change the noise figure value to 11 dB.

Example 2:
What is the carrier-to-noise ratio of the amplifier from Example 1 when the signals are 6 MHz-wide 256-QAM signals operating 6 dB below the peak envelope power of analog TV signal visual carriers? The noise power bandwidth for 6 MHz-wide 256-QAM signals is 5.36 MHz, which means the thermal noise level is –59.16 + [$10\log_{10}$(5.36 MHz/4 MHz)] = –57.89 dBmV (refer to Table 7 and Table 8 for a summary of SC-QAM noise power bandwidths). Since the operating levels are 6 dB lower, the following operating parameters apply.

Thermal noise level N_t = –57.89 dBmV
Noise figure NF = 8 dB
Per-channel RF input I = +9 dBmV

Solution 2:
$CNR_i = 57.89 - 8 + 9$

$CNR_i = 58.89$

Answer: The CNR is 58.89 dB.

14.2. CNR of a cascade of identical amplifiers

The carrier-to-noise ratio of a cascade of identical amplifiers can be calculated using the following formula:

$$CNR_{cascade} = CNR_i - 10log_{10}(N)$$

where
$CNR_{cascade}$ is the carrier-to-noise ratio at the end of a cascade of identical amplifiers
CNR_i is the carrier-to-noise ratio of an individual amplifier
log_{10} is base 10 logarithm
N is the number of identical amplifiers in cascade

Example:
Referring to Figure 15, what is the carrier-to-noise ratio at the end of a cascade of eight identical amplifiers, each of which has an individual carrier-to-noise ratio of 58.89 dB?

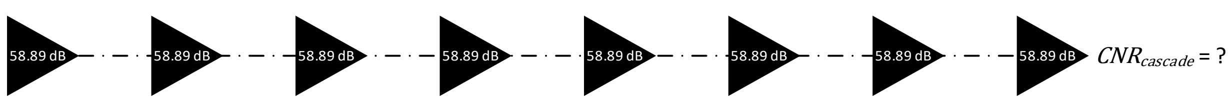


Figure 15 - What is the CNR at the end of this cascade of identical amplifiers?

Solution:
$CNR_{cascade} = CNR_i - 10log_{10}(N)$

$CNR_{cascade} = 58.89\ dB - 10log_{10}(8)$

$CNR_{cascade} = 58.89 - 10 * (0.903)$

$CNR_{cascade} = 58.89 - 9.03$

$CNR_{cascade} = 49.86$

Answer: The carrier-to-noise ratio at the end of the cascade is 49.86 dB.

14.3. Combining different CNRs

The following power addition formula can be used to combine individual carrier-to-noise ratios, for example, those in a cascade of non-identical amplifiers, or carrier-to-noise ratios in different parts of the network (e.g., headend, optical fiber link, and amplifier cascade).

$$CNR_{total} = -10\,log_{10}\left[10^{\frac{-CNR_1}{10}} + 10^{\frac{-CNR_2}{10}} + 10^{\frac{-CNR_3}{10}} \ldots + 10^{\frac{-CNR_n}{10}}\right]$$

where
CNR_{total} is the total carrier-to-noise ratio
log_{10} is base 10 logarithm
CNR_1 is the first carrier-to-noise ratio (individual amplifier, segment of plant such as optical fiber link, etc.)
CNR_2 is the second carrier-to-noise ratio (individual amplifier, segment of plant such as optical fiber link, etc.)
CNR_3 is the third carrier-to-noise ratio (individual amplifier, segment of plant such as optical fiber link, etc.)
CNR_n is the nth carrier-to-noise ratio (individual amplifier, segment of plant such as optical fiber link, etc.)

Example 1:
Referring to Figure 16, what is the combined upstream carrier-to-noise ratio at the input to the cable modem termination system, assuming the CNR from receiver #1 is 32 dB, receiver #2 is 32 dB, receiver #3 is 34 dB, and receiver #4 is 35 dB?

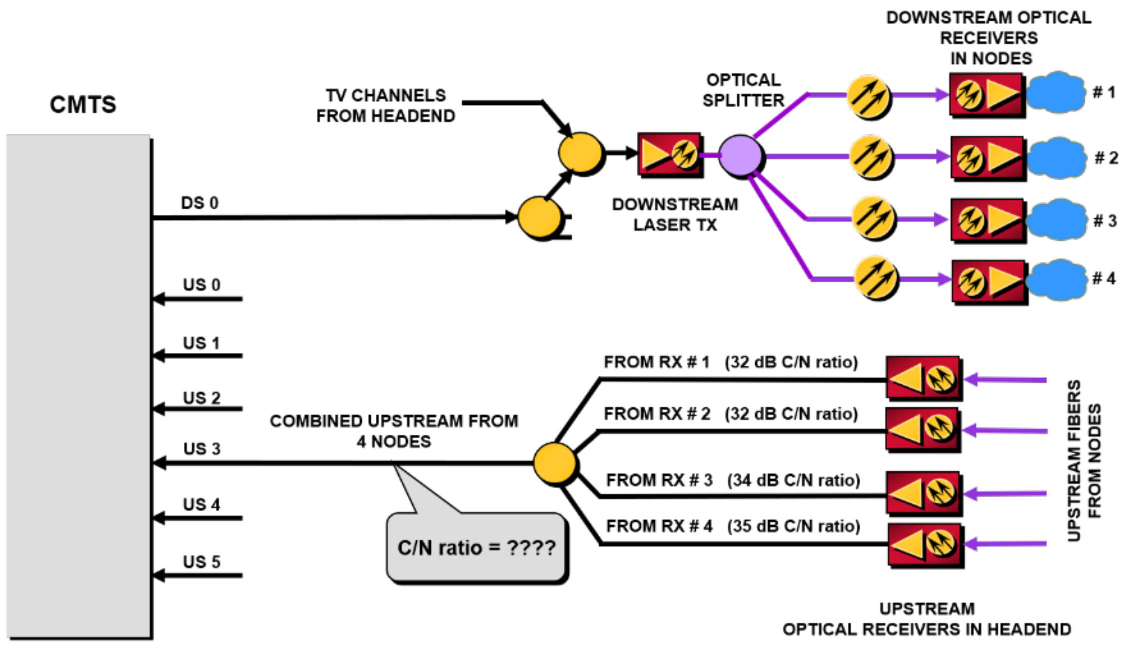


Figure 16 - What is the combined upstream CNR at the CMTS input? (Graphic courtesy of Cisco)

Solution 1:

$$CNR_{total} = -10\,log_{10}\left[10^{\frac{-CNR_1}{10}} + 10^{\frac{-CNR_2}{10}} + 10^{\frac{-CNR_3}{10}} \ldots + 10^{\frac{-CNR_n}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}\left[10^{\frac{-32}{10}} + 10^{\frac{-32}{10}} + 10^{\frac{-34}{10}} + 10^{\frac{-35}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-3.2} + 10^{-3.2} + 10^{-3.4} + 10^{-3.5}]$$

$$CNR_{total} = -10 * log_{10}[0.000631 + 0.000631 + 0.000398 + 0.000316]$$

$$CNR_{total} = -10 * log_{10}[0.001976]$$

$$CNR_{total} = -10 * log_{10}[-2.7042]$$

$$CNR_{total} = 27.04$$

Answer: The combined upstream CNR is 27.04 dB.

Example 2:
What is the end-of-line carrier-to-noise ratio for a cascade of three non-identical amplifiers, the first with a standalone CNR of 47 dB, the second with a standalone CNR of 45 dB, and the third with a standalone CNR of 51 dB?

Solution 2:

$$CNR_{total} = -10\,log_{10}\left[10^{\frac{-CNR_1}{10}} + 10^{\frac{-CNR_2}{10}} + 10^{\frac{-CNR_3}{10}} \ldots + 10^{\frac{-CNR_n}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}\left[10^{\frac{-47}{10}} + 10^{\frac{-45}{10}} + 10^{\frac{-51}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-4.7} + 10^{-4.5} + 10^{-5.1}]$$

$$CNR_{total} = -10 * log_{10}[0.00001995 + 0.00003162 + 0.00000794]$$

$$CNR_{total} = -10 * log_{10}[0.00005952]$$

$$CNR_{total} = -10 * -4.2253$$

$$CNR_{total} = 42.25$$

Answer: The end-of-line CNR is 42.25 dB.

Example 3:
If one knows the headend, optical fiber link, and coax plant carrier-to-noise ratios, these can be combined using the same formula to calculate the end-of-line CNR. Assume the headend, fiber link and coax plant have the following standalone CNRs, as shown in Figure 17. What is the end-of-line CNR?

Headend CNR: 55 dB
Fiber link CNR: 52 dB
Coax plant CNR: 49 dB

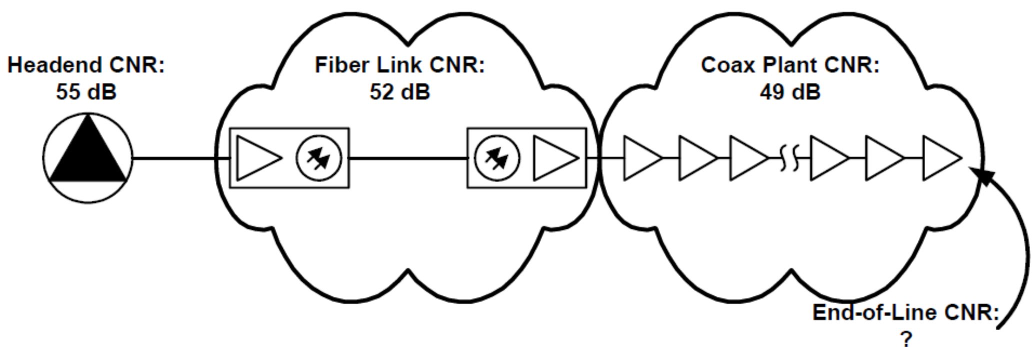


Figure 17 - What is the total (end-of-line) carrier-to-noise ratio?

Solution 3:

$$CNR_{total} = -10\,log_{10}\left[10^{\frac{-CNR_1}{10}} + 10^{\frac{-CNR_2}{10}} + 10^{\frac{-CNR_3}{10}} \ldots + 10^{\frac{-CNR_n}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}\left[10^{\frac{-55}{10}} + 10^{\frac{-52}{10}} + 10^{\frac{-49}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-5.50} + 10^{-5.20} + 10^{-4.90}]$$

$$CNR_{total} = -10 * log_{10}[0.00000316 + 0.00000631 + 0.00001259]$$

$CNR_{total} = -10 * log_{10}[0.00002206]$

$CNR_{total} = -10 * [-4.6564]$

$CNR_{total} = 46.56$

Answer: The end-of-line CNR is 46.56 dB.

Discussion:
If the headend CNR were increased from 55 dB to, say, 60 dB, the end-of-line CNR would improve slightly from 46.56 dB to 47.01 dB.[18] Indeed, excluding the headend CNR contribution from the calculation – that is, calculating the combined CNR for only the fiber link and coax plant – results in a change of less than 1 dB, increasing the end-of-line CNR from 46.56 dB to 47.24 dB:

$$CNR_{total} = -10 * log_{10}\left[10^{\frac{-52}{10}} + 10^{\frac{-49}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-5.20} + 10^{-4.90}]$$

$$CNR_{total} = -10 * log_{10}[0.00000631 + 0.00001259]$$

$$CNR_{total} = -10 * log_{10}[0.00001890]$$

$$CNR_{total} = -10 * [-4.7236]$$

$$CNR_{total} = 47.24$$

What if one wants to calculate the CNR of one of the contributing elements, say, the coax plant, when only the fiber link and end-of-line CNRs are known? This is possible, requiring a slight juggling of the power addition formula (subtraction is used inside the formula's brackets rather than addition). Note that the headend CNR has been excluded. In the following examples, CNR_{total} has been changed to CNR_{EOL}, CNR_2 to CNR_{fiber}, and CNR_3 to $CNR_{coaxplant}$.

$$CNR_{coaxplant} = -10\, log_{10}\left[10^{\frac{-CNR_{EOL}}{10}} - 10^{\frac{-CNR_{fiber}}{10}}\right]$$

$$CNR_{coaxplant} = -10 * log_{10}\left[10^{\frac{-47.24}{10}} - 10^{\frac{-52}{10}}\right]$$

$$CNR_{coaxplant} = -10 * log_{10}[10^{-4.724} - 10^{-5.20}]$$

$$CNR_{coaxplant} = -10 * log_{10}[0.00001888 - 0.00000631]$$

$$CNR_{coaxplant} = -10 * log_{10}[0.00001257]$$

$$CNR_{coaxplant} = -10 * [-4.90065]$$

---

[18]Changing the headend CNR to 60 dB gives $CNR_{total} = -10 * log_{10}\left[10^{\frac{-60}{10}} + 10^{\frac{-52}{10}} + 10^{\frac{-49}{10}}\right] = 47.01\ dB$

$CNR_{coaxplant} = 49.01$

From this, the coax plant's CNR contribution is 49 dB, which agrees with the value used in the earlier example. If one wanted to calculate the fiber link CNR when only the coax plant and end-of-line CNRs were known, the following variation of the formula would be used.

$$CNR_{fiber} = -10\,log_{10}\left[10^{\frac{-CNR_{EOL}}{10}} - 10^{\frac{-CNR_{coaxplant}}{10}}\right]$$

$$CNR_{fiber} = -10 * log_{10}\left[10^{\frac{-47.24}{10}} - 10^{\frac{-49}{10}}\right]$$

$$CNR_{fiber} = -10 * log_{10}[10^{-4.724} - 10^{-4.90}]$$

$$CNR_{fiber} = -10 * log_{10}[0.00001888 - 0.00001259]$$

$$CNR_{fiber} = -10 * log_{10}[0.00000629]$$

$$CNR_{fiber} = -10 *[-5.2013]$$

$$CNR_{fiber} = 52.01$$

Here, the calculated 52 dB matches the original fiber link CNR used previously.

14.4. Correction factor for low CNR measurements

When a signal's amplitude is close to the level of noise surrounding it, the measured signal amplitude can appear too high by up to several decibels (dB). This is because the spectrum analyzer actually measures the amplitude of the signal plus the noise. It does not matter whether the noise is the spectrum analyzer noise floor or the system noise, or a combination of the two. When the signal is much larger in amplitude than the surrounding noise, the contribution of the noise becomes negligible and the measured carrier-to-noise (CNR) approaches its true value.

As a general rule, if the signal amplitude is at least 10 dB above the noise, the measurement offset will be less than about 0.5 dB. If the signal is at least greater than 17 dB above the noise, the measurement offset will be less than about 0.1 dB and can be ignored for all practical purposes (see Figure 18).

The following formula can be used to determine the correction factor to apply to the measured CNR in order to determine the true CNR. Note that care should be taken when the measured CNR is equal to or less than 3 dB, because the correction factor value calculated by the formula will become zero or negative.

$$correction\_factor\_dB = 10log_{10}\left[1 + \frac{1}{10^{(measured\_cnr\_dB/10)} - 1}\right]$$

where
$correction\_factor\_dB$ is the number of decibels to subtract from the measured carrier-to-noise ratio to obtain the true carrier-to-noise ratio
log_{10} is base 10 logarithm
$measured\_cnr\_dB$ is the measured carrier-to-noise ratio in decibels

Example:
You measure a signal which appears to be 5.45 dB above the average noise. What is the correction factor that must applied to obtain the true CNR? What is the true CNR?

Solution:

$$correction\_factor\_dB = 10log_{10}\left[1 + \frac{1}{10^{(measured\_cnr\_dB/10)} - 1}\right]$$

$$correction\_factor\_dB = 10 * log_{10}\left[1 + \frac{1}{10^{(5.45/10)} - 1}\right]$$

$$correction\_factor\_dB = 10 * log_{10}\left[1 + \frac{1}{10^{(0.5450)} - 1}\right]$$

$$correction\_factor\_dB = 10 * log_{10}\left[1 + \frac{1}{3.5075 - 1}\right]$$

$$correction\_factor\_dB = 10 * log_{10}\left[1 + \frac{1}{2.5075}\right]$$

$$correction\_factor\_dB = 10 * log_{10}[1 + 0.3988]$$

$$correction\_factor\_dB = 10 * log_{10}[1.3988]$$

$$correction\_factor\_dB = 10 * 0.1458$$

$$correction\_factor\_dB = 1.4576$$

Answer: The correction factor that must be applied is 1.46 dB.

What is the true CNR?

The true CNR is the $measured\_cnr\_dB - correction\_factor\_dB$

5.45 – 1.46 = 3.99

Answer: The true CNR is 3.99 dB.

Alternatively, the true CNR can be calculated directly for a given measured CNR using the following formula:

$$true\_cnr\_dB = 10log_{10}\left(10^{(measured\_cnr\_dB/10)} - 1\right)$$

where
$true\_cnr\_dB$ is the calculated carrier-to-noise ratio in decibels
log_{10} is base 10 logarithm
$measured\_cnr\_dB$ is the measured carrier-to-noise ratio in decibels

Example:
You measure a signal which appears to be 5.45 dB above the average noise. What is the true CNR?

Solution:
$true\_cnr\_dB = 10log_{10}\left(10^{(measured\_cnr\_dB/10)} - 1\right)$

$true\_cnr\_dB = 10 * log_{10}\left(10^{(5.45/10)} - 1\right)$

$true\_cnr\_dB = 10 * log_{10}\left(10^{(0.5450)} - 1\right)$

$true\_cnr\_dB = 10 * log_{10}(3.5075 - 1)$

$true\_cnr\_dB = 10 * log_{10}(2.5075)$

$true\_cnr\_dB = 10 * 0.3992$

$true\_cnr\_dB = 3.9924$

Answer: The true CNR is 3.99 dB.

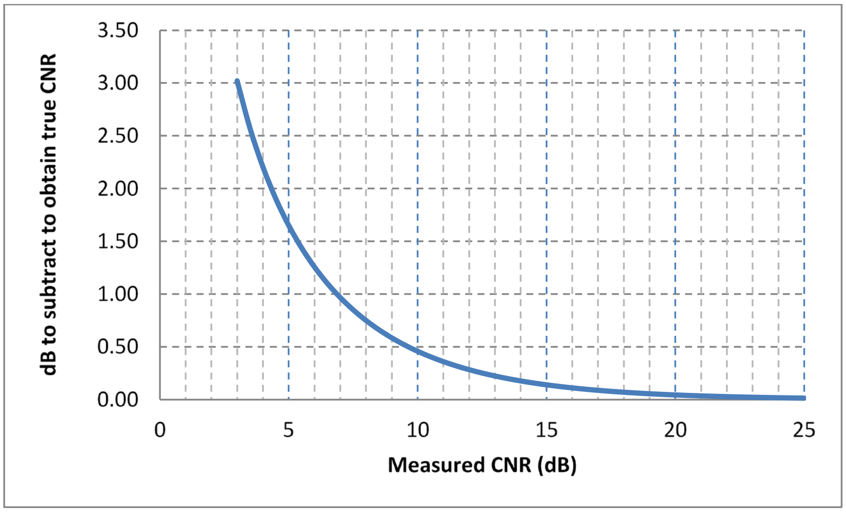


Figure 18 - Correction factor for low CNR measurements.

15. Carrier-to-Distortion Ratio

The measurement of carrier-to-distortion ratio is performed in the frequency domain. Carrier-to-distortion ratio is the difference, in decibels, between the amplitude of an RF signal and the amplitude of intermodulation distortion products (commonly called "beats") present in the RF signal's transmission path. Second and third order distortions[19] tend to be the most prevalent in cable networks (higher order distortions are usually too low in amplitude to be of concern).

In the early days of the cable industry when analog TV channel loading (i.e., the number of active channels) was relatively small, say, 12 NTSC channels, intermodulation distortion products were relatively easily measured discrete beats. Among the dominant distortions produced by older single-ended 12-channel amplifiers was second order distortions, which for the most part fell outside of the active passband. Cross modulation (XMOD), a third order distortion, was the major picture-impacting distortion.

As the number of analog TV channels in cable networks increased over time, the number of distortion products also increased. Dozens or hundreds of beats could be present in each channel, with a cluster of discrete third-order beats falling on the visual carrier frequency and clusters of discrete second order beats falling 1.25 MHz below and 1.25 MHz above the visual carrier.[20] Those beat clusters were given the name *composite distortions*. In cable networks carrying mostly- or all-digital channels, the distortions are noise-like (see Section 15.5).

Composite distortions in cable networks include composite second order (CSO) distortion, composite triple beat (CTB) distortion, and common path distortion (CPD). XMOD and hum modulation (see Section 16), while not considered composite distortions, are also evaluated during cable network engineering analysis. CSO, CTB, XMOD, and hum modulation are generated in active devices; CPD can be generated in passive devices (hum also can be generated in passive devices). This section focuses on distortions in active devices. For an in-depth analysis of distortions in cable networks, see Appendix D.

15.1. Effect of changing amplifier RF output level

Amplifier operating levels have a significant impact on intermodulation distortions. For example, if an amplifier's (absolute) input and output signal levels are increased by 1 dB, the amplitude of second order distortion products at the amplifier's output will increase by 2 dB, but the carrier-to-second order distortion ratio will decrease (i.e., get worse) by 1 dB. Likewise, if an amplifier's (absolute) input and output signal levels are increased by 1 dB, the amplitude of third order distortion products at the amplifier's output will increase by 3 dB, but the carrier-to-third order distortion ratio will decrease (get worse) by 2 dB.

Equipment manufacturers typically specify active device distortion performance with a certain number of active channels, input and output signal levels, and tilt. One can use the formulas in this section to calculate the distortion performance of an individual amplifier when the output level is changed, but the manufacturer's original specified channel loading and tilt are maintained. The formulas are from [5].

---

[19] For any two fundamental frequencies f_1 and f_2, the resulting second order distortions include $2f_1$, $2f_2$, $f_1 + f_2$, and $f_1 - f_2$. For any three fundamental frequencies f_1, f_2, and f_3, the resulting third order distortions include $3f_1$, $3f_2$, $3f_3$, $f_1 \pm f_2 \pm f_3$, $2f_1 + f_2$, and $2f_1 - f_2$.

[20] For example, with 77 analog NTSC TV channels present, the number of triple beats falling in just Ch. 40 is 2,019 (from the System Beats Table in [25]).

15.1.1. Composite second order

The following formula can be used to calculate the new carrier-to-CSO distortion ratio for a single amplifier when output signal level is changed:

$$CSO_{new} = CSO_{ref} - (L_{new} - L_{ref})$$

where
CSO_{new} is the calculated carrier-to-composite second order distortion ratio in decibels
CSO_{ref} is the equipment manufacturer's specified or reference carrier-to-composite second order distortion ratio in decibels (expressed as a positive number)
L_{new} is the new amplifier output RF signal level (e.g., in decibel millivolt)
L_{ref} is the equipment manufacturer's specified or reference amplifier output RF signal level (e.g., in decibel millivolt)

Example:
Assume that a manufacturer's specified carrier-to-CSO distortion ratio for an amplifier is 76 dB with a given channel loading, output signal level, and tilt. If the channel loading and tilt are kept the same but the output signal level is increased by 2 dB, what is the calculated carrier-to-CSO distortion ratio? Use 49 dBmV and 51 dBmV for the reference and new signal levels respectively.

Solution:
$CSO_{new} = CSO_{ref} - (L_{new} - L_{ref})$

$CSO_{new} = 76 - (51 - 49)$

$CSO_{new} = 76 - (2)$

$CSO_{new} = 74$

Answer: The calculated carrier-to-CSO distortion ratio is 74 dB.

15.1.2. Composite triple beat

The following formula can be used to calculate the new carrier-to-CTB distortion ratio for a single amplifier when output signal level is changed:

$$CTB_{new} = CTB_{ref} - 2 * (L_{new} - L_{ref})$$

where
CTB_{new} is the calculated carrier-to-composite triple beat distortion ratio in decibels
CTB_{ref} is the equipment manufacturer's specified or reference carrier-to-composite triple beat distortion ratio in decibels (expressed as a positive number)
L_{new} is the new amplifier output RF signal level (e.g., in decibel millivolt)
L_{ref} is the equipment manufacturer's specified or reference amplifier output RF signal level (e.g., in decibel millivolt)

Example:
Assume that a manufacturer's specified carrier-to-CTB distortion ratio for an amplifier is 81 dB with a given channel loading, output signal level, and tilt. If the channel loading and tilt are kept the same but the output signal level is increased by 2 dB, what is the calculated carrier-to-CTB distortion ratio? Use 49 dBmV and 51 dBmV for the reference and new signal levels respectively.

Solution:
$$CTB_{new} = CTB_{ref} - 2 * (L_{new} - L_{ref})$$

$$CTB_{new} = 81 - 2 * (51 - 49)$$

$$CTB_{new} = 81 - 2 * (2)$$

$$CTB_{new} = 81 - 4$$

$$CTB_{new} = 77$$

Answer: The calculated carrier-to-CTB distortion ratio is 77 dB.

15.1.3. Cross modulation

The following formula can be used to calculate the new carrier-to-XMOD ratio for a single amplifier when output signal level is changed:

$$XMOD_{new} = XMOD_{ref} - 2 * \left(L_{new} - L_{ref}\right)$$

where
$XMOD_{new}$ is the calculated carrier-to-cross modulation ratio in decibels
$XMOD_{ref}$ is the equipment manufacturer's specified or reference carrier-to-cross modulation ratio in decibels (expressed as a positive number)
L_{new} is the new amplifier output RF signal level (e.g., in decibel millivolt)
L_{ref} is the equipment manufacturer's specified or reference amplifier output RF signal level (e.g., in decibel millivolt)

Example:
Assume that a manufacturer's specified carrier-to-XMOD ratio for an amplifier is 76 dB with a given channel loading, output signal level, and tilt. If the channel loading and tilt are kept the same but the output signal level is increased by 2 dB, what is the calculated carrier-to-XMOD ratio? Use 49 dBmV and 51 dBmV for the reference and new signal levels respectively.

Solution:
$$XMOD_{new} = XMOD_{ref} - 2 * \left(L_{new} - L_{ref}\right)$$

$$XMOD_{new} = 76 - 2 * (51 - 49)$$

$$XMOD_{new} = 76 - 2 * (2)$$

$$XMOD_{new} = 76 - 4$$

$$XMOD_{new} = 72$$

Answer: The calculated carrier-to-XMOD ratio is 72 dB.

15.2. Effect of changing amplifier RF output tilt

The amount of tilt from low frequency to high frequency at the output of an active device has an impact on the active device's distortion performance. Tilt in the context used here refers to positive tilt, in which RF signal levels at the upper end of the spectrum are greater than RF signal levels at the lower end of the spectrum. The formulas in this section, from [5], are based upon empirical data. The formulas can be used to calculate distortion performance when tilt is changed, but the RF signal level at the upper end of the spectrum is left as specified in the manufacturer's recommendations. Note: The reader is encouraged to consult with the manufacturer of the active devices in use for additional guidance related to the impact of specific tilt values on overall distortion performance.

15.2.1. Composite second order

The following formula can be used to calculate the new carrier-to-CSO distortion ratio for a single amplifier when output tilt is changed:

$$CSO_{new} = CSO_{ref} + 0.33 * \left(T_{new} - T_{ref}\right)$$

where
CSO_{new} is the calculated carrier-to-composite second order distortion ratio in decibels
CSO_{ref} is the equipment manufacturer's specified or reference carrier-to-composite second order distortion ratio in decibels (expressed as a positive number)
T_{new} is the new amplifier output tilt in decibels
T_{ref} is the equipment manufacturer's specified or reference amplifier output tilt in decibels

Example:
Assume that a manufacturer's specified carrier-to-CSO distortion ratio for an amplifier is 76 dB with a given channel loading, output signal level, and tilt. If the channel loading and the signal level at the upper end of the spectrum are unchanged but the tilt is decreased by 2 dB (i.e., the signal level at the lower end of the spectrum is increased), what is the calculated carrier-to-CSO distortion ratio? Use 14.5 dB and 12 dB for the reference and new tilt values respectively.

Solution:
$CSO_{new} = CSO_{ref} + 0.33 * \left(T_{new} - T_{ref}\right)$

$CSO_{new} = 76 + 0.33 * (12 - 14.5)$

$CSO_{new} = 76 + [0.33 * (-2.5)]$

$CSO_{new} = 76 + [-0.83]$

$CSO_{new} = 75.18$

Answer: The calculated carrier-to-CSO distortion ratio is 75.18 dB.

15.2.2. Composite triple beat

The following formula can be used to calculate the new carrier-to-CTB distortion ratio for a single amplifier when output tilt is changed:

$$CTB_{new} = CTB_{ref} + 0.8 * \left(T_{new} - T_{ref}\right)$$

where
CTB_{new} is the calculated carrier-to-composite triple beat distortion ratio in decibels
CTB_{ref} is the equipment manufacturer's specified or reference carrier-to-composite triple beat distortion ratio in decibels (expressed as a positive number)
T_{new} is the new amplifier output tilt in decibels
T_{ref} is the equipment manufacturer's specified or reference amplifier output tilt in decibels

Example:
Assume that a manufacturer's specified carrier-to-CTB distortion ratio for an amplifier is 81 dB with a given channel loading, output signal level, and tilt. If the channel loading and the signal level at the upper end of the spectrum are unchanged but the tilt is decreased by 2 dB (i.e., the signal level at the lower end of the spectrum is increased), what is the calculated carrier-to-CTB distortion ratio? Use 14.5 dB and 12 dB for the reference and new tilt values respectively.

Solution:
$CTB_{new} = CTB_{ref} + 0.8 * \left(T_{new} - T_{ref}\right)$

$CTB_{new} = 81 + 0.8 * (12 - 14.5)$

$CTB_{new} = 81 + [0.8 * (-2.5)]$

$CTB_{new} = 81 + [-2.0]$

$CTB_{new} = 79$

Answer: The calculated carrier-to-CTB distortion ratio is 79 dB.

15.2.3. Cross modulation

The following formula can be used to calculate the new carrier-to-XMOD ratio for a single amplifier when output tilt is changed:

$$XMOD_{new} = XMOD_{ref} + 0.5 * (T_{new} - T_{ref})$$

where
$XMOD_{new}$ is the calculated carrier-to-cross modulation ratio in decibels
$XMOD_{ref}$ is the equipment manufacturer's specified or reference carrier-to-cross modulation ratio in decibels (expressed as a positive number)
T_{new} is the new amplifier output tilt in decibels
T_{ref} is the equipment manufacturer's specified or reference amplifier output tilt in decibels

Example:
Assume that a manufacturer's specified carrier-to-XMOD ratio for an amplifier is 76 dB with a given channel loading, output signal level, and tilt. If the channel loading and the signal level at the upper end of the spectrum are unchanged but the tilt is decreased by 2 dB (i.e., the signal level at the lower end of the spectrum is increased), what is the calculated carrier-to-XMOD ratio? Use 14.5 dB and 12 dB for the reference and new tilt values respectively.

Solution:
$XMOD_{new} = XMOD_{ref} + 0.5 * (T_{new} - T_{ref})$

$XMOD_{new} = 76 + 0.5 * (12 - 14.5)$

$XMOD_{new} = 76 + [0.5 * (-2.5)]$

$XMOD_{new} = 76 + [-1.25]$

$XMOD_{new} = 74.75$

Answer: The calculated carrier-to-XMOD ratio is 74.75 dB.

15.3. Calculate distortion in a cascade of identical amplifiers

End-of-line distortion performance can be calculated with the formulas in this section. All of the amplifiers in cascade are assumed to be identical, operating with the same channel load, output signal levels, and tilt.[21]

[21] During the cable network design engineering phase, individual amplifier operating parameters can be changed to achieve the desired end-of-line performance. For example, the maximum allowable number of amplifiers in cascade could be reduced. Individual amplifier RF signal levels and tilt also could be reduced ("derated") to improve individual amplifier distortion performance.

15.3.1. Composite second order

The following formula can be used to calculate end-of-line carrier-to-CSO distortion ratio:[22]

$$CSO_{EOL} = CSO_{AMP} - 10log_{10}(N)$$

where
CSO_{EOL} is the calculated carrier-to-composite second order distortion ratio in decibels at the end of a cascade of identical amplifiers
CSO_{AMP} is the carrier-to-composite second order distortion ratio in decibels for an individual amplifier (expressed as a positive number)
log_{10} is base 10 logarithm
N is the number of identical amplifiers in cascade

Example:
Assume that the carrier-to-CSO distortion ratio for an individual amplifier is 76 dB with a given channel loading, output signal level, and tilt. What is the carrier-to-CSO distortion ratio at the end a cascade of 10 of these amplifiers?

Solution:
$CSO_{EOL} = CSO_{AMP} - 10log_{10}(N)$

$CSO_{EOL} = 76 - 10log_{10}(10)$

$CSO_{EOL} = 76 - [10 * log_{10}(10)]$

$CSO_{EOL} = 76 - [10 * (1.00)]$

$CSO_{EOL} = 76 - [10]$

$CSO_{EOL} = 66$

Answer: The carrier-to-CSO distortion ratio at the end of the amplifier cascade is 66 dB.

[22] For very short cascades of identical amplifiers (N < 10 amplifiers), the assumption of $10log_{10}$ power addition for carrier-to-CSO distortion ratio no longer holds; see Appendix D. Some prefer to use $15log_{10}(N)$ in this calculation.

15.3.2. *Composite triple beat*

The following formula can be used to calculate end-of-line carrier-to-CTB distortion ratio:

$$CTB_{EOL} = CTB_{AMP} - 20log_{10}(N)$$

where
CTB_{EOL} is the calculated carrier-to-composite triple beat distortion ratio in decibels at the end of a cascade of identical amplifiers
CTB_{AMP} is the carrier-to-composite triple beat distortion ratio in decibels for an individual amplifier (expressed as a positive number)
log_{10} is base 10 logarithm
N is the number of identical amplifiers in cascade

Example:
Assume that the carrier-to-CTB distortion ratio for an individual amplifier is 81 dB with a given channel loading, output signal level, and tilt. What is the carrier-to-CTB distortion ratio at the end a cascade of 10 of these amplifiers?

Solution:
$CTB_{EOL} = CTB_{AMP} - 20log_{10}(N)$

$CTB_{EOL} = 81 - 20log_{10}(10)$

$CTB_{EOL} = 81 - [20 * log_{10}(10)]$

$CTB_{EOL} = 81 - [20 * (1)]$

$CTB_{EOL} = 81 - [20]$

$CTB_{EOL} = 61$

Answer: The carrier-to-CTB distortion ratio at the end of the amplifier cascade is 61 dB.

15.3.3. Cross modulation

The following formula can be used to calculate end-of-line carrier-to-XMOD ratio:

$$XMOD_{EOL} = XMOD_{AMP} - 20log_{10}(N)$$

where
$XMOD_{EOL}$ is the calculated carrier-to-cross modulation ratio in decibels at the end of a cascade of identical amplifiers
$XMOD_{AMP}$ is the carrier-to-cross modulation ratio in decibels for an individual amplifier (expressed as a positive number)
log_{10} is base 10 logarithm
N is the number of identical amplifiers in cascade

Example:
Assume that the carrier-to-XMOD ratio for an individual amplifier is 76 dB with a given channel loading, output signal level, and tilt. What is the carrier-to-XMOD ratio at the end a cascade of 10 of these amplifiers?

Solution:
$XMOD_{EOL} = XMOD_{AMP} - 20log_{10}(N)$

$XMOD_{EOL} = 76 - 20log_{10}(10)$

$XMOD_{EOL} = 76 - [20 * log_{10}(10)]$

$XMOD_{EOL} = 76 - [20 * (1)]$

$XMOD_{EOL} = 76 - [20]$

$XMOD_{EOL} = 56$

Answer: The carrier-to-XMOD ratio at the end of the amplifier cascade is 56 dB.

15.4. Calculate distortion in a cascade of dissimilar amplifiers (and/or dissimilar operating levels)

Some cable network architectures are designed with different types of active devices in cascade, each with different standalone distortion performance parameters. The formulas in this section can be used to calculate end-of-line carrier-to-distortion performance in a cascade of non-identical amplifiers, and/or in a cascade of identical amplifiers in which some actives operate with different output signal levels than others in the same cascade. The formulas also can be used to combine carrier-to-distortion ratios of different parts of a network, for example, the output of a headend after an active RF management system (i.e., active headend combiner), the node, and a cascade of amplifiers after the node.

15.4.1. Composite second order

The following formula can be used to calculate end-of-line carrier-to-CSO distortion ratio:[23]

$$CSO_{EOL} = -10log_{10}\left[10^{(-CSO_1/10)} + 10^{(-CSO_2/10)} + 10^{(-CSO_3/10)} + \cdots\right]$$

where
CSO_{EOL} is the calculated carrier-to-composite second order distortion ratio at the end-of-line, in decibels
log_{10} is base 10 logarithm
CSO_1 is the carrier-to-composite second order distortion ratio, in decibels, of the first active in cascade (expressed as a positive number)
CSO_2 is the carrier-to-composite second order distortion ratio, in decibels, of the second active in cascade (expressed as a positive number)
CSO_3 is the carrier-to-composite second order distortion ratio, in decibels, of the third active in cascade (expressed as a positive number)
... and so on (i.e., CSO_n, where n is the n^{th} amplifier in cascade)

Example:
Assume a node+2 architecture with the following carrier-to-CSO distortion ratio values for each active in cascade. What is the end-of-line carrier-to-CSO distortion ratio in decibels?

Node: 63 dB
First amplifier: 76 dB
Second amplifier: 66 dB

Solution:
$$CSO_{EOL} = -10log_{10}\left[10^{(-CSO_1/10)} + 10^{(-CSO_2/10)} + 10^{(-CSO_3/10)}\right]$$

$$CSO_{EOL} = -10log_{10}\left[10^{(-63/10)} + 10^{(-76/10)} + 10^{(-66/10)}\right]$$

$$CSO_{EOL} = -10 * log_{10}\left[10^{(-6.30)} + 10^{(-7.60)} + 10^{(-6.60)}\right]$$

$$CSO_{EOL} = -10 * log_{10}[0.00000050 + 0.00000003 + 0.00000025]$$

$$CSO_{EOL} = -10 * log_{10}[0.00000078]$$

$$CSO_{EOL} = -10 * [-6.10930254]$$

$$CSO_{EOL} = 61.09$$

Answer: The end-of-line carrier-to-CSO distortion ratio is 61.09 dB.

---

[23] For very short cascades of identical amplifiers (N < 10 amplifiers), the assumption of $10log_{10}$ power addition for carrier-to-CSO distortion ratio no longer holds; see Appendix D. Some prefer to use $15log_{10}(N)$ for this calculation, in which case the formula would be $CSO_{EOL} = -15log_{10}\left[10^{(-CSO_1/15)} + 10^{(-CSO_2/15)} + 10^{(-CSO_3/15)} + \cdots\right]$.

15.4.2. Composite triple beat

The following formula can be used to calculate end-of-line carrier-to-CTB distortion ratio:

$$CTB_{EOL} = -20log_{10}\left[10^{(-CTB_1/20)} + 10^{(-CTB_2/20)} + 10^{(-CTB_3/20)} + \cdots\right]$$

where
CTB_{EOL} is the calculated carrier-to-composite triple beat distortion ratio at the end-of-line, in decibels
log_{10} is base 10 logarithm
CTB_1 is the carrier-to-composite triple beat distortion ratio, in decibels, of the first active in cascade (expressed as a positive number)
CTB_2 is the carrier-to-composite triple beat distortion ratio, in decibels, of the second active in cascade (expressed as a positive number)
CTB_3 is the carrier-to-composite triple beat distortion ratio, in decibels, of the third active in cascade (expressed as a positive number)
… and so on (i.e., CTB_n, where n is the n^{th} amplifier in cascade)

Example:
Assume a node+2 architecture with the following carrier-to-CTB distortion ratio values for each active in cascade. What is the end-of-line carrier-to-CTB distortion ratio in decibels?

Node: 68 dB
First amplifier: 81 dB
Second amplifier: 66 dB

Solution:
$CTB_{EOL} = -20log_{10}\left[10^{(-CTB_1/20)} + 10^{(-CTB_2/20)} + 10^{(-CTB_3/20)}\right]$

$CTB_{EOL} = -20log_{10}\left[10^{(-68/20)} + 10^{(-81/20)} + 10^{(-66/20)}\right]$

$CTB_{EOL} = -20 * log_{10}\left[10^{(-3.40)} + 10^{(-4.05)} + 10^{(-3.30)}\right]$

$CTB_{EOL} = -20 * log_{10}[0.00039811 + 0.00008913 + 0.00050119]$

$CTB_{EOL} = -20 * log_{10}[0.00098842]$

$CTB_{EOL} = -20 * [-3.00505870]$

$CTB_{EOL} = 60.10$

Answer: The end-of-line carrier-to-CTB distortion ratio is 60.10 dB.

15.4.3. Cross modulation

The following formula can be used to calculate end-of-line carrier-to-XMOD ratio:

$$XMOD_{EOL} = -20log_{10}\left[10^{(-XMOD_1/20)} + 10^{(-XMOD_2/20)} + 10^{(-XMOD_3/20)} + \cdots\right]$$

where
$XMOD_{EOL}$ is the calculated carrier-to-cross modulation ratio at the end-of-line, in decibels
log_{10} is base 10 logarithm
$XMOD_1$ is the carrier-to-cross modulation ratio, in decibels, of the first active in cascade (expressed as a positive number)
$XMOD_2$ is the carrier-to-cross modulation ratio, in decibels, of the second active in cascade (expressed as a positive number)
$XMOD_3$ is the carrier-to-cross modulation ratio, in decibels, of the third active in cascade (expressed as a positive number)
… and so on (i.e., $XMOD_n$, where n is the n^{th} amplifier in cascade)

Example:
Assume a node+2 architecture with the following carrier-to-XMOD ratio values for each active in cascade. What is the end-of-line carrier-to-XMOD ratio in decibels?

Node: 60 dB
First amplifier: 76 dB
Second amplifier: 63 dB

Solution:
$$XMOD_{EOL} = -20log_{10}\left[10^{(-XMOD_1/20)} + 10^{(-XMOD_2/20)} + 10^{(-XMOD_3/20)}\right]$$

$$XMOD_{EOL} = -20log_{10}\left[10^{(-60/20)} + 10^{(-76/20)} + 10^{(-63/20)}\right]$$

$$XMOD_{EOL} = -20 * log_{10}\left[10^{(-3.00)} + 10^{(-3.80)} + 10^{(-3.15)}\right]$$

$$XMOD_{EOL} = -20 * log_{10}[0.00100000 + 0.00015849 + 0.00070795]$$

$$XMOD_{EOL} = -20 * log_{10}[0.00186644]$$

$$XMOD_{EOL} = -20 * [-2.728988711]$$

$$XMOD_{EOL} = 54.58$$

Answer: The end-of-line carrier-to-XMOD ratio is 54.58 dB.

15.5. Distortions in an all-digital network

Contrary to some misconceptions, distortions such as CTB, CSO, and CPD don't go away in an all-digital network. Rather than clusters of discrete beats that occur in a network carrying large numbers of analog TV channels, the "digital distortions" are noise-like. Those noise-like distortion products are variously known as composite intermodulation noise (CIN), composite intermodulation distortion (CID), or intermodulation noise (IMN) – which should not be confused with thermal noise.

Confusion does occur, though. It is well-known that increasing the amplitude of RF signal levels in the plant usually improves the carrier-to-noise ratio (CNR).[24] But in a cable network with mostly- or all-digital channel loading, increasing the signal levels can improve CNR to a point, then the noise floor starts to *increase* and the CNR appears to get worse. That seems counterintuitive, but the now-elevated noise floor is no longer just thermal noise. The noise floor is a combination of thermal noise and the previously mentioned noise-like distortions. When characterizing plant performance in the presence of CIN, the term "carrier-to-composite noise (CCN) ratio" is commonly used. Indeed, CCN is a much more appropriate measurement metric than is CNR under these circumstances, because there is no practical way to differentiate thermal noise from CIN in a cable network without turning off the active RF signals.

The graphic in Figure 19 illustrates the relationship of CSO and CTB to analog NTSC visual and aural carriers.[25] Note that CTB falls on visual carrier frequencies, and CSO falls 1.25 MHz below and above visual carrier frequencies. Thermal noise is represented by a horizontal green line at the base of the carriers. Here, each 1 dB increase in RF signal levels at amplifier inputs and outputs causes the CTB ratio to decrease (degrade) by 2 dB, the CSO ratio to degrade by 1 dB, and the CNR to increase (improve) by 1 dB.

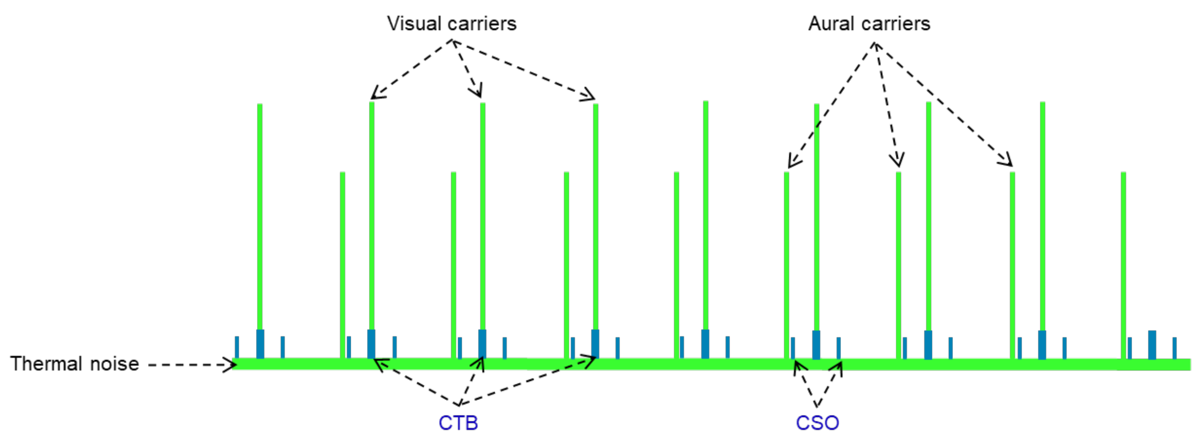


Figure 19 - In a cable network with all-analog TV signals, for each 1 dB increase in RF signal levels: CTB ratio degrades by 2 dB, CSO ratio degrades by 1 dB, CNR improves by 1 dB.

The graphic in Figure 20 illustrates a spectrum with a combination of analog NTSC TV signals and digital signals. In this example the horizontal green line represents thermal noise; the horizontal red line represents CIN (i.e., noise-like distortions); and the horizontal purple line represents composite noise (a mix of thermal noise and the noise-like distortions). As before, each 1 dB increase in amplifier RF input

---

[24] In this context CNR is carrier-to-*thermal* noise ratio.
[25] The assumption in this example is that that the NTSC channels use standard (STD) frequency assignments, as opposed to incremental related carrier (IRC) or harmonic related carrier (HRC) frequencies.

and output signal levels causes the CTB ratio to decrease (degrade) by 2 dB, the CSO ratio to degrade by 1 dB, and the CNR to increase (improve) by 1 dB. However, the carrier-to-CIN ratio degrades by 1 to 2 dB (mix of second and third order components). The CCN ratio degradation depends on the CIN and CNR values.

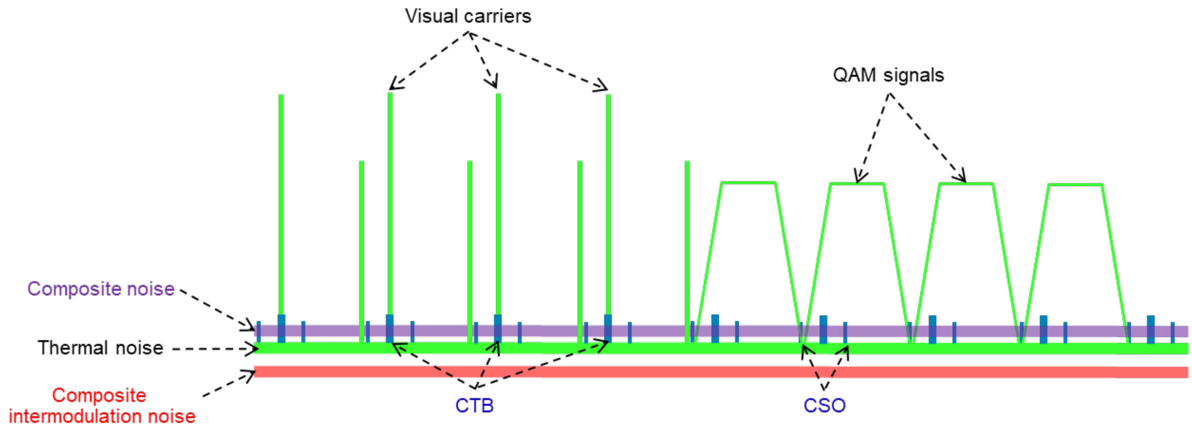


Figure 20 - In a cable network with a mix of analog TV and digital signals, for each 1 dB increase in RF signal levels: CNR, CTB, and CSO ratios behave as before with all-analog operation; CIN ratio degrades by 1 to 2 dB (mix of 2nd and 3rd order components); CCN ratio degradation depends on CIN and CNR values.

The graphic in Figure 21 illustrates all-digital operation. The horizontal green line represents thermal noise; the horizontal red line represents CIN (i.e., noise-like distortions); and the horizontal purple line represents composite noise (a mix of thermal noise and the noise-like distortions). Here, for each 1 dB increase in amplifier input and output RF signal levels, the CNR increases (improves) by 1 dB, the carrier-to-CIN ratio degrades by 1 dB to 2 dB (mix of second and third order components), and the CCN ratio degradation depends on CIN and CNR values.

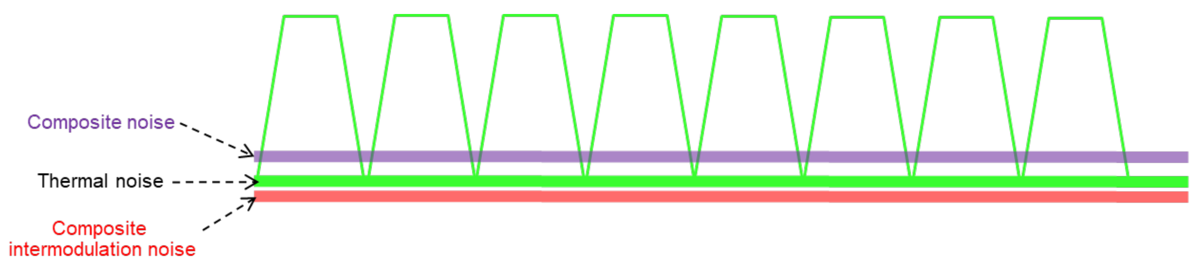


Figure 21 - In a cable network with all-digital operation, for each 1 dB increase in RF signal levels: CNR behaves as before with all-analog operation; CIN ratio degrades by 1 to 2 dB (mix of 2nd and 3rd order components); and CCN ratio degradation depends on CIN and CNR values.

16. Hum Modulation

§76.605(b)(10) of the FCC Rules[26] states the following for analog NTSC television channels: "The peak-to-peak variation in visual signal level caused by undesired low frequency disturbances (hum or repetitive transients) generated within the system, or by inadequate low frequency response, shall not exceed 3 percent of the visual signal level. Measurements made on a single channel using a single unmodulated carrier may be used to demonstrate compliance with this parameter at each test location." The aforementioned is commonly referred to as hum modulation, and the value in the Rules is in percent. However, distribution equipment manufacturers often state hum modulation as a ratio such as 60 dBc. This section includes formulas to convert between hum modulation in percent and as a ratio. This section also includes formulas to calculate end-of-line carrier-to-hum modulation ratio.

16.1. Hum modulation in a cascade of identical amplifiers

The following formula can be used to calculate end-of-line carrier-to-hum modulation ratio:

$$HUM_{EOL} = HUM_{AMP} - 20log_{10}(N)$$

where
HUM_{EOL} is the calculated carrier-to-hum modulation ratio in decibels at the end of a cascade of identical amplifiers
HUM_{AMP} is the carrier-to-hum modulation ratio in decibels for an individual amplifier (expressed as a positive number)
log_{10} is base 10 logarithm
N is the number of identical amplifiers in cascade

Example:
Assume that the carrier-to-hum modulation ratio for an individual amplifier is 65 dB with 12 amperes of steady-state current passing through the amplifier. What is the carrier-to-hum modulation ratio at the end a cascade of 10 of these amplifiers?

Solution:
$HUM_{EOL} = HUM_{AMP} - 20log_{10}(N)$

$HUM_{EOL} = 65 - 20log_{10}(10)$

$HUM_{EOL} = 65 - [20 * log_{10}(10)]$

$HUM_{EOL} = 65 - [20 * (1)]$

$HUM_{EOL} = 65 - [20]$

$HUM_{EOL} = 45$

[26] As of October 2020.

Answer: The carrier-to-hum modulation ratio at the end of the amplifier cascade is 45 dB.[27]

16.2. Hum modulation in a cascade of non-identical active devices

The following formula can be used to calculate end-of-line carrier-to-hum modulation ratio:

$$HUM_{EOL} = -20log_{10}\left[10^{(-HUM_1/20)} + 10^{(-HUM_2/20)} + 10^{(-HUM_3/20)} + \cdots\right]$$

where
HUM_{EOL} is the calculated carrier-to-hum modulation ratio at the end-of-line, in decibels
log_{10} is base 10 logarithm
HUM_1 is the carrier-to-hum modulation ratio, in decibels, of the first amplifier in cascade (expressed as a positive number)
HUM_2 is the carrier-to-hum modulation ratio, in decibels, of the second amplifier in cascade (expressed as a positive number)
HUM_3 is the carrier-to-hum modulation ratio, in decibels, of the third amplifier in cascade (expressed as a positive number)
… and so on (i.e., HUM_n, where n is the n^{th} amplifier in cascade)

Example:
Assume a node+2 architecture with the following carrier-to-hum modulation ratio values for each active in cascade. What is the end-of-line carrier-to-hum modulation ratio in decibels?

Node: 65 dB @ 10 amperes
First amplifier: 60 dB @ 12 amperes
Second amplifier: 70 dB @ 12 amperes

Solution:

$$HUM_{EOL} = -20log_{10}\left[10^{(-HUM_1/20)} + 10^{(-HUM_2/20)} + 10^{(-HUM_3/20)}\right]$$

$$HUM_{EOL} = -20log_{10}\left[10^{(-65/20)} + 10^{(-60/20)} + 10^{(-70/20)}\right]$$

$$HUM_{EOL} = -20 * log_{10}\left[10^{(-3.25)} + 10^{(-3.00)} + 10^{(-3.50)}\right]$$

$$HUM_{EOL} = -20 * log_{10}[0.00056234 + 0.00100000 + 0.00031623]$$

$$HUM_{EOL} = -20 * log_{10}[0.00187857]$$

$$HUM_{EOL} = -20 * [-2.72617283]$$

$$HUM_{EOL} = 54.52$$

[27] This example is intentionally a worst-case scenario. Practically speaking, it is unlikely that each amplifier in a cascade of 10 identical amplifiers would have the manufacturer's maximum steady-state through-current in actual operation (12 amperes in this example).

Answer: The end-of-line carrier-to-hum modulation ratio is 54.58 dB.[28]

16.3. Convert carrier-to-hum ratio to percent

The following formula can be used to convert carrier-to-hum ratio in decibels to hum modulation in percent:

$$hum_{percent} = \left(10^{-CHR/20}\right) * 100$$

where
$hum_{percent}$ is hum modulation in percent
CHR is the carrier-to-hum ratio in dB (expressed as a positive number)

Example:
What is the hum modulation in percent for a carrier-to-hum ratio of 65 dB?

Solution:

$$hum_{percent} = \left(10^{-CHR/20}\right) * 100$$

$$hum_{percent} = \left(10^{-65/20}\right) * 100$$

$$hum_{percent} = (10^{-3.25}) * 100$$

$$hum_{percent} = (0.000562) * 100$$

$$hum_{percent} = 0.0562$$

Answer: The hum modulation is 0.0562 percent.

[28] This example is intentionally a worst-case scenario. Practically speaking, it is unlikely that each active device in a cascade of node+2 amplifiers would have the manufacturer's maximum steady-state through-current in actual operation (≥10 amperes in this example).

16.4. Convert percent to carrier-to-hum ratio

The following formula can be used to convert hum modulation in percent to carrier-to-hum ratio in decibels:

$$CHR = -20log_{10}\left(\frac{hum_{percent}}{100}\right)$$

where
CHR is the carrier-to-hum ratio in dB (expressed as a positive number)
log_{10} is base 10 logarithm
$hum_{percent}$ is hum modulation in percent

Example:
What is 3 percent hum modulation expressed as carrier-to-hum ratio in decibels?

Solution:

$$CHR = -20log_{10}\left(\frac{hum_{percent}}{100}\right)$$

$$CHR = -20 * log_{10}\left(\frac{3}{100}\right)$$

$$CHR = -20 * log_{10}(0.03)$$

$$CHR = -20 * (-1.52)$$

$$CHR = 30.46$$

Answer: The carrier-to-hum ratio is 30.46 dB.

17. Antennas

An antenna is a transducer that converts RF current to electromagnetic waves in transmit applications, or converts electromagnetic waves to RF current in receive applications. Cable operators have long used antennas at headends to receive over-the-air broadcast signals and satellite signals; for base station and vehicle two-way radios; for signal leakage monitoring and measurement; and various other purposes. More information on antenna theory and fundamentals can be found in [1], [2], [3], [7], [11], [13], [14], and [15].

17.1. Isotropic source

From [2], “A source that radiates energy uniformly in all directions is an *isotropic source*.” and “Although the isotropic source is convenient in theory, it is not a physically realizable type.” The directivity of an isotropic source is D = 1, or 0 dBi, where dBi is decibel isotropic (decibels relative to an isotropic source).

An analogy is a light bulb at the center of a sphere, illuminating the surface of that sphere uniformly. While isotropic sources (antennas) do not exist, they are useful for comparisons of real-world antenna performance.

17.2. Directivity and gain

The directivity of an antenna is a measure of the concentration of the radiated energy in a single direction, compared to an isotropic source. The “directivity” is a function of the direction looking from the antenna outward; commonly “the directivity” of an antenna is assigned the value of the maximum directivity.

[2] defines directivity D of an antenna as “the ratio of the maximum radiation intensity (power per unit solid angle) $U(\theta, \phi)_{max}$ to the average radiation intensity U_{av} (averaged over a sphere).”

For practical antennas the concentration of radiation is not uniformly distributed, but concentrated in some directions more than others. Since the directivity (unless otherwise noted) is compared to the uniform radiation pattern of an isotropic source, for some directions the directivity is larger than 1 and for other directions it is less than 1, and by definition averages to 1 over the entire sphere.

It is worth repeating that “the directivity” of an antenna is a measure of the concentration of the radiated energy in the direction of maximum radiation, unless other information is stated. The emphasis on *radiated energy* is to illustrate the distinction between antenna *directivity* and antenna *gain*. In practice, a portion of the power delivered to an antenna is *dissipated* as ohmic loss, and the power which is not dissipated as ohmic loss is *radiated*. The directivity, D, is a measure of the concentration of the radiated energy. The proportion of energy delivered to the antenna which becomes radiated is called the efficiency of the antenna, commonly noted by k.

Directivity and gain are similar, but are often confused. From [2]:

> The *gain* of an antenna (referred to a lossless isotropic source) depends on both its directivity and its efficiency. If the efficiency is not 100 percent, the gain is less than the directivity. Thus, the gain
>
> $G = kD$ (dimensionless)

where k = efficiency factor of antenna ($0 \leq k \leq 1$), dimensionless

This efficiency has to do only with ohmic losses in the antenna.

The directivity and gain for an antenna have been explained in terms of a transmitting (radiating) antenna. The antenna pattern and ohmic loss of an antenna are the same when used to receive, as when transmitting, which is known as reciprocity. The principle of reciprocity is fundamental in antenna theory and practice. The directivity and gain are properties of the antenna.

17.3. Half-wave dipole

The half-wave dipole antenna has long been used for signal leakage measurements by cable operators, and is referenced in the FCC's signal leakage requirements.[29] The dipole antenna most familiar to cable operators is a linear doublet that has a physical end-to-end length equal to slightly less than one-half wavelength at the design frequency.[30] A half-wave dipole comprises two conductive elements in the same plane, fed at the center by a transmission line (see Figure 22).[31] The transmission line can be a balanced transmission line (e.g., twinlead or window line), or an unbalanced transmission line such as coaxial cable. When fed by coaxial cable, a balun is typically used at the antenna's terminals to accommodate the transition from the unbalanced transmission line to the (balanced) dipole antenna.

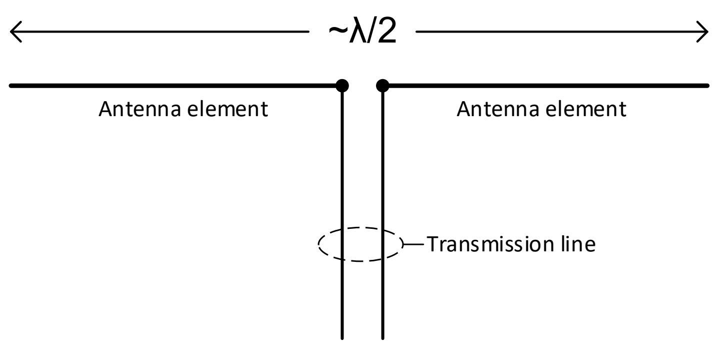


Figure 22 - Half-wave dipole antenna fed by a balanced transmission line.

Figure 23 shows a radiation pattern azimuth plot for a 139 MHz half-wave dipole in free space. The directional nature of the dipole is obvious, with maximum gain broadside to the elements. Nulls in the pattern are evident off of the ends of the antenna. (Visualize the plot as a horizontal slice of a donut standing on its edge, with the dipole poking through the center of the hole in the donut.)

---

[29] See Title 47 of the Code of Federal Regulations, Part 76, §76.609(h).
[30] A thin, linear center-fed dipole antenna that is exactly one-half wavelength long at a given frequency is not resonant at that frequency. The dipole's feedpoint impedance includes some inductive reactance in series with the antenna's radiation resistance. To achieve resonance the dipole's physical length must be shortened a few percent, which brings the reactance to zero, and reduces the radiation resistance somewhat.
[31] Not all half-wave dipoles are fed at the center. A design called an off-center-fed dipole is used in some radio communications applications, and is beyond the scope of this Operational Practice. For more information, see, for example, http://webclass.org/k5ijb/antennas/Windom_off-center-fed-dipole.htm

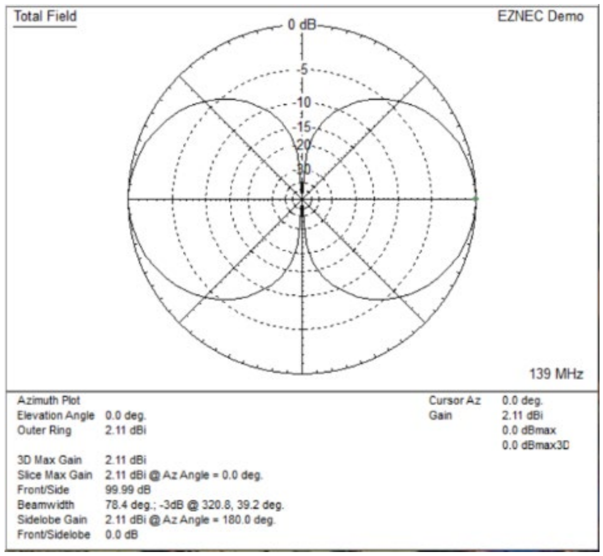


Figure 23. Azimuth plot of a 139 MHz dipole antenna's radiation pattern in free space. Plotted using EZNEC® Demo v. 6.0.30 software.[32]

Figure 24 shows an example of a homemade dipole antenna using adjustable-length elements. The center support section is clear acrylic, and the feedline is a length of 75 ohms impedance mini-coax. Several ferrite beads (#43 mix) were threaded over the coax during assembly; the beads serve as a wide-frequency-range current choke balun. The dipole's resonant frequency is set by adjusting the lengths of the elements to the correct value for the frequency of interest.

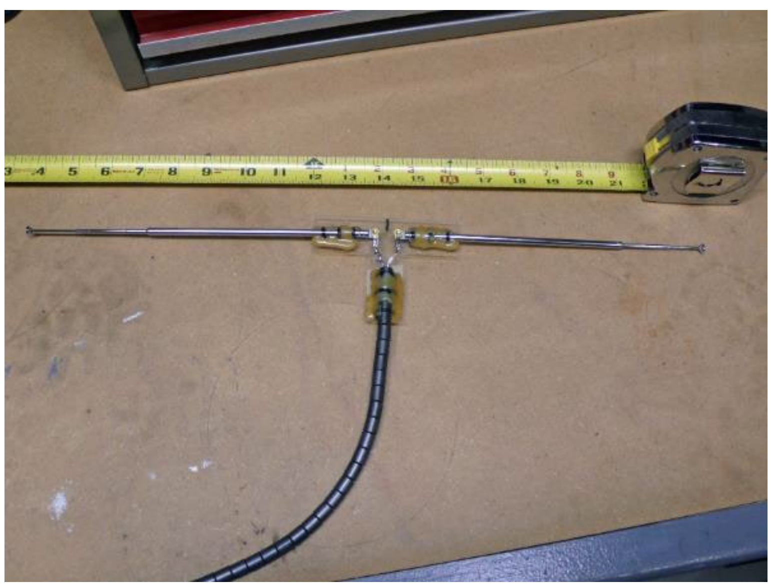


Figure 24. Homemade half-wave dipole antenna using telescoping elements. This particular antenna was designed to be used over a frequency range from about 100 MHz (elements fully extended) to 300 MHz (elements fully collapsed).

[32] EZNEC® is a registered trademark of Roy W. Lewellen.

17.3.1. Dipole antenna end-to-end length

The end-to-end length of a fixed or adjustable half-wave dipole in inches can be calculated using the formula:

$$L_{inches} = \frac{5901.43}{f_{MHz}} * K$$

where
L_{inches} is the dipole's end-to-end length in inches
f_{MHz} is the frequency in megahertz
K is a factor accounting for the antenna element half wavelength-to-diameter ratio (see Section 17.3.2.1)

Example:
What is the end-to-end length of a half-wave dipole antenna for a frequency of 139.25 MHz? Assume the antenna element diameter is 0.25 inch (6.35 mm) and K= 0.967.

Solution:

$$L_{inches} = \frac{5901.43}{f_{MHz}} * K$$

$$L_{inches} = \frac{5901.43}{139.25} * 0.967$$

$$L_{inches} = 42.38 * 0.967$$

$$L_{inches} = 40.98$$

Answer: The end-to-end length is 40.98 inches.

Note: Each element should be of equal length, measured from the center of the support to the element ends. The final end-to-end length will be affected by such things as presence of insulation on the elements, the diameter of the elements, and the proximity to the ground and/or nearby objects. If available, an antenna analyzer should be used for final adjustment of the elements' end-to-end length.

17.3.2. Monopole antenna end-to-end length

A quarter-wavelength monopole antenna, sometimes called a vertical whip antenna, is commonly installed on vehicles. A monopole element functions as half of a half-wave dipole, and the surface on which the monopole is mounted – for instance, a vehicle's roof – functions as the other half, or "image" of the monopole element. For best performance, a monopole should be located on a relatively flat surface such as a vehicle's metallic roof (ideally in the center of the roof), which can function as an effective ground plane. Monopole antennas are often used with two-way radios and for mobile signal leakage monitoring.

The length of a quarter-wavelength monopole in inches can be calculated using the formula:

$$L_{inches} = \frac{2950.71}{f_{MHz}} * K$$

where
L_{inches} is the monopole's end-to-end length in inches
f_{MHz} is the frequency in megahertz
K is a factor accounting for the antenna element half wavelength-to-diameter ratio (see Section 17.3.2.1)

Example:
What is the length of a quarter-wavelength monopole antenna for a frequency of 139.25 MHz? Assume the antenna element diameter is 0.1 inch (2.54 mm) and $K = 0.972$.

Solution:

$$L_{inches} = \frac{2950.71}{f_{MHz}} * K$$

$$L_{inches} = \frac{2950.71}{139.25} * 0.972$$

$$L_{inches} = 21.19 * 0.972$$

$$L_{inches} = 20.60$$

Answer: The length is 20.60 inches.

Note: The final length will be affected by such things as presence of insulation on the element, the diameter of the element, placement on a vehicle, and the proximity to nearby objects on the vehicle (e.g., ladder or lift/bucket, other antennas). If available, an antenna analyzer should be used for final adjustment of the element's length.

17.3.2.1. Calculate K factor for antenna elements

As mentioned in Footnote 30, to achieve resonance a dipole's [or monopole's] physical length must be shortened from the free-space value by a few percent in order to bring the reactance to zero. The antenna element length is affected by the ratio of the half-wavelength to element diameter (for a detailed discussion on this, see [1] and [15]). To account for the element half wavelength-to-diameter ratio, a multiplying factor must be applied to the free-space quarter- or half-wavelength calculation.

The multiplying factor is known as the K factor, and can be calculated with the formula

$$K = 0.978701 + \frac{-11.864971}{\left[1 + \left(\frac{\left(\frac{\lambda/2}{D}\right)}{0.000449}\right)^{1.792529}\right]^{0.3004597}}$$

where
K is the K factor
$\lambda/2$ is the free-space half-wavelength
D is antenna element diameter in the same units as $\lambda/2$

Example:
What is the K factor for a half-wave dipole antenna for 139.25 MHz that uses 0.25 inch diameter elements?

Solution:
First calculate the free-space half-wavelength in inches for 139.25 MHz with the formula $\lambda/2_{inches}$ = 5901.43/f_{MHz} = 5901.43/139.25 = 42.38 inches.

$$K = 0.978701 + \frac{-11.864971}{\left[1 + \left(\frac{\left(\frac{\lambda/2}{D}\right)}{0.000449}\right)^{1.792529}\right]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{\left[1 + \left(\frac{\left(\frac{42.38\ inches}{0.25\ inch}\right)}{0.000449}\right)^{1.792529}\right]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{\left[1 + \left(\frac{(169.52)}{0.000449}\right)^{1.792529}\right]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{[1 + (377550.11)^{1.792529}]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{[1 + 9928615848.95]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{[9928615849.95]^{0.3004597}}$$

$$K = 0.978701 + \frac{-11.864971}{1008.47}$$

$$K = 0.978701 + (-0.01)$$

$$K = 0.967$$

Answer: The K factor is 0.967.

17.3.3. Dipole antenna directivity

The directivity of a half-wave dipole can be calculated as follows:

$$D = \frac{4}{Cin(2\pi)} \approx 1.64$$

where
D is directivity
Cin is cosine integral Cin(x)

Note: The cosine integral Cin(x) is not the same as the cosine integral Ci(x). According to https://en.wikipedia.org/wiki/Dipole_antenna, "Both MATLAB and Mathematica have inbuilt functions which compute Ci(x), but not Cin(x). See the Wikipedia page on cosine integral for the relationship between these functions."

The directivity of a half-wave dipole in decibels can be calculated as follows:

$$D_{dBi} = 10log_{10}(1.64) = 2.15\ dBi$$

where
D_{dBi} is directivity in decibel isotropic
log_{10} is base 10 logarithm

In this document, the gain of a resonant half-wave dipole is considered to be equal to its directivity, or 2.15 dBi, with an implicit assumption that the ohmic losses are negligible. In practice, the real gain of a dipole will be slightly less, depending on the ohmic losses in the antenna.

Note: The literature defines antenna directivity relative to an isotropic source. However, practice has developed such that for antenna gain, a reference must be included. The most common is dBi (decibel isotropic, or decibels relative to an isotropic source) and occasionally also dBd (decibel dipole, or decibels relative to a dipole antenna). From this, one can correctly state that certain design of a Yagi-Uda antenna's gain is, for example, 8.15 dBi or 6 dBd. Gain stated in just decibels is meaningless.

17.3.4. Antenna factor for dipole antennas

In the world of electromagnetic compatibility (EMC) and electromagnetic interference (EMI) testing, a parameter known as *antenna factor* is commonly used. Antenna factor is not widely known in the cable industry, although it is built-in to the calculations used for signal leakage measurements as the "0.021" and "f" (frequency) in the well-known microvolts per meter-to-dBmV conversion formula. For more on the mathematics of field strength measurements, refer to Appendix H.

Antenna factor is the ratio of an electromagnetic field incident upon an antenna to the voltage output at its terminals. In other words, antenna factor is the ratio of the field strength of an electromagnetic field incident upon an antenna to the voltage produced by that field across a load of impedance Z_0 connected to antenna's terminals. The field strength can be calculated by multiplying the voltage at the antenna's terminals by the antenna factor. Many prefer to use logarithms and decibels, and antenna factor is commonly expressed in decibel format rather than the previously discussed linear format. When antenna factor is stated in decibels, field strength in decibel microvolt per meter (dBμV/m) is calculated by adding the signal level at the antenna terminals in decibel microvolt (dBμV)[33] to the antenna factor in decibel/meter (dB/m).

17.3.4.1. Calculate dipole antenna factor for 50 Ω

Half-wave dipole antenna factor in 50 ohms impedance can be calculated with the formula:

$$AF_{50\Omega} = 20log_{10}(f) - 10log_{10}(G) - 29.7707$$

where
$AF_{50\Omega}$ is antenna factor in dB/m for a half-wave dipole connected to a load of Z_0 = 50 ohms
log_{10} is base 10 logarithm
f is frequency in megahertz
G is a dipole antenna's linear gain (G = 1.64)

Example:
What is the antenna factor for a half-wave dipole resonant at 139.25 MHz connected to a load impedance of 50 ohms?

Solution:
$$AF_{50\Omega} \approx 20log_{10}(f) - 10log_{10}(G) - 29.7707$$

$$AF_{50\Omega} \approx 20 * log_{10}(139.25) - 10 * log_{10}(1.64) - 29.7707$$

$$AF_{50\Omega} \approx 20 * (2.144) - 10 * log_{10}(0.215) - 29.7707$$

$$AF_{50\Omega} \approx 42.876 - 2.148 - 29.7707$$

$$AF_{50\Omega} \approx 10.957$$

[33] In North America, signal leakage field strength is usually stated in microvolt per meter (μV/m) rather than decibel microvolt per meter (dBμV/m). Signal levels are usually stated in decibel millivolt (dBmV) rather than decibel microvolt (dBμV). Conversions for all of these are included elsewhere in this document.

Answer: The approximate antenna factor is 10.957 dB/m.

17.3.4.2. Calculate approximate dipole antenna factor for 73 Ω

A thin linear half-wave dipole in free space has an impedance of approximately 73 ohms, a common value used in cable applications (e.g., signal leakage measurements).

The approximate half-wave dipole antenna factor in 73 ohms impedance can be calculated with the formula:

$$AF_{73\Omega} \approx 20log_{10}(f) - 10log_{10}(G) - 31.4142$$

where
$AF_{73\Omega}$ is antenna factor in dB/m for a half-wave dipole connected to a load of Z_0 = 73 ohms
log_{10} is base 10 logarithm
f is frequency in megahertz
G is a dipole antenna's linear gain (G = 1.64)

Example:
What is the approximate antenna factor for a half-wave dipole resonant at 139.25 MHz connected to a load impedance of 73 ohms?

Solution:
$AF_{73\Omega} \approx 20log_{10}(f) - 10log_{10}(G) - 31.4142$

$AF_{73\Omega} \approx 20 * log_{10}(139.25) - 10 * log_{10}(1.64) - 31.4142$

$AF_{73\Omega} \approx 20 * (2.144) - 10 * log_{10}(0.215) - 31.4142$

$AF_{73\Omega} \approx 42.876 - 2.148 - 31.4142$

$AF_{73\Omega} \approx 9.313$
Answer: The approximate antenna factor is 9.313 dB/m.

17.3.4.3. Calculate approximate dipole antenna factor for 75 Ω

The approximate half-wave dipole antenna factor in 75 ohms impedance can be calculated with the following formula:

$$AF_{75\Omega} \approx 20log_{10}(f) - 10log_{10}(G) - 31.5315$$

where
$AF_{75\Omega}$ is antenna factor in dB/m for a half-wave dipole connected to a load of Z_0 = 75 ohms
log_{10} is base 10 logarithm
f is frequency in megahertz
G is a dipole antenna's linear gain (G = 1.64)

Example:
What is the approximate antenna factor for a half-wave dipole resonant at 139.25 MHz connected to a load impedance of 75 ohms?

Solution:
$AF_{75\Omega} \approx 20log_{10}(f) - 10log_{10}(G) - 31.5315$

$AF_{75\Omega} \approx 20 * log_{10}(139.25) - 10 * log_{10}(1.64) - 31.5315$

$AF_{75\Omega} \approx 20 * (2.144) - 10 * log_{10}(0.215) - 31.5315$

$AF_{75\Omega} \approx 42.876 - 2.148 - 31.5315$

$AF_{75\Omega} \approx 9.196$

Answer: The approximate antenna factor is 9.196 dB/m.

17.4. Effective aperture

Effective aperture is the geometric area over which an antenna receives power from an incident RF signal and delivers that power to a connected load. If the antenna is considered lossless, effective aperture is called maximum effective aperture (A_{em}). For a half-wave dipole antenna, A_{em} can be approximated by a rectangle that has dimensions of 0.5λ by 0.25λ, or an ellipse whose area is 0.13λ^2. See Figure 25.

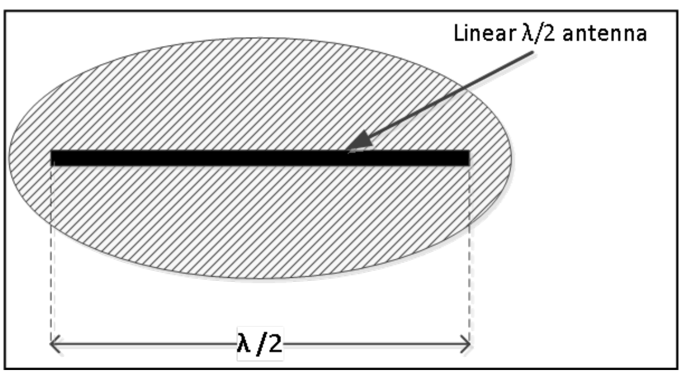


Figure 25 - A linear half-wave dipole's maximum effective aperture A_{em} can be represented by an ellipse with an area of 0.13λ^2. Adapted from [2].

17.4.1. Calculate the effective aperture of an antenna (linear)

$$A_e = \left[\frac{\lambda^2}{4 * \pi}\right] * G$$

where
A_e is the effective aperture of the antenna in square meters (m^2)
λ is the wavelength in meters
G is the linear gain of the antenna

Example 1:
What is the effective aperture of a half-wave dipole antenna with a linear gain of 1.64 and tuned to receive 139.25 MHz?

Solution 1:

$$A_e = \left[\frac{\lambda^2}{4 * \pi}\right] * G$$

$$A_e = \left[\frac{\left(\frac{299.79}{139.25}\right)^2}{4 * 3.14}\right] * 1.64$$

$$A_e = \left[\frac{4.63}{12.56}\right] * 1.64$$

$$A_e = 0.37 * 1.64$$

$$A_e = 0.61$$

Answer: The affective aperture is 0.61 m^2.

Example 2:
What is the effective aperture of a C-band earth station antenna with a linear gain of 1,585 receiving a satellite signal at 4,000 MHz?

Solution 2:

$$A_e = 10log_{10}\left[\frac{\lambda^2}{4 * \pi}\right] * G$$

$$A_e = 10log_{10}\left[\frac{\left(\frac{299.79}{4{,}000}\right)^2}{4 * 3.14}\right] * 1{,}585$$

$$A_e = \left[\frac{0.0056}{12.56}\right] * 1{,}585$$

$$A_e = 0.00047 * 1{,}585$$

$A_e = 0.71$

Answer: The affective aperture is 0.71 m^2.

17.4.2. Calculate the effective aperture of an antenna (logarithmic)

$$A_e = 10log_{10}\left[\frac{\left(\frac{c}{f}\right)^2}{4 * \pi}\right] + G$$

where
A_e is the effective aperture of the antenna in dB per square meter (dB/m^2)
c is the speed of light (299.79 megameters per second)
f is the frequency in megahertz
G is the gain of the antenna in dBi

Example 1:
What is the effective aperture of a half-wave dipole antenna with a gain of 2.15 dBi and tuned to receive 139.25 MHz?

Solution 1:

$$A_e = 10log_{10}\left[\frac{\left(\frac{c}{f}\right)^2}{4 * \pi}\right] + G$$

$$A_e = 10log_{10}\left[\frac{\left(\frac{299.79}{139.25}\right)^2}{4 * 3.14}\right] + 2.15$$

$$A_e = 10log_{10}\left[\frac{4.63}{12.56}\right] + 2.15$$

$$A_e = -4.33 + 2.15$$

$$A_e = -2.18$$

Answer: The affective aperture is –2.18 dB/m^2.

Example 2:
What is the effective aperture of a C-band earth station antenna with a gain of 32 dBi receiving a satellite signal at 4000 MHz?

Solution 2:

$$A_e = 10log_{10}\left[\frac{\left(\frac{c}{f}\right)^2}{4 * \pi}\right] + G$$

$$A_e = 10log_{10}\left[\frac{\left(\frac{299.79}{4000}\right)^2}{4 * 3.14}\right] + 32$$

$$A_e = 10log_{10}\left[\frac{0.0056}{12.56}\right] + 32$$

$$A_e = -33.49 + 32$$

$$A_e = -1.49$$

Answer: The affective aperture is –1.49 dB/m^2.

17.5. Horizontal antenna spacing to reduce off-axis interference

A pair of identical antennas can be spaced horizontally on a tower or similar support structure to reduce off-axis interference such as a reflection (multipath) off of a building, hillside, etc., or co-channel interference from another signal on the same frequency. This method works because the off-axis interference arrives at one antenna a half wavelength before it reaches the other antenna, resulting in a 180° phase shift and cancellation of the interference when the antenna outputs are combined (see Figure 26).

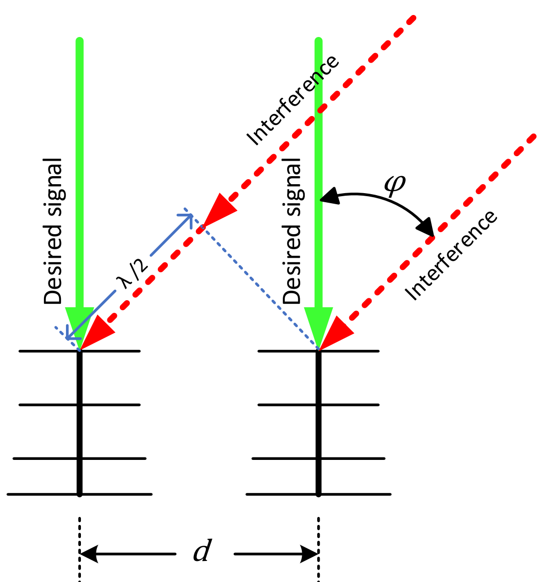


Figure 26 - Horizontal antenna spacing to cancel off-axis interference.

The horizontal spacing in wavelengths can be calculated with the formulas (from [13] and [14]):

$$d_\lambda = \frac{1}{2 * sin\varphi}$$

where
d_λ is the spacing between the antennas, in wavelengths
φ is the angle in degrees between the desired signal and the interference

A variation of the formula to calculate horizontal spacing is

$$d = \frac{\lambda_I}{2 * sin\varphi}$$

where
d is the spacing between the antennas (same units as λ_I)
λ_I is the wavelength of the interfering signal
φ is the angle in degrees between the desired signal and the interfering signal

Example 1:
Assume interference to UHF Ch. 14 (470 MHz to 476 MHz) is being received at a headend site 35° off axis from the desired signal (refer to Figure 27), and that the interfering signal is on the same frequency as the desired signal. What is the horizontal spacing in wavelengths between a pair of identical antennas to reduce or eliminate the off-axis interference?

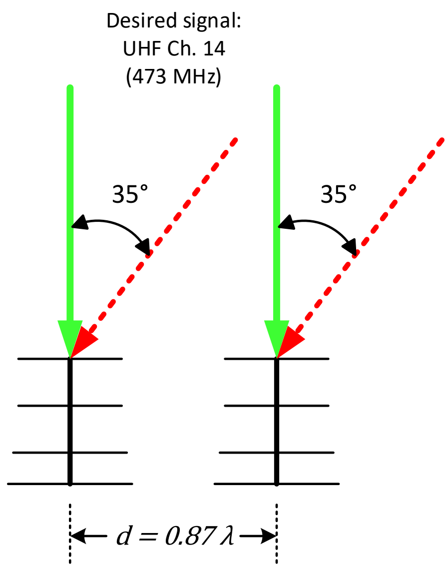


Figure 27 - Calculated horizontal spacing for 35° off-axis interference on UHF Ch. 14 is 0.87 λ.

Solution 1:
First calculate the wavelength of UHF Ch. 14 (473 MHz center frequency), in feet, with the formula $\lambda = 983.57/f_{MHz}$.

$\lambda = 983.57/473$

$\lambda = 2.08$ feet

Next, calculate the necessary antenna separation in wavelengths:

$$d_\lambda = \frac{1}{2 * sin\varphi}$$

$$d_\lambda = \frac{1}{2 * sin35°}$$

$$d_\lambda = \frac{1}{2 * 0.57}$$

$$d_\lambda = \frac{1}{1.15}$$

$$d_\lambda = 0.87$$

Answer: The separation is 0.87 wavelength. Since one wavelength at 473 MHz = 2.08 feet, the antenna separation in feet is 0.87 $*$ 2.08 feet = 1.81 feet.

Example 2:
Use the second formula to calculate the separation. Assume the same parameters as Example 1.

Solution 2:
First calculate the wavelength of UHF Ch. 14 (473 MHz center frequency), in feet, with the formula $\lambda = 983.57/f_{MHz}$. In this example $\lambda = \lambda_I$.

$\lambda = 983.57/473$

$\lambda = 2.08$ feet

Using the second formula

$$d = \frac{\lambda_I}{2 * sin\varphi}$$

$$d = \frac{2.08}{2 * sin35°}$$

$$d = \frac{2.08}{2 * 0.57}$$

$$d = \frac{2.08}{1.15}$$

$$d = 1.81$$

The answer is 1.81 feet.

17.6. Parabolic antennas

Parabolic or "dish" antennas are commonly used for satellite and microwave applications because of their relatively high gain and narrow beamwidth. The cable industry has long used parabolic antennas for television receive only (TVRO) satellite downlinks, typically in the C-band (4.0 GHz to 4.2 GHz). Parabolic antennas are also used for point-to-point and point-to-multipoint microwave service, such as CARS-band (12.7 GHz to 13.2 GHz) microwave installations.

17.6.1. Calculate the gain of a parabolic antenna

The formulas in this section are used to calculate the main beam gain of a parabolic antenna, excluding the effect of sidelobes. When doing path or link analysis, one should use antenna gain values available from the manufacturer. The manufacturer's gain specifications are often based upon actual measurements, and take into account the actual antenna efficiency. If the manufacturer's gain figures are not available, the formulas included here will provide calculated gain values that are satisfactory for modeling.

The main beam gain of a parabolic antenna with a diameter in meters can be calculated using the following formula:

$$G = 10log_{10}(D^2 * f^2 * k)$$

where
G is the main beam gain of the antenna in dBi
log_{10} is base 10 logarithm
D is the antenna diameter in meters
f is the frequency in gigahertz
k is the antenna efficiency as a percentage (typ. 55% to 70%)

Example 1:
Assuming an antenna efficiency of 55%, what is the main beam gain of a 4.5 meters diameter earth station antenna at a C-band frequency of 4 GHz?

Solution:
$G = 10log_{10}(D^2 * f^2 * k)$

$G = 10 * log_{10}(4.5^2 * 4^2 * 55)$

$G = 10 * log_{10}(20.25 * 16 * 55)$

$G = 10 * log_{10}(17820)$

$G = 10 * (4.25)$

$G = 42.51$

Answer: The main beam gain is 42.51 dBi

Example 2:
Assuming an antenna efficiency of 62%, what is the main beam gain of a 2.4 meters (8 ft.) diameter microwave antenna at the CARS-band frequency of 12.7 GHz?

Solution:
$G = 10log_{10}(D^2 * f^2 * k)$

$G = 10 * log_{10}(2.4^2 * 12.7^2 * 62)$

$G = 10 * log_{10}(5.76 * 161.29 * 62)$

$G = 10 * log_{10}(57599.88)$

$G = 10 * (4.76)$

$G = 47.6$

Answer: The main beam gain is 47.6 dBi

The main beam gain of a parabolic antenna with a diameter in feet can be calculated using the following formula:

$$G = 20log_{10}D + 10log_{10}(10.2E) + 20log_{10}f$$

where
G is the main beam gain of the antenna in dBi
log_{10} is base 10 logarithm
D is the antenna diameter in feet
E is the antenna efficiency (typ. 55% to 70%) in decimal form
f is the frequency in gigahertz

Example:
Assuming an antenna efficiency of 55%, what is the main beam gain of an 8 ft. diameter microwave antenna at the CARS-band frequency 12.7 GHz?

Solution:
$G = 20log_{10}D + 10log_{10}(10.2E) + 20log_{10}f$

$G = 20log_{10}8 + 10log_{10}(10.2 * 0.55) + 20log_{10}12.7$

$G = 20 * log_{10}(8) + 10 * log_{10}(10.2 * 0.55) + 20 * log_{10}(12.7)$

$G = 20 * (0.90) + 10 * (0.75) + 20 * (1.10)$

$G = 18.06 + 7.49 + 22.08$

$G = 47.63$

The answer is 47.63 dBi.

17.6.2. Parabolic antenna half-power beamwidth

A parabolic antenna's half-power beamwidth (HPBW) – also called the 3 dB beamwidth – can be calculated with the formulas in this section. When doing path or link analysis, one should use HPBW values available from the antenna manufacturer. The manufacturer's HPBW specifications are often based upon measurements, and take into account the actual antenna efficiency. If the manufacturer's HPBW figures are not available, the formulas included here will provide calculated values that are satisfactory for modeling. Expressed mathematically:

$$HPBW = \frac{70}{D * f_{GHz}}$$

where
$HPBW$ is the half-power (-3 dB) beamwidth in degrees
D is the antenna's diameter in feet
f_{GHz} is the frequency in gigahertz

$$HPBW = 70 * \left(\frac{\lambda}{D}\right)$$

where
$HPBW$ is the half-power (-3 dB) beamwidth of the antenna in degrees
λ is the wavelength in meters ($\lambda = 299.79/f_{MHz}$)
D is the antenna's diameter in meters

Example 1 (using the first formula):
What is the half-power beamwidth of an 8 ft. diameter CARS band parabolic antenna at 12.7 GHz?

Solution 1:

$$HPBW = \frac{70}{D * f_{GHz}}$$

$$HPBW = \frac{70}{8 * 12.7}$$

$$HPBW = \frac{70}{101.6}$$

$$HPBW = 0.69$$

Answer: The half-power beamwidth is 0.69 degree.

Example 2 (using the second formula):
What is the half-power beamwidth of a 6 meters (19.7 ft.) diameter earth station antenna at the C-band frequency 4,180 MHz?

Solution 2:

$$HPBW = 70 * \left(\frac{\lambda}{D}\right)$$

$$HPBW = 70 * \left[\frac{\left(\frac{299.79}{4,180}\right)}{6}\right]$$

$$HPBW = 70 * \left[\frac{0.072}{6}\right]$$

$$HPBW = 70 * 0.012$$

$$HPBW = 0.84$$

Answer: The half-power beamwidth is 0.84 degree.

18. Calculate Power Density

Power density (sometimes referred to as "power flux density") is the measure of power per unit area normal to the direction of electromagnetic wave propagation and a certain distance from the source, usually expressed in units of watts per square meter (W/m^2).

18.1. Power density in watt per square meter

Power density in watt per square meter (W/m^2) at a certain distance from a source (e.g., transmitter) and an antenna with a known linear gain can be calculated using the following formula:

$$P_d = \left(\frac{G_t * P_t}{4 * \pi * D^2}\right)$$

where
P_d is the power density in watts per square meter (W/m^2)
G_t is the linear gain of the source antenna
P_t is the input power in watts to the source antenna
D is the distance in meters

Example:
What is the power density in W/m^2 at a distance of 10 meters from a source with a power of 1 watt at the input to the source antenna and a linear antenna gain of 4?

Solution:
$$P_d = \left(\frac{G_t * P_t}{4 * \pi * D^2}\right)$$

$$P_d = \left(\frac{4 * 1}{4 * 3.1416 * 10^2}\right)$$

$$P_d = \left(\frac{4}{12.56 * 100}\right)$$

$$P_d = \left(\frac{4}{1256}\right)$$

$$P_d = 0.003183$$

Answer: The power density is 0.003183 W/m^2

18.2. Power density in decibel watt per square meter

The power density can be also expressed in decibel watt per square meter (dBW/m^2) using the following formula:

$$P_d = 10log_{10}(P_t) + 10log_{10}(G_t) - 20log_{10}(D) - 10log_{10}(4 * \pi)$$

where
P_d is the power density in decibel watt per square meter (dBW/m^2)
log_{10} is base 10 logarithm
P_t is the input power in watts to the source antenna
G_t is the linear gain of the source antenna
D is the distance in meters

Example:
What is the power density in dBW/m^2 at a distance of 10 meters from a source with a power of 1 watt at the input to the source antenna and a linear antenna gain of 4?

Solution:

$$P_d = 10log_{10}(P_t) + 10log_{10}(G_t) - 20log_{10}(D) - 10log_{10}(4 * \pi)$$

$$P_d = [10 * log_{10}(1)] + [10 * log_{10}(4)] - [20 * log_{10}(10)] - [10 * log_{10}(12.56)]$$

$$P_d = [10 * (0)] + [10 * (0.602)] - [20 * (1)] - [10 * (1.099)]$$

$$P_d = 0 + 6.02 - 20 - 10.99$$

$$P_d = -24.97$$

Answer: The power density is –24.97 dBW/m^2

18.3. Convert power density to received power

The power intercepted by a receive antenna can be found by multiplying the power density present at the antenna by the antenna's effective aperture:

$$P_r = P_d * A_e$$

where
P_r is the power at the output of the receive antenna in watts
P_d is the power density present at the receive antenna in watts per square meter (W/m^2)
A_e is the effective aperture of the receive antenna in square meters (m^2)

Example:
Assuming a power density of 0.003183 W/m^2 present at a receive antenna with an effective aperture of 0.0004323 m^2, what is the power at the output of the antenna in watts?

Solution:
$P_r = P_d * A_e$

$P_r = 0.003183 * 0.0004323$

$P_r = 1.376 * 10^{-6}$

Answer: The power at the output of the receiving antenna is 1.376 ∗ 10^{-6} (1.376E–6) watts.

The power at the output of the receive antenna can be also expressed in decibel watts (dBW) using the following formula:

$$P_r = P_d - 20log_{10}(f) + (G_r) + 38.54$$

where
P_r is the power at the output of the receiving antenna in decibel watts (dBW)
P_d is the power density present at the receive antenna in decibel watts per square meter (dBW/m^2)
log_{10} is base 10 logarithm
f is the frequency in megahertz
G_r is the gain of the receiving antenna in dBi

Example:
Assuming a power density of –24.97 dBW/m^2, what is the power at the output of a receive antenna with with a gain of 3.00 dBi and tuned to receive 5,745 MHz?

Solution:
$P_r = P_d - 20log_{10}(f) + (G_r) + 38.54$

$P_r = -24.97 - [20 * log_{10}(5{,}745)] + (3.00) + 38.54$

$P_r = -24.97 - [20 * (3.759)] + (3.00) + 38.54$

$P_r = -24.97 - 75.18 + 3.00 + 38.54$

$P_r = -58.61$

Answer: The power at the output of the receiving antenna is –58.61 dBW.

19. Signal Leakage Formulas

The following formulas are used to calculate various signal leakage-related parameters and to convert between various signal leakage-related units. When dealing with leakage measurements and distance(s) from a leakage source, it is assumed that all field strength measurements are in the far-field.

19.1. Calculate wavelength (λ)

Wavelength in meters and feet can be calculated with the following formulas:

$$wavelength_{meters} = \frac{299.792458}{f_{MHz}}, and$$

$$wavelength_{feet} = \frac{983.571056}{f_{MHz}}$$

where
$wavelength_{meters}$ is wavelength in meters
f_{MHz} is frequency in megahertz
$wavelength_{feet}$ is wavelength in feet

Example:
What is the approximate length of a half-wave dipole tuned to receive 139.25 MHz?

Solution in meters:

$$wavelength_{meters} = \frac{299.792458}{f_{MHz}}$$

$$wavelength_{meters} = \frac{299.792458}{139.25}$$

$$wavelength_{meters} = 2.15$$

Answer: Divide the free-space wavelength by 2 to get the free-space half wavelength: 2.15/2 = 1.08 meters. A half-wave dipole's physical length is approximately 95% of the free-space half wavelength value, or 1.02 meters in this example. A more accurate end-to-end length can be determined by multiplying the free-space half-wavelength by a K factor, as discussed in Section 17.3.2.1.

Solution in feet:

$$wavelength_{feet} = \frac{983.571056}{f_{MHz}}$$

$$wavelength_{feet} = \frac{983.571056}{139.25}$$

$wavelength_{feet} = 7.06$

Answer: Divide the free-space wavelength (7.06 feet) by 2 to get the free-space half wavelength: 7.06/2 = 3.53 feet. A half-wave dipole's physical length is approximately 95% of the free-space half wavelength value, or 3.35 feet in this example. A more accurate end-to-end length can be determined by multiplying the free-space half-wavelength by a K factor, as discussed in Section 17.3.2.1.

19.2. Calculate the radiating near-field, far-field boundary

$$R = \frac{2D^2}{\lambda}$$

where
R = distance from the antenna elements
D = largest dimension of the antenna aperture (for a resonant half-wave dipole, D is equal to approximately 0.5λ to 0.6λ[34])
λ = wavelength

Note: All variables must be in the same units (feet, meters, etc.)

Example:
What is the approximate distance defining the radiating near-field and radiating far-field boundary for a half-wave dipole tuned for resonance at 139.25 MHz? Assume the free-space wavelength is 7.06 feet, 0.5λ is 3.53 feet, and 0.6λ is 4.24 feet.

Solution:

$$R = \frac{2D^2}{\lambda}$$

$$R = \frac{2(3.53^2)}{7.06}$$

$$R = \frac{2(12.46)}{7.06}$$

$$R = \frac{24.92}{7.06}$$

$$R = 3.53$$

to

---

[34] In [2], the maximum effective aperture of a dipole "…is approximately represented by a rectangle ½ by ¼λ on a side." Using this definition, a half wavelength is the largest dimension of a dipole antenna's aperture, so D is 0.5 λ. The author also says maximum effective aperture can be "…represented by elliptical area of 0.13λ^2." Here the largest dimension of the aperture (width of the ellipse) is approximately 0.6 λ.

$$R = \frac{2D^2}{\lambda}$$

$$R = \frac{2(4.24^2)}{7.06}$$

$$R = \frac{2(17.98)}{7.06}$$

$$R = \frac{35.96}{7.06}$$

$$R = 5.09$$

Answer: The approximate distance defining the radiating near-field and radiating far-field boundary for a half-wave dipole tuned for resonance at 139.25 MHz 3.5 feet to 5.1 feet.

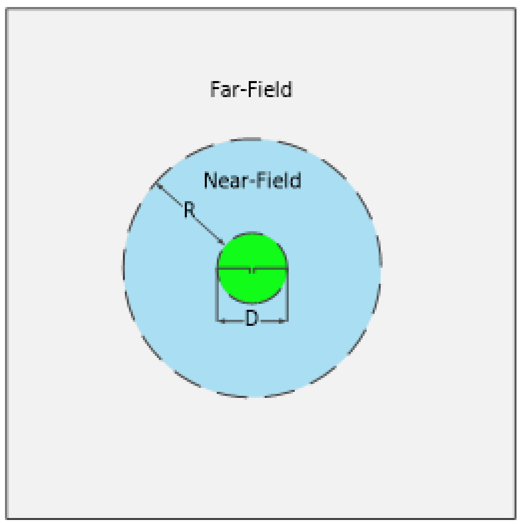


Figure 28 - Approximate distance from dipole to near-field/far-field boundary

19.3. Calculate free space path loss

$$Loss_{dB} = 20\,log_{10}(f_{MHz}) + 20\,log_{10}(d_{km}) + 32.45$$

where
$Loss_{dB}$ is free space path loss in decibels
log_{10} is base 10 logarithm
f_{MHz} is frequency in megahertz
d_{km} is path length in kilometers (1 meter = 0.001 km)

Example 1:
What is the free-space path loss at 139.25 MHz between a leakage source and an antenna 3 meters away from the leak? (Note: 3 meters is equal to 0.003 kilometer)

Solution 1:
$Loss_{dB} = 20\,log_{10}(f) + 20\,log_{10}(d_{km}) + 32.45$

$Loss_{dB} = 20\,log_{10}(139.25) + 20\,log_{10}(0.003) + 32.45$

$Loss_{dB} = (20 * [log_{10}(139.25)]) + (20 * [log_{10}(0.003)]) + 32.45$

$Loss_{dB} = (20 * [2.14]) + (20 * [-2.52]) + 32.45$

$Loss_{dB} = (42.88) + (-50.46) + 32.45$

$Loss_{dB} = 24.87$

Answer: The free-space path loss at 139.25 MHz between a leakage source and an antenna 3 meters away from the leak is 24.87 dB.

Where the distance is in feet:

$$Loss_{dB} = 20\,log_{10}(f_{MHz}) + 20\,log_{10}(d_{feet}) - 37.89$$

where
$Loss_{dB}$ is free space path loss in decibels
log_{10} is base 10 logarithm
f_{MHz} is frequency in megahertz
d_{feet} is path length in feet

Example 2:
What is the free-space loss at 139.25 MHz between a leakage source and an antenna 9.84 feet away from the leak?

Solution 2:
$Loss_{dB} = 20\,log_{10}(f_{MHz}) + 20\,log_{10}(d_{feet}) - 37.89$

$Loss_{dB} = 20\,log_{10}(139.25) + 20\,log_{10}(9.84) - 37.89$

$Loss_{dB} = (20 * [log_{10}(139.25)]) + (20 * [log_{10}(9.84)]) - 37.89$

$Loss_{dB} = (20 * [2.14]) + (20 * [0.99]) - 37.89$

$Loss_{dB} = (42.88) + (19.86) - 37.89$

$Loss_{dB} = 24.85$

Answer: The free-space loss at 139.25 MHz between a leakage source and an antenna 9.84 feet away from the leak is 24.85 dB.

19.4. Convert microvolt (μV) to microvolt per meter (μV/m)

$$E_{\mu V/m} = \mu V * 0.021 * f$$

where
$E_{\mu V/m}$ is field strength in microvolt per meter
μV is RF signal level in microvolt at the terminals of a resonant half-wave dipole
f is frequency in megahertz

Example:
What is the μV/m equivalent of 4 μV at 139.25 MHz?

Solution:
$E_{\mu V/m} = \mu V * 0.021 * f$

$E_{\mu V/m} = \mu V * 0.021 * 139.25$

$E_{\mu V/m} = 4 * 2.92$

$E_{\mu V/m} = 11.7$

Answer: The μV/m equivalent of 4 μV at 139.25 MHz is 11.7 μV/m.

19.5. Convert microvolt per meter (μV/m) to microvolt (μV)

$$\mu V = \frac{E_{\mu V/m}}{0.021 * f}$$

where
μV is RF signal level in microvolt at the terminals of a resonant half-wave dipole
$E_{\mu V/m}$ is field strength in microvolt per meter
f is frequency in megahertz

Example:
What is the voltage equivalent of 11.7 μV/m at 139.25 MHz?

Solution:
$\mu V = \frac{E_{\mu V/m}}{0.021 * f}$

$\mu V = \frac{11.7}{0.021 * 139.25}$

$$\mu V = \frac{11.7}{2.92}$$

$$\mu V = 4$$

Answer: The voltage equivalent of 11.7 µV/m at 139.25 MHz is 4 µV.

19.6. Convert microvolt per meter (µV/m) to decibel millivolt (dBmV)

$$dBmV = 20log\left[\frac{\left(\frac{E_{\mu V/m}}{0.021 * f}\right)}{1{,}000}\right]$$

where
$dBmV$ is RF signal level in decibel millivolt at the terminals of a resonant half-wave dipole antenna
$E_{\mu V/m}$ is field strength in microvolt per meter
f is frequency in megahertz

Example:
What is the power, in dBmV, delivered to the terminals of a resonant half-wave dipole antenna by a 139.25 MHz leak whose field strength is 20 µV/m at the point of measurement?

Solution:

$$dBmV = 20log_{10}\left[\frac{\left(\frac{E_{\mu V/m}}{0.021 * f}\right)}{1{,}000}\right]$$

$$dBmV = 20 * log_{10}\left[\frac{\left(\frac{20}{0.021 * 139.25}\right)}{1{,}000}\right]$$

$$dBmV = 20 * log_{10}\left[\frac{\left(\frac{20}{2.92}\right)}{1{,}000}\right]$$

$$dBmV = 20 * log_{10}\left[\frac{(6.84)}{1{,}000}\right]$$

$$dBmV = 20 * log_{10}[0.006839]$$

$$dBmV = 20 * [-2.16]$$

$$dBmV = -43.30$$

Answer: The power delivered to the terminals of the dipole antenna is –43.3 dBmV.

19.7. Convert decibel millivolt (dBmV) to microvolt per meter (µV/m)

$$E_{\mu V/m} = 21 * f * 10^{\frac{dBmV}{20}}$$

where
$E_{\mu V/m}$ is field strength in microvolt per meter
f is frequency in megahertz
$dBmV$ is RF signal level in decibel millivolt at the terminals of a resonant half-wave dipole antenna

Example:
What is the field strength in microvolts per meter when the power delivered to the terminals of a resonant half-wave dipole is –48 dBmV at 139.25 MHz?

Solution:

$$E_{\mu V/m} = 21 * f * 10^{\frac{dBmV}{20}}$$

$$E_{\mu V/m} = 21 * 139.25 * 10^{\frac{dBmV}{20}}$$

$$E_{\mu V/m} = 21 * 139.25 * 10^{\frac{-48}{20}}$$

$$E_{\mu V/m} = 21 * 139.25 * 0.004$$

$$E_{\mu V/m} = 11.7$$

Answer: The field strength is 11.7 µV/m.

19.8. Using a link budget to calculate received signal power at a resonant half-wave dipole antenna's terminals

$$P_{receive} = transmit\ power(dBm) - transmit\ feedline\ loss(dB) + transmit\ antenna\ gain(dBi) - free\ space\ path\ loss(dB) + receive\ antenna\ gain(dBi)$$

where
$P_{receive}$ is the RF power in decibel milliwatt (dBm) at the terminals of a receive antenna
$transmit\ power(dBm)$ is the transmitter's output power in decibel milliwatt
$transmit\ feedline\ loss(dB)$ is the attenuation in decibels of the feedline between the transmitter and its antenna (if a filter is used between the transmitter and antenna, its loss in decibels should be added to the feedline loss)
$transmit\ antenna\ gain(dBi)$ is the transmitter's antenna gain in decibel isotropic
$free\ space\ path\ loss(dB)$ is the free space path loss in decibels between the transmit antenna and receive antenna
$receive\ antenna\ gain(dBi)$ is the receiver's antenna gain in decibel isotropic (2.15 dBi for a resonant half-wave dipole)

Example:
What is the received power at the terminals of a resonant half-wave dipole given the following:

- Transmit power = 46 dBm @ 752 MHz
- Transmit feedline loss = 2 dB
- Transmit antenna gain = 18 dBi
- Free-space path loss = 108 dB
- Receive antenna gain = 2.15 dBi

Solution:
$P_{receive}$ = transmit power (dBm) – transmit feedline loss (dB) + transmit antenna gain (dBi) – free space path loss (dB) + receive antenna gain (dBi)

$P_{receive}$ = 46 – 2 + 18 – 108 + 2.15

$P_{receive}$ = –43.85

Answer: The received power at the dipole terminals is –43.85 dBm.

19.9. Convert microvolt per meter (µV/m) to decibel microvolt per meter (dBµV/m)

$$dB\mu V/m = 20log_{10}\left(E_{\mu V/m}\right)$$

where
$dB\mu V/m$ is field strength in decibel microvolt per meter
log_{10} is base 10 logarithm
$E_{\mu V/m}$ is field strength in microvolt per meter

Example:
What is the dBµV/m equivalent of 50 µV/m?

Solution:
$dB\mu V/m = 20log_{10}\left(E_{\mu V/m}\right)$

$dB\mu V/m = 20 * log_{10}(50)$

$dB\mu V/m = 20 * 1.70\, dB\mu V/m = 34$

Answer: The dBµV/m equivalent of 50 µV/m is 34 dBµV/m.

19.10. Convert decibel microvolt per meter (dBµV/m) to microvolt per meter (µV/m)

$$E_{\mu V/m} = 10^{\frac{dB\mu V/m}{20}}$$

where
$E_{\mu V/m}$ is field strength in microvolt per meter
$dB\mu V/m$ is field strength in decibel microvolt per meter

Example:
What is the µV/m equivalent of 34 dBµV/m?

Solution:
$E_{\mu V/m} = 10^{\frac{dB\mu V/m}{20}}$

$E_{\mu V/m} = 10^{\frac{34}{20}}$

$E_{\mu V/m} = 10^{1.70}$

$E_{\mu V/m} = 50.12$

Answer: The µV/m equivalent of 34 dBµV/m is 50.12 µV/m.

19.11. Convert leakage field strength at 30 meters measurement distance to an equivalent field strength at 3 meters measurement distance

$$E_{\mu V/m}\ at\ 3\ meters = E_{\mu V/m}\ at\ 30\ meters * \left(\frac{30}{3}\right)$$

where
$E_{\mu V/m}\ at\ 3\ meters$ is field strength in microvolt per meter at a 3 meters measurement distance
$E_{\mu V/m}\ at\ 30\ meters$ is field strength in microvolt per meter at a 30 meters measurement distance

Example:
What would be the equivalent field strength at 3 meters given a measured value of 15 µV/m at 30 meters?

Solution:

$$E_{\mu V/m}\ at\ 3\ meters = E_{\mu V/m}\ at\ 30\ meters * \left(\frac{30}{3}\right)$$

$$E_{\mu V/m}\ at\ 3\ meters = 15 * \left(\frac{30}{3}\right)$$

$$E_{\mu V/m}\ at\ 3\ meters = 150$$

Answer: The equivalent field strength at 3 meters is 150 µV/m.

19.12. Convert leakage field strength at 3 meters measurement distance to an equivalent field strength at 30 meters measurement distance

$$E_{\mu V/m}\ at\ 30\ meters = E_{\mu V/m}\ at\ 3\ meters * \left(\frac{3}{30}\right)$$

where
$E_{\mu V/m}\ at\ 30\ meters$ is field strength in microvolt per meter at a 30 meters measurement distance
$E_{\mu V/m}\ at\ 3\ meters$ is field strength in microvolt per meter at a 3 meters measurement distance

Example:
What is the equivalent field strength at 30 meters given a measured value of 150 µV/m at 3 meters?

Solution:

$$E_{\mu V/m}\ at\ 30\ meters = E_{\mu V/m}\ at\ 3\ meters * \left(\frac{3}{30}\right)$$

$E_{\mu V/m}\ at\ 30\ meters = 150 * \left(\frac{3}{30}\right)$

$E_{\mu V/m}\ at\ 30\ meters = \ 15$

Answer: The equivalent field strength at 30 meters is 15 µV/m.

19.13. Calculate leakage field strength difference in decibels at new measurement distance versus reference measurement distance

$$C_{dB} = 20log_{10}\left(\frac{d_{new}}{d_{ref}}\right)$$

where
C_{dB} is the correction factor in decibels
log_{10} is base 10 logarithm
d_{new} is the new measurement distance
d_{ref} is the reference measurement distance (e.g., 3 meters)

Example:
What is the difference in decibels between a field strength of 15 µV/m measured at 30 meters and a field strength of 150 µV/m measured at 3 meters?

Solution:

$C_{dB} = 20log_{10}\left(\frac{d_{new}}{d_{ref}}\right)$

$C_{dB} = 20 * log_{10}\left(\frac{30}{3}\right)$

$C_{dB} = 20$

Answer: The difference is 20 dB.

19.14. Calculate leakage field strength difference in decibels between two values of the same signal measured at different distances from the source

$$C_{dB} = 20log_{10}\left(\frac{\mu V/m_{new}}{\mu V/m_{ref}}\right)$$

where
C_{dB} is the correction factor in decibels
log_{10} is base 10 logarithm
$\mu V/m_{new}$ is the new measured value
$\mu V/m_{ref}$ is the measured reference value

Example:
What is the difference in decibels between a field strength of 150 µV/m measured at 3 meters and field strength of 15 µV/m measured at 30 meters?

Solution:

$$C_{dB} = 20log_{10}\left(\frac{\mu V/m_{new}}{\mu V/m_{ref}}\right)$$

$$C_{dB} = 20 * log_{10}\left(\frac{150}{15}\right)$$

$$C_{dB} = 20$$

Answer: The difference is 20 dB.

19.15. Convert the FCC's 25 kHz and 30 kHz bandwidths to a wider equivalent bandwidth

$$\Delta dB = 10log_{10}\left(\frac{BW_{new}}{BW_{FCC}}\right)$$

where
ΔdB is the correction factor in decibels to add to the FCC's power threshold
log_{10} is base 10 logarithm
BW_{new} is the new bandwidth (e.g., 6 MHz), expressed in Hz
BW_{FCC} is the bandwidth used in Part 76 of the FCC Rules (i.e., 25 kHz or 30 kHz), expressed in Hz

The values in the following table assume 75 ohms impedance:

Table 9 - Power conversion

Power (exp.)	Power (µW)	Power (dBmV)
10^{-4} watt	100	38.750613
–	75.85	37.550168
10^{-5} watt	10	28.750613

Example 1:
What is the FCC's 100 microwatts (µW) power threshold across a 25 kHz bandwidth [ref. §76.610] in a 6 MHz equivalent bandwidth?

Solution 1:

$$\Delta dB = 10log_{10}\left(\frac{BW_{new}}{BW_{FCC}}\right)$$

$$\Delta dB = 10 * log_{10}\left(\frac{6000000}{25000}\right)$$

$$\Delta dB = 23.80$$

From the table, 100 µW is 38.75 dBmV. Add 23.80 dB to 38.75 dBmV. The answer is 62.55 dBmV.

Example 2:
What is the FCC's 10^{-5} watt power threshold across a 30 kHz bandwidth [ref. §76.616(b)] in a 6 MHz equivalent bandwidth?

Solution 2:

$$\Delta dB = 10log_{10}\left(\frac{BW_{new}}{BW_{FCC}}\right)$$

$$\Delta dB = 10 * log_{10}\left(\frac{6000000}{30000}\right)$$

$$\Delta dB = 23.01$$

From the table, 10^{-5} watt is 28.75 dBmV. Add 23.01 dB to 28.75 dBmV. The answer is 51.76 dBmV.

19.16. Calculate cumulative leakage index

The FCC Rules on signal leakage require that cable systems demonstrate compliance with the signal leakage performance criteria associated with either a drive-out or a flyover test method.[35] This section covers how to determine whether or not a cable network has met the signal leakage performance criteria associated with the drive-out test method.

With respect to the drive-out test method, §76.611(a)(1) of the FCC Rules states that "[P]rior to carriage of signals in the aeronautical radio bands and at least once each calendar year, with no more than 12 months between successive tests thereafter, based on a sampling of at least 75% of the cable strand, and

[35] See Title 47 of the Code of Federal Regulations, Part 76, §76.611(a).

including any portion of the cable system which are known to have or can reasonably be expected to have less leakage integrity than the average of the system, the cable operator demonstrates compliance with a cumulative signal leakage index by showing that 10 log I∞ is equal to or less than 64..."[36]

The following formula can be used to calculate the cumulative leakage index:[37]

$$I_\infty = \frac{1}{\theta}\sum_{i=1}^{n} E_i^2$$

where
I_∞ is the value used to compute the cumulative leakage index by showing that $10\log_{10}(I_\infty)$ is equal to or less than 64.
θ is the fraction of the system cable length actually examined for leakage sources and is equal to the strand miles of plant tested divided by the total strand miles in the plant
E_i is the electric field strength in microvolts per meter (µV/m) measured 3 meters from the leak i
n is the number of leaks found of field strength equal to or greater than 50 µV/m

More simply stated:

$$CLI(I_\infty) = 10log_{10}\left[\left(\frac{total\ plant\ miles}{plant\ miles\ tested}\right) * \ (sum\ of\ each\ leak^2)\right]$$

where
$CLI(I_\infty)$ is cumulative leakage index (I_∞, sometimes called the "I sub infinity method")
log_{10} is base 10 logarithm

Example:
A small cable system has 500 total plant miles. Field technicians have driven out 475 miles testing for signal leakage and have collected the following list of leaks measuring 50 µV/m or more:

Table 10 - Signal leakage data used for this example.

Number of signal leaks	Leakage level (µV/m)
20	50
15	75
10	100
5	500

Calculate the cumulative leakage index for this system based on this list of measured leaks.

[36] As of November 2020.
[37] Cumulative leakage index (CLI) is a commonly misused term. CLI is not the same as signal leakage. CLI is a figure of merit that provides a snapshot of the magnitude of a cable system's overall signal leakage. It is not possible to measure or test CLI; one must measure signal leakage in order to calculate CLI.

Solution:

$$CLI_{\infty} = 10log_{10}\left[\left(\frac{total\ plant\ miles}{plant\ miles\ tested}\right) * \ (sum\ of\ each\ leak^2)\right]$$

$$CLI_{\infty} = 10 * log_{10}\left[\left(\frac{500}{475}\right) * ([20 * 50^2] + [15 * 75^2] + [10 * 100^2] + [5 * 500^2])\right]$$

$$CLI_{\infty} = 10 * log_{10}\,[(1.05263158) * ([20 * 2500] + [15 * 5625] + [10 * 10000] + [5 * 250000])]$$

$$CLI_{\infty} = 10 * log_{10}\,[(1.05263158) * ([50000] + [84375] + [100000] + [1250000])]$$

$$CLI_{\infty} = 10 * log_{10}\,[(1.05263158) * (1484375)]$$

$$CLI_{\infty} = 10 * log_{10}\,(1562500)$$

$$CLI_{\infty} = 10 * (6.19382003)$$

$$CLI_{\infty} = 61.94$$

Answer: The cumulative leakage index for this system is 61.94.

Note: The FCC provides an online cumulative leakage index calculator at
https://www.fcc.gov/media/cumulative-leakage-index-calculator

20. Coaxial Cable

20.1. Characteristic impedance

The characteristic impedance, Z_0, of coaxial cable is expressed in ohms, and is related to the outside diameter of the inner or center conductor, the inside diameter of the outer conductor or shield, and the dielectric constant[38] (relative permittivity) of the insulating material (dielectric) separating the two conductors.

$$Z_0 = \frac{138}{\sqrt{\varepsilon}} log_{10}\left(\frac{D}{d}\right)$$

where
Z_0 is the cable's characteristic impedance in ohms
ε is the dielectric constant
log_{10} is base 10 logarithm
D is the inside diameter of the outer conductor or shield
d is the outside diameter of the inner or center conductor

Example:
What is the characteristic impedance of coaxial cable whose center conductor diameter *d* is 0.109 inch, inside diameter of the shield *D* is 0.452 inch, and has a dielectric constant *ε* of 1.32?

Solution:

$$Z_0 = \frac{138}{\sqrt{1.32}} log_{10}\left(\frac{0.452}{0.109}\right)$$

$$Z_0 = \frac{138}{1.149} * log_{10}(4.147)$$

$$Z_0 = 120.114 * 0.618$$

$$Z_0 = 74.2$$

Answer: The characteristic impedance of the coaxial cable is 74.2 ohms.

Note: Most coaxial cable manufacturers specify a nominal value for characteristic impedance, such as 75 ohms ±2 ohms. In this example, the calculated characteristic impedance is within the tolerance common for nominal values.

[38] Dielectric constant is generally not specified by coaxial cable manufacturers, but can be derived from the cable's published velocity factor. See Section 20.4.

20.2. Skin depth

Skin depth, denoted by the symbol δ, is a measure of skin effect[39] and is the depth at which the current density is 1/*e* of the current density at the surface of the conductor. Note: "*e*" is the mathematical constant that is the base of the natural logarithm ("LN" on some scientific calculators) and is equal to about 2.718, so 1/*e* ≈ 37%.

Figure 29 illustrates skin depth δ in a metallic conductor, the depth at which current density is approximately 37% of the value at the surface. RF current doesn't stop at the dashed line, but decreases logarithmically with respect to depth in the conductor.

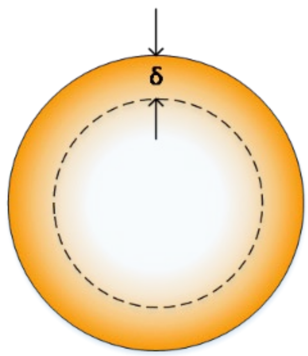


Figure 29 - Concept of skin depth (not to scale).

Skin depth can be calculated using the following formula.

$$\delta = \sqrt{\frac{2\rho}{(2\pi f)(\mu_0 \mu_r)}}$$

where
δ is skin depth in meters
ρ is resistivity of the conductor in ohm-meter (Ω·m), $1.678 * 10^{-8}$ for copper at room temperature[40]
f is frequency in hertz
μ_0 is the permittivity of free space ($4\pi * 10^{-7}$)
μ_r is relative permittivity of the conductor (0.999991 for copper)

Example 1:
What is the skin depth in a copper conductor for a 60 Hz alternating current?

Solution 1:

[39] For alternating current (AC) applications – which includes RF – conduction of current is largely confined to a region at and near the surface of the conductor. The higher the frequency, the shallower the region (and closer to the surface) in which the current is conducted. This phenomenon is known as skin effect.
[40] The resistivity for copper at 20 °C (68 ˚F) is about $1.68 * 10^{-8}$ Ω·m, and the resistivity for annealed copper is about $1.72 * 10^{-8}$ Ω·m. For this example, $1.68 * 10^{-8}$ Ω·m is being used.

$$\delta = \sqrt{\frac{2 * (1.678 * 10^{-8})}{(2 * \pi * 60)(4\pi * 10^{-7})(0.999991)}}$$

$$\delta = \sqrt{\frac{3.356 * 10^{-8}}{(376.991)(1.257 * 10^{-6})(0.999991)}}$$

$$\delta = \sqrt{\frac{3.356 * 10^{-8}}{4.737 * 10^{-4}}}$$

$$\delta = \sqrt{7.082 * 10^{-5}}$$

$$\delta = 0.00841$$

Answer: The skin depth in a copper conductor for 60 Hz AC is 0.00841 meter, or 8.41 millimeters (about 1/3 inch).

Example 2:
What is the skin depth in a copper conductor for a 100 MHz RF signal?

Solution 2:

$$\delta = \sqrt{\frac{2 * (1.678 * 10^{-8})}{(2 * \pi * 100{,}000{,}000)(4\pi * 10^{-7})(0.999991)}}$$

$$\delta = \sqrt{\frac{3.356 * 10^{-8}}{(628{,}318{,}530.718)(1.257 * 10^{-6})(0.999991)}}$$

$$\delta = \sqrt{\frac{3.356 * 10^{-8}}{789.789}}$$

$$\delta = \sqrt{4.249 * 10^{-11}}$$

$$\delta = 6.519 * 10^{-6}$$

Answer: The skin depth in a copper conductor for a 100 MHz RF signal is 6.519 micrometers (µm), or about 0.00025 inch.

20.3. Coaxial cable attenuation

20.3.1. Attenuation versus frequency

Coaxial cable attenuation can be calculated using the following formula:[41]

$$\alpha = \left[\frac{3296}{Z} * \left(\frac{\sqrt{P_i}}{d * k_s} + \frac{\sqrt{P_o}}{D}\right)\right] * \sqrt{f} + \left(\frac{0.884 * \pi * \sigma}{VF}\right) * f$$

where
α is attenuation in decibels per 100 feet
Z is nominal impedance in ohms
P_i is resistivity of the inner conductor in Ω·m
P_o is resistivity of the outer conductor in Ω·m
d is the outside diameter of the inner (center) conductor
D is the inside diameter of the outer conductor (shield)
k_s is a stranding factor (1.0 for a solid shield, and a slightly smaller value for braided drop cable)
f is frequency in megahertz
σ is dissipation factor for the dielectric
VF is the cable's velocity factor

Example:
What is the attenuation of 0.500 hardline coaxial cable at 1 GHz?

Solution:
Assume the following values for the formula's terms.

Z = 75 ohms
$P_i = 1.7241 * 10^{-8}$ Ω·m
$P_o = 2.828 * 10^{-8}$ Ω·m
d = 0.109 inch
D = 0.450 inch
k_s = 1.0
f = 1,000 MHz
π = 3.1416
$\sigma = 7 * 10^{-5}$
VF = 0.8825

$$\alpha = \left[\frac{3296}{75} * \left(\frac{\sqrt{1.7241 * 10^{-8}}}{0.109 * 1.0} + \frac{\sqrt{2.828 * 10^{-8}}}{0.450}\right)\right] * \sqrt{1000} + \left(\frac{0.884 * 3.1416 * (7 * 10^{-5})}{0.8825}\right) * 1000$$

$$\alpha = \left[43.947 * \left(\frac{1.313 * 10^{-4}}{0.109} + \frac{1.682 * 10^{-4}}{0.450}\right)\right] * 31.623 + \left(\frac{1.944 * 10^{-4}}{0.8825}\right) * 1000$$

---

[41] The coaxial attenuation-versus-frequency formula and assumptions used in the example are courtesy of Amphenol Broadband Solutions.

$\alpha = [43.947 * (0.0012 + (3.737 * 10^{-4}))] * 31.623 + (2.203 * 10^{-4}) * 1000$

$\alpha = [43.947 * (0.001578)] * 31.623 + (0.2203)$

$\alpha = 0.069 * 31.623 + (0.2203)$

$\alpha = 2.193 + (0.2203)$

$\alpha = 2.4137$

Answer: The cable's attenuation at 1 GHz is about 2.41 dB/100 ft.

Note: Manufacturers sometimes specify typical and maximum attenuation values, and sometimes just maximum values. The calculated value is closer to what can be considered a typical value. Maximum values are generally a few percent higher than the typical values. For instance, adding 4% to 5% to the calculated value here gives 2.51 dB to 2.53 dB, which are closer to published maximum values for 0.500 hardline coax.

20.3.2. Cable loss ratio (square root of frequency method)

The ratio of coaxial cable attenuation, in decibels, at two frequencies is approximately equal to the square root of the ratio of the two frequencies. From this, one can calculate the approximate loss at one frequency when the loss at another frequency is known, using the following formulas:

$$L_H \cong L_L * \sqrt{\left(\frac{f_H}{f_L}\right)}$$

where
L_H is the approximate loss in decibels at frequency f_H
L_L is the known loss in decibels at frequency f_L
f_H is the high frequency of interest
f_L is the low frequency of interest

$$L_L \cong L_H * \sqrt{\left(\frac{f_L}{f_H}\right)}$$

where
L_L is the approximate loss in decibels at frequency f_L
L_H is the known loss in decibels at frequency f_H
f_L is the low frequency of interest
f_H is the high frequency of interest

Example 1:
If a 100 ft. length of widely used 0.500 hardline coax has a published loss of 1.82 dB at 550 MHz, what is the approximate loss at 1002 MHz?

Solution 1:

$$L_H \cong 1.82 * \sqrt{\left(\frac{1002}{550}\right)}$$

$$L_H \cong 1.82 * \sqrt{(1.822)}$$

$$L_H \cong 1.82 * 1.350$$

$$L_H \cong 2.457$$

Answer: The approximate loss at 1002 MHz is 2.46 dB.

Note: The published loss for the example 0.500 hardline coax at 1002 MHz is 2.54 dB, showing that this approach can provide a reasonable approximation of the loss at the higher frequency.

Example 2:
If a 100 ft. length of widely used 0.500 hardline coax has a published loss of 2.54 dB at 1002 MHz, what is the approximate loss at 550 MHz?

Solution 2:

$$L_L \cong 2.54 * \sqrt{\left(\frac{550}{1002}\right)}$$

$$L_H \cong 2.54 * \sqrt{(0.549)}$$

$$L_H \cong 2.54 * 0.741$$

$$L_H \cong 1.882$$

Answer: The approximate loss at 550 MHz is 1.88 dB.

Note: The published loss for the example 0.500 hardline coax at 550 MHz is 1.82 dB, showing that this approach can provide a reasonable approximation of the loss at the lower frequency.

20.3.3. Convert cable tilt to cable loss

The following formula can be used to calculate the approximate cable loss at a higher frequency f_2 when the cable's attenuation-related tilt between a lower frequency f_1 and the higher frequency f_2 is known.

$$loss_{f_2} \cong \frac{tilt}{1-\sqrt{\frac{f_1}{f_2}}}$$

where
$loss_{f_2}$ is the cable's approximate loss at a higher frequency f_2
$tilt$ is the cable's attenuation-related tilt in decibels between frequencies f_1 and f_2
f_1 is the lower frequency
f_2 is the higher frequency

Example 1:
What is the loss in a length of feeder cable at 750 MHz (f_2) when the tilt is 15 dB between 55 MHz (f_1) and 750 MHz? Refer to Figure 30.

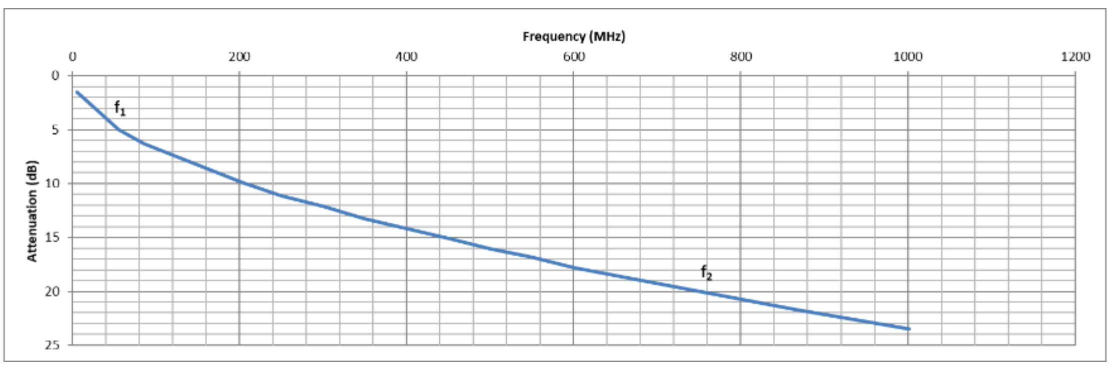


Figure 30. Attenuation vs. frequency for a length of 0.500 feeder cable.

Solution:

$$loss_{f_2} \cong \frac{tilt}{1-\sqrt{\frac{f_1}{f_2}}}$$

$$loss_{f_2} \cong \frac{15}{1-\sqrt{\frac{55}{750}}}$$

$$loss_{f_2} \cong \frac{15}{1-\sqrt{0.0733}}$$

$$loss_{f_2} \cong \frac{15}{1-0.2708}$$

$$loss_{f_2} \cong \frac{15}{0.7292}$$

$$loss_{f_2} \cong 20.57$$

Answer: The approximate loss at 750 MHz (f_2) is 20.57 dB.

20.3.4. Attenuation versus temperature

Coaxial cable attenuation in decibels changes about 1% for every 10 °F ambient temperature change. As the temperature increases, cable attenuation increases; as the temperature decreases, cable attenuation decreases. Both downstream and upstream RF signal levels are affected by temperature-related cable attenuation variations.

If one knows the coaxial cable's attenuation at the manufacturer's reference temperature, the attenuation at a different temperature can be calculated using the following formulas:

$$A(°\mathrm{F})_{new} = A_{ref} * \left(1 + \left[0.0011 * \left(T_{new} - T_{ref}\right)\right]\right)$$

where
$A(°\mathrm{F})_{new}$ is attenuation in decibels at the new temperature in degrees Fahrenheit
A_{ref} is attenuation in decibels at the cable manufacturer's reference temperature (typ. 68 ˚F)
T_{new} is the new temperature
T_{ref} is the cable manufacturer's reference temperature (typ. 68 ˚F)

$$A(°\mathrm{C})_{new} = A_{ref} * \left(1 + \left[0.002 * \left(T_{new} - T_{ref}\right)\right]\right)$$

where
$A(°\mathrm{C})_{new}$ is attenuation in decibels at the new temperature in degrees Celsius
A_{ref} is attenuation in decibels at the cable manufacturer's reference temperature (typ. 20 ˚C)
T_{new} is the new temperature
T_{ref} is the cable manufacturer's reference temperature (typ. 20 ˚C)

Example 1:
Assume a coaxial cable manufacturer's published attenuation at 870 MHz for a length of express feeder is 15 dB at 68 ˚F. What is the attenuation at 870 MHz of that express feeder at –10 ˚F?

Solution 1:

$$A(°\mathrm{F})_{new} = A_{ref} * \left(1 + \left[0.0011 * \left(T_{new} - T_{ref}\right)\right]\right)$$

$$A(°\mathrm{F})_{new} = 15 * (1 + [0.0011 * (-10 - 68)])$$

$$A(°\mathrm{F})_{new} = 15 * (1 + [0.0011 * (-78)])$$

$$A(°\mathrm{F})_{new} = 15 * (1 + [-0.0858])$$

$A(°F)_{new} = 15 * (0.9142)$

$A(°F)_{new} = 13.71$

Answer: The attenuation at 870 MHz of the span of express feeder cable at –10 ˚F is 13.71 dB.

Example 2:
Assume a coaxial cable manufacturer's published attenuation at 1002 MHz for a length of express feeder is 16.1 dB at 20 ˚C. What is the attenuation at 1002 MHz of that express feeder at 40 ˚C?

Solution 2:
$A(°C)_{new} = A_{ref} * \left(1 + \left[0.002 * \left(T_{new} - T_{ref}\right)\right]\right)$

$A(°C)_{new} = 16.1 * (1 + [0.002 * (40 - 20)])$

$A(°C)_{new} = 16.1 * (1 + [0.002 * (20)])$

$A(°C)_{new} = 16.1 * (1 + [0.0400])$

$A(°C)_{new} = 16.1 * (1.0400)$

$A(°C)_{new} = 16.74$

Answer: The attenuation at 1002 MHz of the length of express feeder cable at 40 ˚C is 16.74 dB.

20.4. Coaxial cable velocity factor and velocity of propagation

Velocity factor is the ratio – in decimal form – of the speed of electromagnetic signals propagating through coaxial cable to the speed of light in a vacuum.

Velocity factor is calculated with the formula

$$VF = \frac{1}{\sqrt{\varepsilon}}$$

where
VF is the velocity factor
ε is the dielectric constant

Dielectric constant is calculated with the formula

$$\varepsilon = \frac{1}{VF^2}$$

Example 1:
What is the velocity factor for coaxial cable with a dielectric constant of 1.32?

Solution 1:

$$VF = \frac{1}{\sqrt{1.32}}$$

$$VF = \frac{1}{1.149}$$

$$VF = 0.87$$

Example 2:
What is the dielectric constant for coaxial cable with a velocity factor of 0.92?

Solution 2:

$$\varepsilon = \frac{1}{0.92^2}$$

$$\varepsilon = \frac{1}{0.846}$$

$$\varepsilon = 1.181$$

Velocity of propagation is velocity factor expressed as a percentage:

$$VoP = VF * 100$$

where
VoP is velocity of propagation
VF is the velocity factor

Example:
What is the velocity of propagation for coaxial cable with a velocity factor of 0.87?

Solution:
$VoP = 0.87 * 100$
$VoP = 87\%$

20.5. Wavelength in coaxial cable

The wavelength in feet in coaxial cable can be calculated using the following formulas:

$$\lambda_{feet} = \frac{(983.571 * VF)}{f}$$

where
λ_{feet} is wavelength in feet
VF is velocity factor
f is frequency in MHz

$$\lambda_{feet} = \frac{983.571}{f * \sqrt{\varepsilon}}$$

where
λ_{feet} is wavelength in feet
f is frequency in MHz
ε is the cable's dielectric constant

Example:
What is the wavelength of a 100 MHz signal in coaxial cable with a velocity factor of 0.87?

Solution:

$$\lambda_{feet} = \frac{(983.571 * 0.87)}{100}$$

$$\lambda_{feet} = \frac{(855.707)}{100}$$

$$\lambda_{feet} = 8.557$$

Answer: One wavelength is 8.557 feet.

20.6. Coaxial cable TE mode cutoff frequency

The desired mode of propagation in coaxial cable is known as the transverse electromagnetic (TEM) mode. Higher modes such as transverse electric (TE) are undesirable, in part because the electric and magnetic fields are non-uniform, and the interactions between the fundamental TEM mode and higher modes can cause unwanted problems. The first higher order mode, called TE_{11}, can propagate above the TE mode cutoff frequency f_c. Ideally, the maximum operating frequency in coaxial cable should not exceed f_c, such that only TEM mode is supported. The following formula can be used to calculate f_c for TE_{11} mode in coaxial cable.

$$f_c = \frac{11.8}{\sqrt{\varepsilon} * \pi * \left(\frac{D + d}{2}\right)}$$

where
f_c is the TE_{11} mode cutoff frequency in gigahertz (GHz)
ε is the cable's dielectric constant
D is the inner diameter of the cable's shield, in inches
d is the outer diameter of the cable's center conductor, in inches

Example:
What is f_c for .500 hardline coaxial cable, assuming the center conductor diameter is 0.109 inch, the inner diameter of the shield is 0.452 inch, and the dielectric constant is 1.32?

Solution:

$$f_c = \frac{11.8}{\sqrt{\varepsilon} * \pi * \left(\frac{D + d}{2}\right)}$$

$$f_c = \frac{11.8}{\sqrt{1.32} * \pi * \left(\frac{0.452 + 0.109}{2}\right)}$$

$$f_c = \frac{11.8}{1.1489 * \pi * \left(\frac{0.5610}{2}\right)}$$

$$f_c = \frac{11.8}{1.1489 * \pi * (0.2805)}$$

$$f_c = \frac{11.8}{1.0124}$$

$$f_c = 11.6551$$

Answer: The TE_{11} mode cutoff frequency f_c for .500 hardline coax is about 11.66 GHz.

Note: The calculated f_c for .750 hardline coaxial cable (D = 0.680 inch, d = 0.167 inch, and ε = 1.32) is about 7.7 GHz, and f_c for Series 6 drop cable (D = 0.18 inch, d = 0.04 inch, and ε = 1.38) is about 29.06 GHz.

20.7. Equalizer loss

Equalizers are passive circuits used to compensate for the attenuation characteristics of coaxial cable at different frequencies. The following formula can be used to calculate the loss of an equalizer at any frequency:

$$Loss\ at\ f_1 = Eq - \left[\left(Eq\sqrt{\frac{f_1}{f_2}}\right) - 1\right]$$

where
$Loss\ at\ f_1$ is the equalizer's loss at frequency f_1
Eq is the equalizer value in decibels at frequency f_2

Example:
What is the loss in decibels at 54 MHz (f_1) of an equalizer for 20 dB of cable at 750 MHz (f_2)?

Solution:

$$Loss\ at\ f_1 = Eq - \left[\left(Eq\sqrt{\frac{f_1}{f_2}}\right) - 1\right]$$

$$Loss\ at\ f_1 = 20 - \left[\left(20\sqrt{\frac{54}{750}}\right) - 1\right]$$

$$Loss\ at\ f_1 = 20 - \left[\left(20 * \sqrt{0.0720}\right) - 1\right]$$

$$Loss\ at\ f_1 = 20 - [(20 * 0.2683) - 1]$$

$$Loss\ at\ f_1 = 20 - [(5.3666) - 1]$$

$$Loss\ at\ f_1 = 20 - [4.3666]$$

$$Loss\ at\ f_1 = 15.63$$

Answer: The equalizer's loss at 54 MHz is 15.63 dB.

20.8. Calculate RF signal levels in coax plant

The cable network design process is a complex subject that involves many facets, and a detailed overview is beyond the scope of this Operational Practice. This section includes simplified examples of downstream and upstream RF signal level calculations in the coaxial cable portion of an HFC network (refer to Section 25.8 for an example fiber optic link loss budget analysis). Note: Modern cable network designs generally use conditioned taps, that is, taps that support plug-in reverse attenuators, forward equalizers, and forward inverse equalizers. Those plug-ins allow for optimization of tap output levels, and manage reverse path tap losses to help achieve a narrow window of cable modem transmit levels. The examples here do NOT use conditioned taps.

The power of the decibel allows the math to be mostly addition and subtraction.

In the example shown in Figure 31, the task is to calculate 1) the downstream RF signal levels at the cable modem input, and 2) the cable modem's upstream RF transmit level, given the provided parameters and specifications.

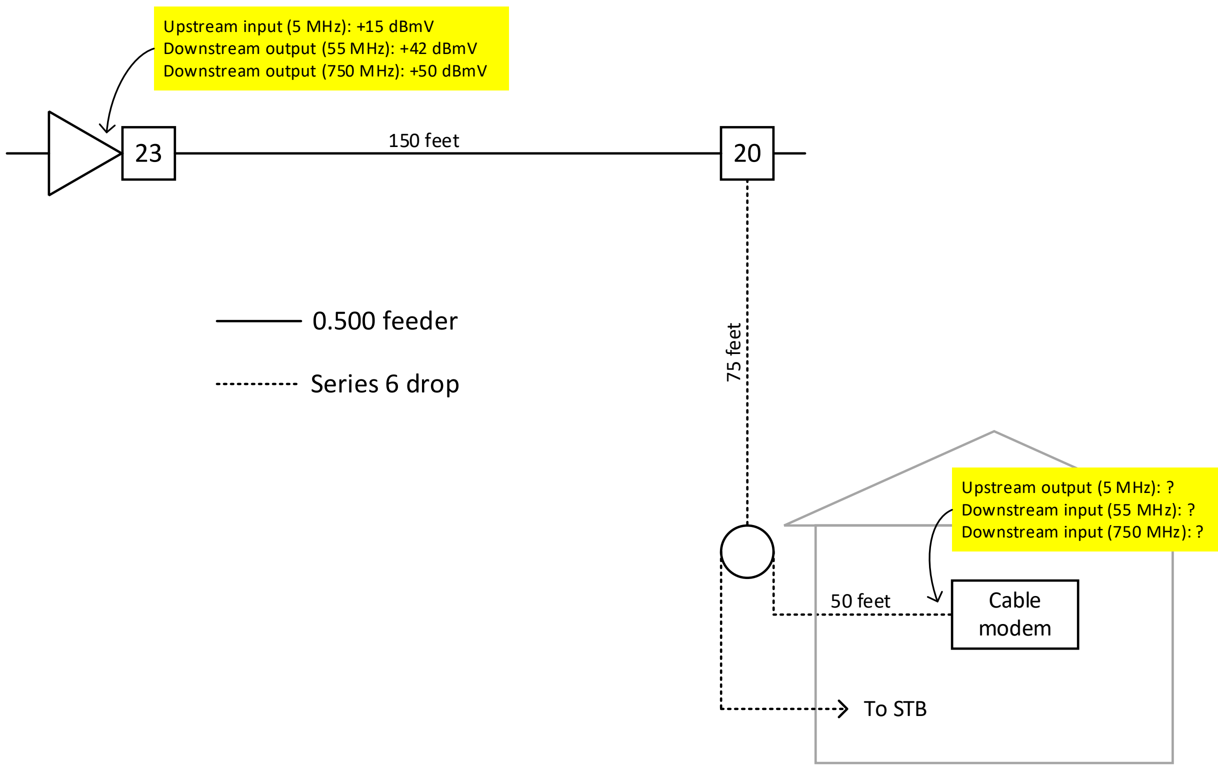


Figure 31 - Cable network details for examples in this section.

20.8.1. Cable and passive device specifications

Assume the following specifications for the half-inch hardline feeder coaxial cable, Series 6 subscriber drop coaxial cable, distribution passives (taps), and the two-way splitter used at the subscriber premises.

Table 11 - Coaxial cable attenuation specifications (at 20 ˚C or 68 ˚F)

Frequency (MHz)	0.500 feeder cable attenuation (dB/100 ft)	Series 6 drop cable attenuation (dB/100 ft)
5	0.16	0.58
55	0.54	1.60
750	2.16	5.65

Table 12 - Tap specifications

Frequency (MHz)	23 dB tap insertion loss (dB)	20 dB tap insertion loss (dB)*
5	0.16	0.58
55	0.54	1.60
750	2.16	5.65

*Note to table: The 20 dB tap's insertion (through) loss values are not used in the accompanying examples, but are provided for reference.

Table 13 - Drop splitter specifications

Frequency (MHz)	2-way splitter insertion loss (dB)
5	3.6
55	3.6
750	4.5

20.8.2. Calculate coaxial cable attenuation

The first step is to calculate the attenuation of the three lengths of coaxial cable at the three frequencies of interest (5 MHz, 55 MHz, and 750 MHz), which can be done using the following formula:

$$loss_{dB} = \left(\frac{dB_{spec}}{100}\right) * length_{ft}$$

where
$loss_{dB}$ is the attenuation at the frequency of interest in the given length of coaxial cable, in decibels (dB)
dB_{spec} is the published coaxial cable attenuation at the frequency of interest, in dB per 100 feet
$length_{ft}$ is the length of the coaxial cable, in feet

Example:
What is the attenuation of the 150 ft. span of half-inch hardline feeder cable at 5 MHz, 55 MHz, and 750 MHz? Refer to Table 11 for the published attenuation specifications.

Solution (5 MHz):

$$loss_{dB} = \left(\frac{dB_{spec}}{100}\right) * length_{ft}$$

$$loss_{dB} = \left(\frac{0.16}{100}\right) * 150$$

$$loss_{dB} = (0.0016) * 150$$

$$loss_{dB} = 0.24$$

Answer: The attenuation at 5 MHz in 150 feet of half-inch hardline coaxial cable is 0.24 dB.

Solution (55 MHz):

$$loss_{dB} = \left(\frac{0.54}{100}\right) * 150$$

$$loss_{dB} = (0.0054) * 150$$

$$loss_{dB} = 0.81$$

Answer: The attenuation at 55 MHz in 150 feet of half-inch hardline coaxial cable is 0.81 dB.

Solution (750 MHz):

$$loss_{dB} = \left(\frac{2.16}{100}\right) * 150$$

$$loss_{dB} = (0.0216) * 150$$

$$loss_{dB} = 3.24$$

Answer: The attenuation at 750 MHz in 150 feet of half-inch hardline coaxial cable is 3.24 dB.

Example:
What is the attenuation of the 75 ft length of Series 6 drop cable at 5 MHz, 55 MHz, and 750 MHz? Refer to Table 11 for the published attenuation specifications.

Solution (5 MHz):

$$loss_{dB} = \left(\frac{dB_{spec}}{100}\right) * length_{ft}$$

$$loss_{dB} = \left(\frac{0.58}{100}\right) * 75$$

$$loss_{dB} = (0.0058) * 75$$

$$loss_{dB} = 0.435$$

Answer: The attenuation at 5 MHz in 75 feet of Series 6 drop cable is 0.435 dB.

Solution (55 MHz):

$$loss_{dB} = \left(\frac{1.60}{100}\right) * 75$$

$$loss_{dB} = (0.0160) * 75$$

$$loss_{dB} = 1.20$$

Answer: The attenuation at 55 MHz in 75 feet of Series 6 drop cable is 1.20 dB.

Solution (750 MHz):

$$loss_{dB} = \left(\frac{5.65}{100}\right) * 75$$

$$loss_{dB} = (0.0565) * 75$$

$$loss_{dB} = 4.2375$$

Answer: The attenuation at 55 MHz in 75 feet of Series 6 drop cable is 4.2375 dB.

Example:
What is the attenuation of the 50 ft length of Series 6 drop cable at 5 MHz, 55 MHz, and 750 MHz? Refer to Table 11 for the published attenuation specifications.

Solution (5 MHz):

$$loss_{dB} = \left(\frac{dB_{spec}}{100}\right) * length_{ft}$$

$$loss_{dB} = \left(\frac{0.58}{100}\right) * 50$$

$$loss_{dB} = (0.0058) * 50$$

$$loss_{dB} = 0.29$$

Answer: The attenuation at 5 MHz in 50 feet of Series 6 drop cable is 0.29 dB.

Solution (55 MHz):

$$loss_{dB} = \left(\frac{1.60}{100}\right) * 50$$

$$loss_{dB} = (0.0160) * 50$$

$$loss_{dB} = 0.80$$

Answer: The attenuation at 55 MHz in 50 feet of Series 6 drop cable is 0.80 dB.

Solution (750 MHz):

$$loss_{dB} = \left(\frac{5.65}{100}\right) * 50$$

$$loss_{dB} = (0.0565) * 50$$

$$loss_{dB} = 2.8250$$

Answer: The attenuation at 55 MHz in 50 feet of Series 6 drop cable is 2.825 dB.

20.8.3. Calculate cable modem input signal level

The following formula can be used to calculate the downstream RF input signal level to the cable modem at each frequency of interest:

$$CM_{in} = amp_{out} - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$$

where
CM_{in} is the cable modem's downstream RF input signal level, in decibel millivolt (dBmV)
amp_{out} is the amplifier's downstream RF output signal level at the specified frequency, in dBmV
L_1 is the insertion loss (through loss) of the 23 dB tap
L_2 is the attenuation in the span of hardline coaxial cable between the two taps, in decibels (dB)
L_3 is the tap loss of the 20 dB tap
L_4 is the attenuation in the subscriber drop cable between the tap and splitter, in dB
L_5 is the insertion loss of the two-way splitter at the subscriber premises, in dB
L_6 is the attenuation in the length of subscriber drop cable between the splitter and cable modem, in dB

Example:
What is the cable modem's downstream RF input signal level at 55 MHz? From Figure 31, amp_{out} is +42 dBmV; from Table 12, L_1 is 0.54 dB; from the previous calculations, L_2 is 0.81 dB; L_3 is 20 dB; from the previous calculations L_4 is 1.20 dB; from Table 13 L_5 is 3.6 dB; and from the previous calculations, L_6 is 0.80 dB.

Solution:
$CM_{in} = amp_{out} - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$

$CM_{in} = 42 - (0.54 + 0.81 + 20 + 1.20 + 3.60 + 0.80)$

$CM_{in} = 42 - (26.95)$

$CM_{in} = 15.05$

Answer: The cable modem's downstream RF input signal level at 55 MHz is +15.05 dBmV.

Example:
What is the cable modem's downstream RF input signal level at 750 MHz? From Figure 31, amp_{out} is +50 dBmV; from Table 12, L_1 is 2.16 dB; from the previous calculations, L_2 is 3.24 dB; L_3 is 20 dB; from the previous calculations L_4 is 4.2375 dB; from Table 13 L_5 is 4.50 dB; and from the previous calculations, L_6 is 2.825 dB.

Solution:
$CM_{in} = amp_{out} - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$

$CM_{in} = 50 - (2.16 + 3.24 + 20 + 4.2375 + 4.50 + 2.825)$

$CM_{in} = 50 - (36.9625)$

$CM_{in} = 13.04$

Answer: The cable modem's downstream RF input signal level at 750 MHz is +13.04 dBmV.

20.8.4. Calculate cable modem upstream transmit signal level

The following formula can be used to calculate the cable modem's upstream transmit level at the frequency of interest:

$$CM_{out} = amp_{in} + (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$$

where
CM_{out} is the cable modem's upstream RF output signal level, in decibel millivolt (dBmV)
amp_{in} is the amplifier's upstream RF input signal level at the specified frequency, in dBmV
L_1 is the insertion loss (through loss) of the 23 dB tap
L_2 is the attenuation in the span of hardline coaxial cable between the two taps, in decibels (dB)
L_3 is the tap loss of the 20 dB tap
L_4 is the attenuation in the subscriber drop cable between the tap and splitter, in dB
L_5 is the insertion loss of the two-way splitter at the subscriber premises, in dB
L_6 is the attenuation in the length of subscriber drop cable between the splitter and cable modem, in dB

Example:
What is the cable modem's upstream RF output signal level at 5 MHz? From Figure 31, amp_{in} is +15 dBmV; from Table 12, L_1 is 0.16 dB; from the previous calculations, L_2 is 0.24 dB; L_3 is 20 dB; from the previous calculations L_4 is 0.435 dB; from Table 13 L_5 is 3.6 dB; and from the previous calculations, L_6 is 0.29 dB.

Solution:
$CM_{out} = amp_{in} + (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$

$CM_{out} = 15 + (0.16 + 0.24 + 20 + 0.435 + 3.6 + 0.29)$

$CM_{out} = 15 + (24.725)$

$CM_{out} = 39.725$

Answer: The cable modem's upstream transmit signal level at 5 MHz is +39.73 dBmV.

21. Characterizing Impedance Mismatches: Reflection Coefficient, Reflection Loss, Standing Wave Ratio, and Return Loss

The coaxial cable portion of our networks has long been designed and built to have a nominal impedance of 75 ohms. The word "nominal" is used to describe the impedance, because it is impossible for the impedance to be exactly 75 ohms, especially across a wide frequency range. Another way to look at this is to understand that all connectors, passive and active devices, and even the coaxial cable itself represent an impedance mismatch of some sort.

When the impedance of a load equals the characteristic impedance of the transmission line connected to that load, an incident wave is completely absorbed by the load. In the real world, there are no perfectly reflectionless loads, which means impedance mismatches exist. Impedance mismatches cause reflections. Reflected waves interact with incident waves to produce a distribution of fields in the transmission line known as standing waves.

The question is just how severe is each impedance mismatch? There are several ways to characterize impedance mismatches, among them reflection coefficient, reflection loss, standing wave ratio, and return loss. Formulas for each of the aforementioned parameters and examples of their use are included in this section.

21.1. Reflection coefficient

Reflection coefficient is the ratio of reflected voltage to incident voltage, and is commonly represented by the Greek letter gamma (Γ), and sometimes by the Greek letter rho (ρ). The magnitude of reflection coefficient, |Γ|, can have values from 0 (indicating a reflectionless load – that is, all of the incident energy is absorbed by the load) to 1 (indicating that all of the incident energy is reflected by the load). For the latter, the load would be an open, short, or pure reactance. Reflection coefficient for voltage is expressed mathematically as:

$$\Gamma = \frac{E_{reflected}}{E_{incident}}$$

where
Γ is reflection coefficient
$E_{reflected}$ is reflected voltage in units of volts
$E_{incident}$ is incident voltage in the same units of volts as $E_{reflected}$

Example:
What is the reflection coefficient when the incident voltage is 1.2 volts and the reflected voltage is 0.7 volt?

Solution:
$\Gamma = \frac{0.7}{1.2}$

$\Gamma = 0.58$
Answer: The reflection coefficient is 0.58.

21.1.1. Other formulas for reflection coefficient

The following formulas can also be used to calculate reflection coefficient:

$$\Gamma = \frac{1}{10^{(R/20)}}$$

where
Γ is reflection coefficient
R is return loss in decibels

$$\Gamma = 10^{(-R/20)}$$

where
Γ is reflection coefficient
R is return loss in decibels

$$\Gamma = \sqrt{\frac{P_{reflected}}{P_{incident}}}$$

where
Γ is reflection coefficient
$P_{reflected}$ in reflected power in units of watts
$P_{incident}$ is incident power in the same units of watts as $P_{reflected}$

Example 1:
What is the reflection coefficient when the return loss is 14 dB?

Solution 1:

$$\Gamma = \frac{1}{10^{(R/20)}}$$

$$\Gamma = \frac{1}{10^{(14/20)}}$$

$$\Gamma = \frac{1}{10^{(0.7)}}$$

$$\Gamma = \frac{1}{5.0119}$$

$$\Gamma = 0.1995$$

Answer: The reflection coefficient is 0.1995.

Example 2:
What is the reflection coefficient when the return loss is 14 dB?

Solution 2:
$\Gamma = 10^{(-R/20)}$

$\Gamma = 10^{(-14/20)}$

$\Gamma = 10^{(-0.7)}$

$\Gamma = 0.1995$

Answer: The reflection coefficient is 0.1995.

Example 3:
What is the reflection coefficient when the incident power is 5 watts and the reflected power is 0.19905 watt?

Solution 3:
$\Gamma = \sqrt{\frac{P_{reflected}}{P_{incident}}}$

$\Gamma = \sqrt{\frac{0.19905}{5}}$

$\Gamma = \sqrt{0.0398}$

$\Gamma = 0.1995$

Answer: The reflection coefficient is 0.1995.

21.1.2. Calculate magnitude of complex reflection coefficient

Reflection coefficients are complex quantities, which means they have magnitude and phase. Considering reflection coefficients as complex quantities, the reflection coefficient for voltage can be calculated using the following formulas:

$$\Gamma_E = \frac{Z_L - Z_0}{Z_L + Z_0}$$

where
Γ_E is reflection coefficient for voltage
Z_0 is the characteristic impedance of the transmission line
Z_L is the load impedance

Note: The above formula is equal to

$$\Gamma = \frac{(R_L \pm jX_L) - (R_0 \pm jX_0)}{(R_L \pm jX_L) + (R_0 \pm jX_0)}$$

where
Γ is reflection coefficient
j represents the imaginary unit, satisfying the equation $j^2 = -1$
R_L is the load resistance
X_L is the load reactance
R_0 is the real part of the transmission line characteristic impedance
X_0 is the reactive part of the transmission line characteristic impedance

From [15], "For high-quality, low-loss transmission lines at low frequencies, the characteristic impedance Z_0 is almost completely resistive, meaning that $Z_0 \cong R_0$ and $X_0 \cong 0$. The magnitude of the complex reflection coefficient … then simplifies to:"

$$|\Gamma| = \sqrt{\frac{(R_L - R_0)^2 + {X_L}^2}{(R_L + R_0)^2 + {X_L}^2}}$$

where
$|\Gamma|$ is the magnitude of the complex reflection coefficient
R_L is the load resistance
R_0 is the real part of the transmission line characteristic impedance
X_L is the load reactance

Example:
What is the magnitude of the reflection coefficient when the characteristic impedance of a transmission line is Z_0 = 75 ohms and the load impedance is $Z_L = 125 - j80$ ohms (that is, 125 ohms in series with a capacitive reactance of –80 ohms)?

Solution:

$$|\Gamma| = \sqrt{\frac{(R_L - R_0)^2 + {X_L}^2}{(R_L + R_0)^2 + {X_L}^2}}$$

$$|\Gamma| = \sqrt{\frac{(75 - 125)^2 + (-80)^2}{(75 + 125)^2 + (-80)^2}}$$

$$|\Gamma| = \sqrt{\frac{(-50)^2 + (-80)^2}{(200)^2 + (-80)^2}}$$

$$|\Gamma| = \sqrt{\frac{2500 + 6400}{40000 + 6400}}$$

$$|\Gamma| = \sqrt{\frac{8900}{46400}}$$

$$|\Gamma| = \sqrt{0.1918}$$

$$|\Gamma| = 0.438$$

Answer: The magnitude of the reflection coefficient is 0.438.

21.1.3. Convert SWR to magnitude of reflection coefficient

A common way to describe the magnitude of the reflection is standing wave ratio (discussed later). The SWR is related to the magnitude $|\Gamma|$ of the reflection coefficient with the following formula:

$$|\Gamma| = \frac{(SWR - 1)}{(SWR + 1)}$$

where
$|\Gamma|$ is the magnitude of the reflection coefficient
SWR is standing wave ratio

Example:
What is the magnitude of the reflection coefficient when the standing wave ratio is 1.5?

Solution:

$$|\Gamma| = \frac{(SWR - 1)}{(SWR + 1)}$$

$$|\Gamma| = \frac{(1.5 - 1)}{(1.5 + 1)}$$

$$|\Gamma| = \frac{(0.5)}{(2.5)}$$

$$|\Gamma| = 0.20$$

Answer: The magnitude of the reflection coefficient is 0.20.

21.2. Reflection loss

Reflection loss (sometimes called transmission loss) is the loss, as the result of a reflection, in the power absorbed by a load. Expressed mathematically:

$$L_{reflection} = 10log_{10}\left(\frac{P_{incident}}{P_{absorbed}}\right)$$

where
$L_{reflection}$ is reflection loss in decibels
log_{10} is base 10 logarithm
$P_{incident}$ is incident power in watts
$P_{absorbed}$ is power in watts absorbed by the load or termination

Another formula for reflection loss is

$$L_{reflection} = 10log_{10}\left(\frac{1}{1 - |\Gamma|^2}\right)$$

where
$L_{reflection}$ is reflection loss in decibels
log_{10} is base 10 logarithm
$|\Gamma|$ is the magnitude of the reflection coefficient

Note: If a load absorbs 100% of the incident power, there is no reflection loss.

Example 1:
What is the reflection loss when the incident power is 0.01333 watt (13.33 mW) and the power absorbed by the load is 0.0100 watt (10 mW)?

Solution 1:

$$L_{reflection} = 10log_{10}\left(\frac{P_{incident}}{P_{absorbed}}\right)$$

$$L_{reflection} = 10log_{10}\left(\frac{0.0133\ watt}{0.0100\ watt}\right)$$

$$L_{reflection} = 10 * log_{10}(13.3300)$$

$$L_{reflection} = 10 * (0.1239)$$

$$L_{reflection} = 1.24$$

Answer: The reflection loss is 1.24 dB.

Example 2:
What is the reflection loss when the reflection coefficient Γ = 0.8?

Solution 2:

$$L_{reflection} = 10log_{10}\left(\frac{1}{1-|\Gamma|^2}\right)$$

$$L_{reflection} = 10log_{10}\left(\frac{1}{1-|0.8|^2}\right)$$

$$L_{reflection} = 10 * log_{10}\left(\frac{1}{1-0.8^2}\right)$$

$$L_{reflection} = 10 * log_{10}\left(\frac{1}{1-0.64}\right)$$

$$L_{reflection} = 10 * log_{10}\left(\frac{1}{0.36}\right)$$

$$L_{reflection} = 10 * log_{10}(2.78)$$

$$L_{reflection} = 10 * (0.4437)$$

$$L_{reflection} = 4.437$$

Answer: The reflection loss is 4.44 dB.

21.3. Standing wave ratio

When the load impedance does not equal the characteristic impedance of a transmission line, a reflection occurs. As mentioned previously, reflected waves interact with incident waves to produce a distribution of fields in the transmission line known as standing waves. Another consequence of an impedance mismatch is variation of the signal amplitude as a function of frequency, aka amplitude ripple.[42] The standing wave ratio (SWR) is the ratio of maximum voltage to minimum voltage,[43] and depends on the magnitude of the reflection coefficient. Figure 32 illustrates a standing wave in a transmission line. Note that the distance between adjacent minima or adjacent maxima is a half wavelength.

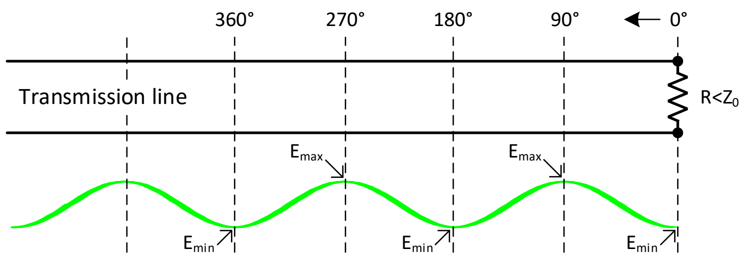


Figure 32. Standing wave of voltage (green trace) along a transmission line in which the load resistance R is less than the transmission line's characteristic impedance Z_0 (adapted from [15]).

Standing wave ratio can be calculated using the following formulas:

$$\text{SWR} = \frac{E_{max}}{E_{min}}$$

where
SWR is standing wave ratio
E_{max} in the maximum voltage in the standing wave
E_{min} is the minimum voltage in the standing wave

$$SWR = \frac{(1 + |\Gamma|)}{(1 - |\Gamma|)}$$

where
SWR is standing wave ratio
$|\Gamma|$ is the magnitude of the reflection coefficient

[42] Amplitude ripple in the frequency response as seen on a broadband sweep receiver display is commonly (but incorrectly) called a standing wave; amplitude ripple is technically the correct terminology to describe that frequency response variation.
[43] An impedance mismatch also results in a current standing wave. Indeed, either voltage or current can be used to determine the standing wave ratio, since SWR = E_{max}/E_{min} = I_{max}/I_{min}. For more on this, see [15] and [18].

Example 1:
What is the standing wave ratio when the maximum and minimum voltages in the standing wave are 0.8000 volt and 0.5339 volt respectively?

Solution 1:

$$\text{SWR} = \frac{E_{max}}{E_{min}}$$

$$\text{SWR} = \frac{0.8000}{0.5339}$$

$$\text{SWR} = 1.4984$$

Answer: The SWR is about 1.5 (sometimes written as 1.5:1).

Example 2:
What is the standing wave ratio when the magnitude of the reflection coefficient is 0.1995?

Solution 2:

$$SWR = \frac{(1 + |\Gamma|)}{(1 - |\Gamma|)}$$

$$SWR = \frac{(1 + |0.1995|)}{(1 - |0.1995|)}$$

$$SWR = \frac{(1.1995)}{(0.8005)}$$

$$SWR = 1.4984$$

Answer: The SWR is about 1.5 (sometimes written as 1.5:1).

21.3.1. Calculate SWR for purely resistive load

When the load contains no reactance and the characteristic impedance of the transmission line is assumed to be essentially resistive, SWR can be calculated as follows:

When $R > Z_0$:

$$\text{SWR} = \frac{R}{Z_0}$$

When $R < Z_0$:

$$\text{SWR} = \frac{Z_0}{R}$$

where
SWR is standing wave ratio
R is the load resistance
Z_0 is the transmission line characteristic impedance

Example 1:
What is the standing wave ratio when a 75 ohms impedance transmission line is connected to a resistive load of 100 ohms?

Solution 1:
Since $R > Z_0$, use the formula

$$\text{SWR} = \frac{R}{Z_0}$$

$$\text{SWR} = \frac{100}{75}$$

$$\text{SWR} = 1.3333$$

Answer: The standing wave ratio is 1.33 (sometimes written as 1.33:1).

Example 2:
What is the standing wave ratio when a 75 ohms impedance transmission line is connected to a resistive load of 50 ohms?

Solution 2:
Since $R < Z_0$, use the formula

$$\text{SWR} = \frac{Z_0}{R}$$

$$\text{SWR} = \frac{75}{50}$$

$$\text{SWR} = 1.50$$

Answer: The standing wave ratio is 1.50 (sometimes written as 1.50:1).

21.3.2. Distance to impedance mismatch or length of echo tunnel

When amplitude ripple is present on a broadband sweep receiver display or other test equipment, a technician can calculate the distance to an impedance mismatch, or more commonly, the length of an echo tunnel[44] that produces the amplitude ripple, using the following formula:

$$\text{D} = 492 * \left(\frac{VF}{f_{MHz}}\right)$$

where
D is distance in feet
VF is the coaxial cable's velocity factor
f_{MHz} is the frequency separation in megahertz between the amplitude ripple's adjacent peaks or adjacent valleys

Example:
What is the length of an echo tunnel that produces the amplitude ripple shown in Figure 33, where the frequency separation between adjacent peaks is 45 MHz? Assume the coaxial cable has a velocity factor of 0.87.

[44] An echo tunnel (sometimes called an echo cavity) is the span of coaxial cable between two impedance mismatches that result in amplitude ripple in the frequency response. One end of the echo tunnel might be an amplifier or similar, and the other end a damaged device such as a corroded tap.

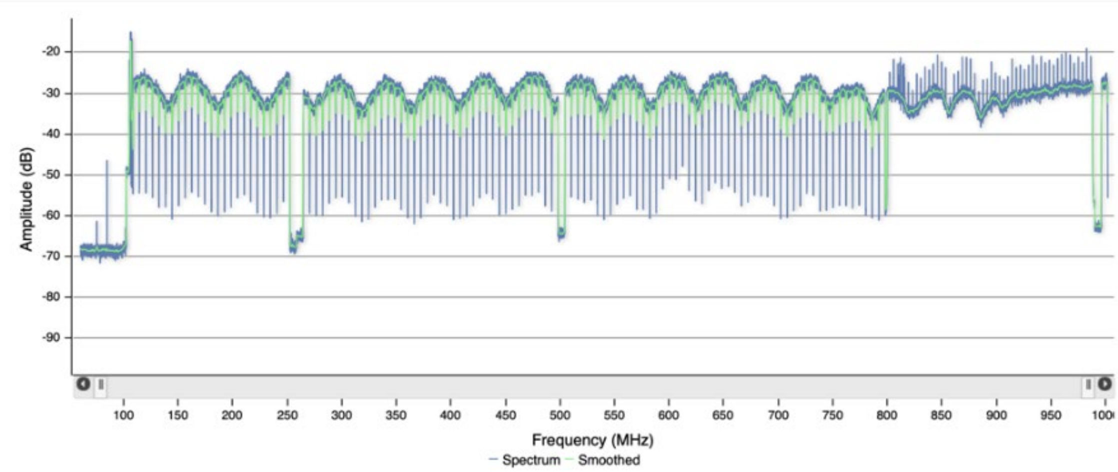


Figure 33 - Full band capture spectrum display showing amplitude ripple, with a frequency separation of 45 MHz between adjacent peaks or adjacent valleys (courtesy of Akleza).

Solution:

$$D = 492 * \left(\frac{VF}{f_{MHz}}\right)$$

$$D = 492 * \left(\frac{0.87}{45}\right)$$

$$D = 492 * (0.0193)$$

$$D = 9.512$$

Answer: The length of the echo tunnel is 9.5 feet.

21.3.2.1. Calculate length of echo tunnel from upstream adaptive pre-equalization graph

Many cable operators use proactive network maintenance (PNM) tools to remotely identify and locate outside plant and subscriber drop problems. One of the earliest PNM features was the ability to derive an upstream adaptive pre-equalizer graph, along with a plot of in-channel frequency response (ICFR),[45] from the adaptive pre-equalization coefficients.

Figure 34 shows an example of an adaptive pre-equalizer graph for an upstream 6.4 MHz-wide single carrier quadrature amplitude modulation (SC-QAM) channel. The vertical axis is relative amplitude in decibels, and the horizontal axis shows the pre-equalizer tap numbers. The horizontal axis can be converted to units of time if the tap spacing is known.

[45] Unless indicated otherwise, the ICFR provided by many PNM tools is actually the frequency response of the adaptive pre-equalizer, which is the inverse of the upstream channel's frequency response.

The tap spacing of a DOCSIS 2.0 and later cable modem's 24 upstream pre-equalizer taps is symbol-spaced, (also referred to as T-spaced), which means the time delay per tap is equal to the symbol period (T). The symbol period is the reciprocal of the symbol rate – that is, 1/T. See Table 19 for a list of upstream channel bandwidths and symbol rates (aka modulation rates) for commonly used DOCSIS upstream SC-QAM channels.

Referring to Figure 34, the graph shows the 24 taps (represented by the vertical red bars) used in DOCSIS 2.0 and 3.0 upstream pre-equalization. The main tap in this example is tap #8. The elevated post-main tap (tap #13) to the right of the main tap indicates the presence of a micro-reflection. Since the fifth tap to the right of the main tap is elevated relative to the other post-main taps, this example could also indicate what is called a 5T echo.

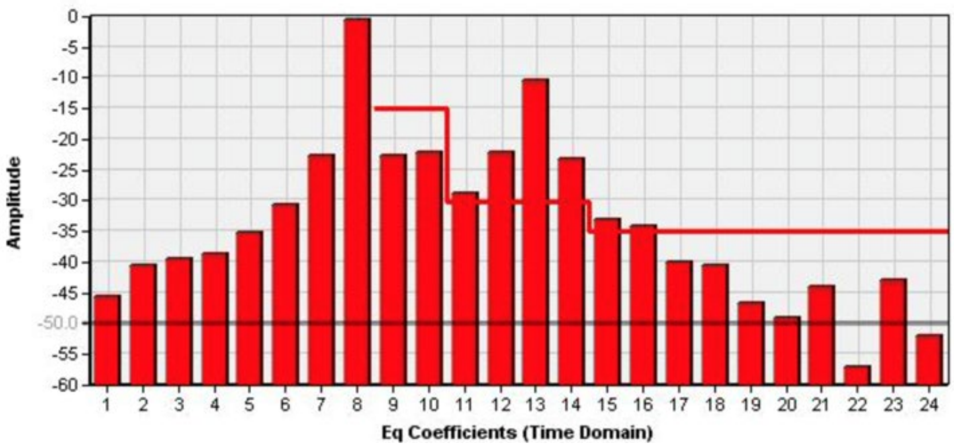


Figure 34 - Adaptive pre-equalizer graph for a 6.4 MHz-wide upstream SC-QAM channel, derived from pre-equalization coefficients. This example shows what is commonly called a 5T echo.

The following formula can be used to calculate the approximate length of the echo tunnel (or echo cavity) using the adaptive pre-equalizer graph.[46]

$$D_{ft} \cong \left(\frac{T_{ns}}{2} * VF\right) * tap_{\#}$$

where
D_{ft} is the approximate length of the echo tunnel (or echo cavity) in feet
T_{ns} is the symbol period in nanoseconds
VF is the coaxial cable's velocity factor
$tap_{\#}$ is number of the post-main tap that is elevated, relative to the main tap

[46] In the formula, the symbol period T_{ns} is divided by 2 to account for the echo's round-trip within the echo tunnel.

Example:
Referring to Figure 34, what is the approximate length of the echo tunnel, assuming a 6.4 MHz-wide upstream SC-QAM signal (5.12 Msym/s symbol rate), 0.87 for the cable's velocity factor, and an elevated fifth post-main tap?

Solution:
Since the symbol rate is 5.12 Msym/s, the symbol period is $T = 1/5{,}120{,}000 = 1.953125 * 10^{-7}$ second, or 195.3125 nanoseconds.

$$D_{ft} \cong \left(\frac{T_{ns}}{2} * VF\right) * tap_{\#}$$

$$D_{ft} \cong \left(\frac{195.3125}{2} * 0.87\right) * 5$$

$$D_{ft} \cong (97.6563 * 0.87) * 5$$

$$D_{ft} \cong (84.9609) * 5$$

$$D_{ft} \cong 424.8$$

Answer: The approximate length of the echo tunnel is 425 feet.

Figure 35 shows the ICFR derived from the same pre-equalization coefficients as the graph in Figure 34. The spacing, in megahertz, between adjacent peaks or adjacent valleys is about 1 MHz.

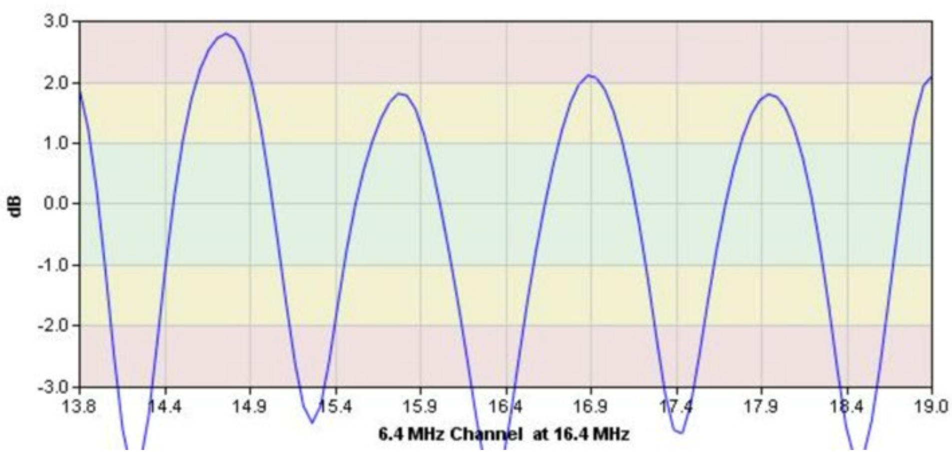


Figure 35 - In-channel frequency response plot for a 6.4 MHz-wide upstream SC-QAM channel, derived from pre-equalization coefficients. This example shows what is commonly called a 5T echo, easily determined here by counting the number of ripples.

Using the formula from Section 21.3.2, the approximate length of the echo tunnel can be calculated:

$$D = 492 * \left(\frac{0.87}{1}\right)$$

$$D = 492 * (0.87)$$

$D = 428$

Answer: The calculated length of the echo tunnel is 428 feet, which is close to the 424.8 feet calculated in the earlier example.

Note: It is important to understand that the method described in this section provides an approximation of the echo tunnel length, largely because of the limited horizontal axis resolution of the adaptive pre-equalizer graph. Another factor is that the micro-reflection rarely has a time delay corresponding exactly with the adaptive pre-equalizer tap spacing in nanoseconds. This is illustrated in Figure 36, which shows a 3T echo (each of the adaptive pre-equalizer graph's taps, or vertical bars, corresponds to about 85 feet for a symbol rate of 5.12 Msym/s, and about 170 feet per post-main tap for a symbol rate of 2.56 Msym/s. The 3T echo is roughly equivalent to an echo tunnel length of $3 * 85$ ft = 255 ft). In this case, tap #11 (the third post-main tap) is elevated more than the other post-main taps, but tap #10 (the second post-main tap) is also elevated, although not as much as tap #11. A mathematical method called parabolic interpolation can be used to further refine the resolution in a scenario like this, and that capability is included in some PNM tools.

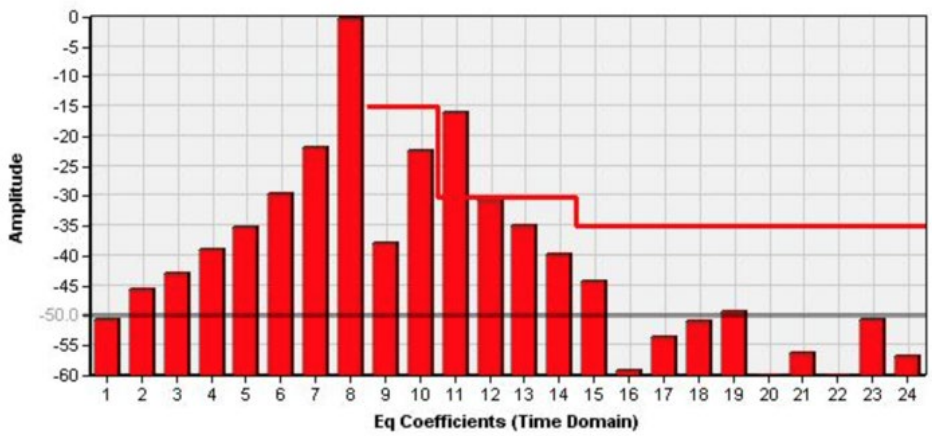


Figure 36 - Adaptive pre-equalizer graph for a 6.4 MHz-wide upstream SC-QAM channel, derived from pre-equalization coefficients. This example shows a 3T echo.

21.4. Return loss

Return loss (which is not the same thing as attenuation in the return or upstream spectrum of a cable network) is the ratio, in decibels, of the power incident upon an impedance discontinuity to the power reflected from the impedance discontinuity. Note: When $P_{reflected} < P_{incident}$ return loss is a positive number. Expressed mathematically:

$$R = 10log_{10}\left(\frac{P_{incident}}{P_{reflected}}\right)$$

where
R is return loss in decibels
log_{10} is base 10 logarithm
$P_{incident}$ is incident power in watts
$P_{reflected}$ is reflected power in watts

Example:
What is the return loss of a device under test when incident power is 0.01333 watt (13.33 mW) and reflected power is 0.00042 watt (0.42 mW)?

Solution:

$$R = 10log_{10}\left(\frac{P_{incident}}{P_{reflected}}\right)$$

$$R = 10log_{10}\left(\frac{0.01333\ watt}{0.00042\ watt}\right)$$

$$R = 10 * log_{10}\left(\frac{0.01333}{0.00042}\right)$$

$$R = 10 * log_{10}(31.7381)$$

$$R = 10 * (1.5016)$$

$$R = 15.02$$

Answer: The return loss is 15.02 dB.

Return loss can also be calculated with the following two formulas:

$$R = 20log_{10}\left(\frac{E_{incident}}{E_{reflected}}\right)$$

where
R is return loss in decibels
log_{10} is base 10 logarithm
$E_{incident}$ is incident voltage in volts
$E_{reflected}$ is reflected voltage in volts

$$R = P_{incident} - P_{reflected}$$

where
R is return loss in decibels
$P_{incident}$ is incident power in dBmV
$P_{reflected}$ is reflected power in dBmV

Example:
What is the return loss of a device under test when the incident power is +30 dBmV and the reflected power is +12 dBmV?

Solution:
$R = P_{incident} - P_{reflected}$

$R = 30\ dBmV - 12\ dBmV$

$R = 30 - 12$

$R = 18$

Answer: The return loss is 18 dB.

21.4.1. Convert reflection coefficient to return loss

Reflection coefficient can be converted to return loss in decibels using the following formula:

$$R = 20log_{10}\left(\frac{1}{|\Gamma|}\right)$$

where
R is return loss in decibels
log_{10} is base 10 logarithm
$|\Gamma|$ is the magnitude of the reflection coefficient

Example:
What is the return loss when the reflection coefficient is 0.2?

Solution:

$$R = 20log_{10}\left(\frac{1}{|\Gamma|}\right)$$

$$R = 20log_{10}\left(\frac{1}{|0.2|}\right)$$

$$R = 20 * log_{10}\left(\frac{1}{0.2}\right)$$

$$R = 20 * log_{10}(5.0)$$

$$R = 20 * (0.6990)$$

$$R = 13.98$$

Answer: The return loss is 13.98 dB.

21.4.2. Convert standing wave ratio to return loss

Standing wave ratio can be converted to return loss in decibels using the following formula:

$$R = -20log_{10}\left[\frac{(SWR-1)}{(SWR+1)}\right]$$

where
R is return loss in decibels
log_{10} is base 10 logarithm
SWR is standing wave ratio

Note the minus sign in front of the "20" in the above formula. If the minus sign is not included, the answer will be a negative number, which is incorrect for return loss when $P_{reflected} < P_{incident}$. That said, some references wrongly state return loss in negative numbers. What's going on? The confusion generally arises because the decibel representation of reflection coefficient is a negative number, but return loss is a positive number (as long as $P_{reflected} < P_{incident}$).

Another source of confusion comes from measurements made on network analyzers. Most vector network analyzers incorporate S-parameter test sets and display $20log_{10}(|S_{11}|)$ and $20log_{10}(|S_{22}|)$ rather than return loss (see Section 22 for an explanation of S parameters). Scalar network analyzers use RF bridges and display the magnitude of reflected power in dB with respect to incident power in dB rather than the other way around as in the definition of return loss in Section 21.4.

Example:
What is the return loss for a standing wave ratio of 3 (also written as 3:1)?

Solution:

$$R = -20log_{10}\left[\frac{(SWR-1)}{(SWR+1)}\right]$$

$$R = -20log_{10}\left[\frac{(3-1)}{(3+1)}\right]$$

$$R = -20 * log_{10}\left[\frac{(2)}{(4)}\right]$$

$$R = -20 * log_{10}[0.50]$$

$$R = -20 * [-0.301]$$

$$R = 6.02$$

Answer: The return loss is 6.02 dB.

21.4.3. Calculate cascaded return loss

The return loss in decibels of a cascade of two components or devices ("SYSTEM 1" and "SYSTEM 2"), can be calculated using the following formula (see Appendix D for the derivation of the formula):

$$R_{cascade} = -10log_{10}\left[\Gamma_1{}^2 + \left(\Gamma_2{}^2 * L_1{}^2\right)\right]$$

where
$R_{cascade}$ is the cascaded return loss in decibels of SYSTEM 1 and SYSTEM 2
log_{10} is base 10 logarithm
Γ_1 is the reflection coefficient
Γ_2 is the reflection coefficient
L_1 is the insertion loss of SYSTEM 1 in decibels ($L_1 = 10^{-IL1/10}$)

Example:
Assume two taps ("SYSTEM 1" and "SYSTEM 2") installed in the feeder back-to-back with a housing-to-housing connector each have a return loss of 18 dB, and the insertion loss IL1 of the first tap ("SYSTEM 1") is 0.5 dB. What is the cascaded return loss?

Solution:
First convert each tap's 18 dB return loss to reflection coefficient:

$$\Gamma = 10^{(-R/20)}$$

$$\Gamma = 10^{(-18/20)}$$

$$\Gamma = 10^{(-0.90)}$$

$$\Gamma = 0.12589$$

Then calculated the cascaded return loss (be sure to substitute $10^{-0.5/10}$ for L_1):

$$R_{cascade} = -10log_{10}\left[\Gamma_1{}^2 + \left(\Gamma_2{}^2 * L_1{}^2\right)\right]$$

$$R_{cascade} = -10 * log_{10}\left[0.12589^2 + \left(0.12589^2 * \left(10^{-0.5/10}\right)^2\right)\right]$$

$$R_{cascade} = -10 * log_{10}[0.12589^2 + (0.12589^2 * (10^{-0.05})^2)]$$

$$R_{cascade} = -10 * log_{10}[0.01585 + (0.01585 * (0.89125)^2)]$$

$$R_{cascade} = -10 * log_{10}[0.01585 + (0.01585 * 0.79433)]$$

$$R_{cascade} = -10 * log_{10}[0.01585 + (0.01259)]$$

$$R_{cascade} = -10 * log_{10}[0.02844]$$

$R_{cascade} = -10 * [-1.54610]$

$R_{cascade} = 15.46$

Answer: The cascaded return loss is 15.46 dB.

21.5. Comparison of reflection coefficient, reflection loss, SWR, and return loss

Values in the following table are derived using the formulas in this section.

Table 14. Reflection coefficient, reflection loss, standing wave ratio, and return loss

Magnitude of reflection coefficient |Γ|	Reflection loss (dB)	SWR	Return loss (dB)
1.0	∞	∞	0
0.9	7.21	19.00	0.92
0.8	4.44	9.00	1.94
0.7	2.92	5.67	3.10
0.6	1.94	4.00	4.44
0.5	1.25	3.00	6.02
0.4	0.76	2.33	7.96
0.3	0.41	1.86	10.46
0.2	0.18	1.50	13.98
0.1	0.04	2.20	20.00
0	0	1.0	∞

22. Scattering Parameters

Material in this section is excerpted and/or adapted from [16]. Used with the author's permission.

Consider a component or device being evaluated as a "network" with some number of ports N, and a characteristic impedance Z_0. For instance, a terminator can be considered a one-port network and an in-line attenuator or a line extender amplifier can be considered two-port network. Among the various metrics that can be used to characterize N-port networks are scattering parameters, also called S-parameters.

S-parameters "…describe the electrical behavior of linear electrical networks when undergoing various steady state stimuli by electrical signals."[47] S-parameters are a simplified representation of a black box network, and are complex numbers – that is, S-parameters have a magnitude and phase component. S-parameters are expressed in the format S_{mn} where "m" is the output port number during the measurement and "n" is the input port number during the measurement. For a two-port network, either port can be an input or output port, depending on the measurement; so, in most cases it's better to refer to port numbers. The "n" subscript also represents the port to which the incident (test) signal is applied. Figure 37 provides a high-level graphical representation of S-parameters in a two-port network.

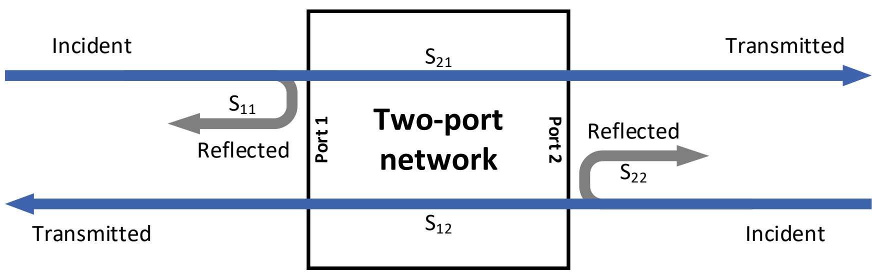


Figure 37 - High-level representation of S-parameters in a two-port network.

22.1. One- and two-port S-parameters

The following table summarizes S-parameters for a one- or two-port network. For a one-port network, only S_{11} is applicable. For a two-port network, S_{11}, S_{12}, S_{21}, and S_{22} are applicable. Note: S_{11} and S_{22} are sometimes confused with return loss, but they are not the same thing. Return loss can, however, be derived from S_{11} and S_{22}, as discussed later in this section.

Table 15 - Summary of S-parameters for one- and two-port networks.

One- and Two-Port S-Parameters	
S_{11}	Port 1 voltage reflection coefficient, $E_{Reflected}/E_{Incident}$
S_{12}	Reverse voltage gain or reverse transmission coefficient, $E_{Transmitted}/E_{Incident}$
S_{21}	Forward voltage gain or forward transmission coefficient, $E_{Transmitted}/E_{Incident}$
S_{22}	Port 2 voltage reflection coefficient, $E_{Reflected}/E_{Incident}$

For S_{11} and S_{22}, the voltage reflection coefficient is the ratio of reflected voltage $E_{Reflected}$ to the incident voltage $E_{Incident}$. For S_{12} and S_{21}, the transmission coefficient is the ratio of transmitted voltage $E_{Transmitted}$ to incident voltage $E_{Incident}$.

[47] From Wikipedia (https://en.wikipedia.org/wiki/Scattering_parameters)

An important point: S-parameters are frequency dependent – that is, the ratio that describes, say, S_{21}, is valid only for a given frequency. As well, S-parameters are referenced at the measurement or interface plane of a network; examples of that plane are Port 1 and Port 2 of a two-port network.

Figure 38 shows the relationships of $E_{Incident}$, $E_{Reflected}$, and $E_{Transmitted}$ when measuring from Port 1 to Port 2. Here, $E_{Transmitted}/E_{Incident} = S_{21}$ and $E_{Reflected}/E_{Incident} = S_{11}$. Note: Port 2 would normally be terminated in the characteristic impedance of the network under test when measuring S_{11}.

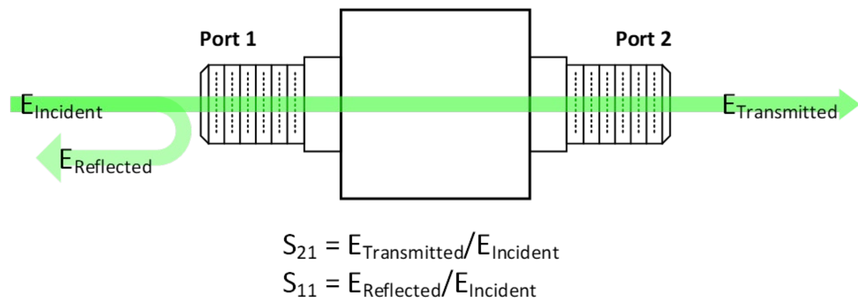


Figure 38 - Relationships of $E_{Incident}$, $E_{Transmitted}$, and $E_{Reflected}$ when measuring from Port 1 to Port 2.

Going the other direction, from Port 2 to Port 1, $E_{Transmitted}/E_{Incident} = S_{12}$ and $E_{Reflected}/E_{Incident} = S_{22}$. See Figure 39. Note: Port 1 would normally be terminated in the characteristic impedance of the network under test when measuring S_{22}.

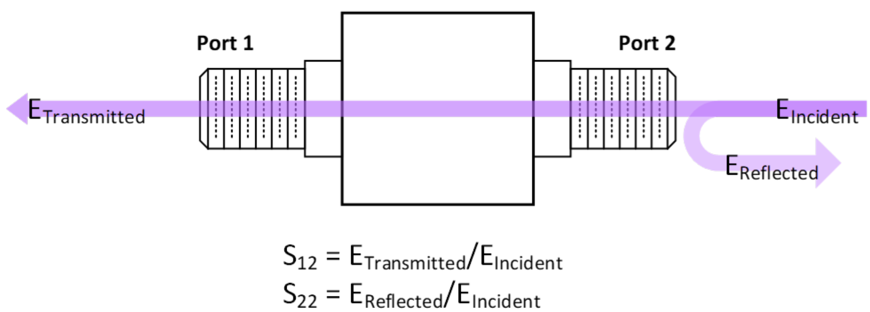


Figure 39 - Relationships of $E_{Incident}$, $E_{Transmitted}$, and $E_{Reflected}$ when measuring from Port 2 to Port 1.

Example 1:
Assume a voltage $E_{incident}$ of 0.8 volt at a specific test frequency is applied to Port 1 of a one-port network, and a voltage $E_{reflected}$ of 0.1 volt is reflected by that port. Which S-parameter is applicable, and what is its value?

Solution 1:
Referring to Figure 38 or Table 15, the S-parameter in this example is the Port 1 voltage reflection coefficient $E_{reflected}/E_{incident} = S_{11}$.

S_{11} = 0.1 volt/0.8 volt

S_{11} = 0.125

Answer: The applicable S-parameter is S_{11} and the value is 0.125.

Example 2:
Assume a voltage $E_{incident}$ of 0.2 volt at a specific test frequency is applied to Port 1 of a two-port network (an amplifier in this example) and a voltage $E_{transmitted}$ of 1.2 volt is measured at Port 2. Which S-parameter is applicable, and what is its value?

Solution 2:
Referring to Figure 38 or Table 15, the S-parameter in this example is the forward transmission coefficient $E_{Transmitted}/E_{Incident} = S_{21}$.

S_{21} = 1.2 volt/0.2 volt

S_{21} = 6.0

Answer: The applicable S-parameter is S_{21} and the value is 6.0.

22.1.1. Convert S-parameters to return loss, gain, and insertion loss

Cable operators are usually more familiar with characteristics such as gain, insertion loss, and return loss, all expressed in decibels. The following formulas are used to derive those metrics from S-parameters (referenced to a two-port network, and assuming Port 1 is the input port and Port 2 is the output port).

Table 16 - S-parameter conversions

Measurement Metric	Formula
Input return loss in decibels	$R_{in} = -20\log_{10}\|S_{11}\|$
Output return loss in decibels	$R_{out} = -20\log_{10}\|S_{22}\|$
Gain in decibels	$G_{dB} = 20\log_{10}\|S_{21}\|$
Insertion loss in decibels	$L_{dB} = -20\log_{10}\|S_{21}\|$

Example 3:
What is the return loss, in decibels, for Example 1 above?

Solution:
$R_{in} = -20 * \log_{10}|0.125|$

$R_{in} = -20 * -0.903$

$R_{in} = 18.06$

Answer: The return loss at the test frequency is 18.06 dB.

Example 4:
What is the gain, in decibels, of the two-port device in Example 2?

Solution 4:
$G_{dB} = 20 * \log_{10}|6.0|$

$G_{dB} = 20 * 0.778$

$G_{dB} = 15.56$

Answer: The gain at the test frequency is 15.56 dB.

23. Satellite Communications Formulas

Cable operators have used earth station antennas at headends since the 1970s, in large part for the reception of many of the program services carried on their networks. This section includes some of the more commonly used formulas for satellite communications system link budget analysis. For more information about satellite-related calculations and additional formulas, see [17].

23.1. Calculate gain to noise temperature ratio (G/T)

The gain to noise temperature ratio (G/T) of a satellite receive system is the ratio of the gain, at a specified reference point in the system, to the system's noise temperature at that same reference point. The main elements determining G/T are the gain of the receive antenna and combined noise temperature of the antenna and low noise amplifier (LNA) or low noise block converter (LNB). The antenna noise temperature is relatively constant with diameter, but increases as the elevation angle of the antenna decreases. LNA/LNB noise temperature is a function of device's design and manufacture using available low noise components (see Section 12 for more information on noise temperature and noise figure). The G/T of the receive system can be considered as a figure of merit for its performance.
The G/T of the receive system can be calculated using the following formula:

$$G/T_{sys} = G_{ant} - 10\,log_{10}(Temp_{ant} + Temp_{LNB})$$

where
G/T_{sys} is the gain to noise temperature ratio of the receive system in decibels
G_{ant} is the gain of the receive antenna in dBi
log_{10} is base 10 logarithm
$Temp_{ant}$ is the noise temperature of the receive antenna in kelvin (K)
$Temp_{LNB}$ is the noise temperature of the LNA/LNB in kelvin (K)

Example:
A C-band earth station antenna has a gain of 44 dBi and uses an LNB with a noise temperature of 25 K. The satellite viewed requires an elevation angle of 16.26 degrees. The noise temperature of the antenna, found in the manufacturer's specifications, is approximately 31 K at an antenna elevation angle of 16.26 degrees. Calculate the G/T of this system.

Solution:
$G/T_{sys} = G_{ant} - 10\,log_{10}(Temp_{ant} + Temp_{LNB})$

$G/T_{sys} = 44 - [10 * \ log_{10}(31 + 25)]$

$G/T_{sys} = 44 - [10 * \ log_{10}(56)]$

$G/T_{sys} = 44 - [10 * (1.748)]$

$G/T_{sys} = 44 - 17.48$

$G/T_{sys} = 26.52$

Answer: The G/T of the receive system is 26.52 dB.

23.2. Calculate free space path loss between the satellite and the antenna site

Free space path loss describes the RF signal attenuation between a satellite in orbit and an earth station antenna on the surface of the Earth. Free space path loss is an important part of link budget analysis for satellite communications systems, and assumes an unobstructed path. Calculated free space path loss should be considered an approximation, since a variety of factors can affect the actual received signal power at an earth station antenna. Examples of some of those factors include the previously mentioned path obstruction; satellite transmit power uncertainty, especially transponder-to-transponder power variations; weather effects (e.g., precipitation); Faraday rotation; and so forth.

The free space loss in decibels between the satellite of interest and the earth station antenna can be calculated using the following formula:

$$FSPL = 20\,log_{10}(f) + 20\,log_{10}(D) + 32.45$$

where
$FSPL$ is the free space loss from the satellite antenna to the earth station antenna in decibels
log_{10} is base 10 logarithm
f is the frequency in megahertz
D is the slant range distance[48] from the satellite antenna to the earth station antenna in kilometers

Example:
Calculate the free space path loss at 4,200 MHz for a slant range distance of 39,931 kilometers (about 24,813 miles) between a C-band earth station antenna and a geostationary satellite of interest.

Solution:
$FSPL = 20\,log_{10}(f) + 20\,log_{10}(D) + 32.45$

$FSPL = [20 * \ log_{10}(4{,}200)] + [20 * \ log_{10}(39{,}931)] + 32.45$

$FSPL = [20 * (3.623)] + [20 * (4.601)] + 32.45$

$FSPL = 72.46 + 92.02 + 32.45$

$FSPL = 196.93$

Answer: The free space path loss between the satellite antenna and the earth station antenna is 196.93 dB.

23.3. Calculate satellite downlink carrier-to-noise ratio

The carrier-to-noise ratio (CNR) at the earth station receive system is the difference, in decibels, between the strength of the satellite downlink carrier signal received and the strength of the noise present in the receive system. The CNR is an important factor in determining the quality of the demodulated signal. The CNR can be calculated using the following formula:

[48] Slant range distance is the line-of-sight distance along a slant direction between two points which are not at the same level relative to each other. As used in this section, slant range distance is the distance between a satellite in geostationary orbit above the equator and an earth station (usually) north or south of the equator.

$$CNR = EIRP - FSPL + G/T_{sys} - 10log_{10}(B * k)$$

where
CNR is the downlink carrier-to-noise ratio in decibels
$EIRP$ is the equivalent isotropic radiated power output of the satellite in decibel watts (dBW)
$FSPL$ is the free space loss from the satellite antenna to the earth station antenna in decibels
G/T_{sys} is the gain to noise temperature ratio of the receive system in decibels
log_{10} is base 10 logarithm
B is the carrier bandwidth in hertz (Hz)
k is Boltzmann's Constant ($1.38 * 10^{-23}$ joules/kelvin)

Example:
The C-band satellite of interest has an output of 41 dBW EIRP and a carrier bandwidth of 36 MHz. The earth station antenna has a G/T of 26.52 dB. The free space path loss between the satellite and the earth station antenna is 196.93 dB. Calculate the CNR of this receive system.

Solution:
$CNR = EIRP - FSPL + G/T_{sys} - 10log_{10}(B * k)$

$CNR = 41 - 196.93 + 26.52 - [10 * log_{10}([36 * 10^6] * [1.38 * 10^{-23}])]$

$CNR = 41 - 196.93 + 26.52 - [10 * log_{10}(4.968 * 10^{-16})]$

$CNR = 41 - 196.93 + 26.52 - [10 * (-15.304)]$

$CNR = 41 - 196.93 + 26.52 - (-153.04)$

$CNR = 41 - 196.93 + 26.52 + 153.04$

$CNR = 23.63$

Answer: The CNR of this receive system is 23.63 dB.

23.4. Calculate E_b/N_0

The energy per bit to noise power spectral density ratio (E_b/N_0) is a normalized signal-to-noise ratio (SNR) measure, often referred to as the "SNR per bit." The E_b/N_0 is commonly used to evaluate digital communication systems, and is especially useful when comparing the bit error ratio (BER) performance of different digital modulation schemes. The E_b/N_0 is closely related to the carrier-to-noise ratio (CNR) of the received signal.

The E_b/N_0 can be calculated using the following formula:

$$E_b/N_0 = CNR + 10log_{10}\left[\frac{bandwidth}{net\ bit\ rate}\right]$$

where
E_b/N_0 is the energy per bit-to-noise power density ratio in decibels
CNR is the carrier-to-noise ratio in decibels
log_{10} is the base 10 logarithm
$net\ bit\ rate$ is the net bit rate in megabits per second (Mbps, sometimes as $Mbps_{net}$), excluding FEC and other overhead[49]
$bandwidth$ is the transmission bandwidth in megahertz (MHz)

Example:
A C-band satellite transponder has a bandwidth of 36 MHz and is transmitting a net bit rate of 72 Mbps. The CNR of the earth station receive system is 14.6 dB. Calculate the E_b/N_0 of the received signal.

Solution:

$$E_b/N_0 = CNR + 10log_{10}\left[\frac{bandwidth}{net\ bit\ rate}\right]$$

$$E_b/N_0 = 14.6 + 10log_{10}\left[\frac{36}{72}\right]$$

$$E_b/N_0 = 14.6 + 10log_{10}[0.5]$$

$$E_b/N_0 = 14.6 + 10 * (log_{10}[0.5])$$

$$E_b/N_0 = 14.6 + 10 * (-0.301)$$

$$E_b/N_0 = 14.6 + (-3.01)$$

$$E_b/N_0 = 14.6 - 3.01$$

$$E_b/N_0 = 11.59$$

Answer: The E_b/N_0 of this receive system is 11.59 dB.

[49] When calculating E_b/N_0, bit rate is the transmission net bit rate (excluding FEC coding and physical layer protocol overhead if present) in megabits per second. The transmitted symbols are a physical entity in the channel, and it is customary to define energy-per-symbol to noise-density ratio (E_S/N_0) without mention of PHY overhead. There is no notion or distinction of "net symbol." However, E_b/N_0 *should* state if net (information) or gross (channel) bit rate is used. In many cases in literature and text E_b/N_0 may be discussed without mention of any FEC or PHY overhead, in which case the gross (channel) bit rate is the assumption. In cases in literature comparing modulation schemes where FEC and/or overhead of protocols are involved, such as different proposals in a standards body, the context will normally make it clear that the net (information) bit rate is intended. Unfortunately, it is common in literature to not be explicit, and the distinction between "net" and "gross" is often not provided, but rather has to be understood from context.

23.4.1. Convert E_b/N_0 to carrier-to-noise ratio

The following formula can be used to convert E_b/N_0 to CNR:

$$CNR = E_b/N_0 + 10log_{10}(bps) - 10log_{10}(N_{BW})$$

where
CNR is carrier-to-noise ratio
E_b/N_0 is energy per bit-to-noise power density ratio in decibels
log_{10} is base 10 logarithm
bps is the bit rate in bits per second
N_{BW} is the noise bandwidth in hertz (Hz)

Example:
What is the CNR when E_b/N_0 is 11.59 dB, the bit rate is 72 Mbps (72,000,000 bits per second), and the bandwidth is 36 MHz (36,000,000 Hz)?

Solution:
$$CNR = E_b/N_0 + 10log_{10}(bps) - 10log_{10}(N_{BW})$$

$$CNR = 11.59 + 10 * log_{10}(72{,}000{,}000) - 10 * log_{10}(36{,}000{,}000)$$

$$CNR = 11.59 + 10 * (7.8573) - 10 * (7.5563)$$

$$CNR = 11.59 + 78.5733 - 75.5630$$

$$CNR = 14.6$$

Answer: The CNR is 14.6 dB.

23.5. Earth station antenna pointing calculations

The satellites used to deliver programming to cable systems are located in geostationary orbits, which means the satellites orbit the Earth at the same rate as the Earth rotates. As such, the satellites appear to be stationary in the sky. Cable operators need to know where to point the earth station antennas, in terms of azimuth relative to true north and elevation relative to a flat horizon.

23.5.1. Calculate antenna azimuth

The azimuth angle is one of two calculations (the other calculation is elevation angle) needed to accurately align the earth station antenna to the satellite of interest.

Once the geographic coordinates of the earth station antenna (Lat_{site} and $Long_{site}$) and the satellite longitude ($Long_{sat}$) are known, the antenna azimuth can be calculated using the following formulas.

Note: North latitude and east longitude are expressed as positive numbers, and south latitude and west longitude are expressed as negative numbers when calculating the antenna azimuth and elevation angles.

$$Y = cos^{-1}[\cos(Long_{sat} - Long_{site}) * cos(Lat_{site})]$$

$$Az_{ant} = 360 - cos^{-1}\left[\frac{-tan(Lat_{site})}{tan(Y)}\right]$$

where
Y is a calculated value used in antenna azimuth and elevation angle calculations
Az_{ant} is the antenna azimuth in decimal degrees relative to true north
Lat_{site} is antenna site latitude in decimal degrees
$Long_{site}$ is the antenna site longitude in decimal degrees
$Long_{sat}$ is the satellite longitude in decimal degrees

Example:
A C-band earth station antenna is located at a north latitude (Lat_{site}) of 37.635587 decimal degrees and a west longitude ($Long_{site}$) of –76.104050 decimal degrees. The satellite of interest is in an orbital position located at –135 decimal degrees west longitude ($Long_{sat}$). Determine the azimuth angle of the earth station antenna relative to true north.

Solution:
First, calculate the value of Y:

$$Y = cos^{-1}[\cos(Long_{sat} - Long_{site}) * \cos(Lat_{site})]$$

$$Y = cos^{-1}[\cos(-135 - (-76.104050)) * \cos(37.635587)]$$

$$Y = cos^{-1}[\cos(-135 + 76.104050) * \cos(37.635587)]$$

$$Y = cos^{-1}[\cos(-58.89595) * \cos(37.635587)]$$

$$Y = cos^{-1}[(0.51659385) * (0.79191052)]$$

$$Y = cos^{-1}(0.40909610)$$

$$Y = 65.85193414$$

Next, calculate the azimuth angle:

$$Az_{ant} = 360 - cos^{-1}\left[\frac{-tan(Lat_{site})}{tan(Y)}\right]$$

$$Az_{ant} = 360 - cos^{-1}\left[\frac{-tan(37.635587)}{tan(65.85193414)}\right]$$

$$Az_{ant} = 360 - cos^{-1}\left[\frac{-0.77109361}{2.230506}\right]$$

$Az_{ant} = 360 - cos^{-1}(-0.34570345)$

$Az_{ant} = 360 - 110.2247429$

$Az_{ant} = 249.78$

Answer: The azimuth angle of the earth station antenna relative to true north is 249.78 decimal degrees.

23.5.2. Calculate antenna elevation

The elevation angle is one of two calculations (the other calculation is azimuth angle) needed to accurately align the earth station antenna to the satellite of interest.

Once the geographic coordinates of the earth station antenna (Lat_{site} and $Long_{site}$) and the satellite longitude ($Long_{sat}$) are known, the antenna elevation can be calculated using the following formulas.

Note: North latitude and east longitude are expressed as positive numbers, and south latitude and west longitude are expressed as negative numbers when calculating the antenna azimuth and elevation angles.

$$Y = cos^{-1}[\cos(Long_{sat} - Long_{site}) * cos(Lat_{site})]$$

$$El_{ant} = tan^{-1}\left[\frac{\cos(Y) - 0.15116}{\sin(Y)}\right]$$

where
Y is a calculated value used in antenna azimuth and elevation angle calculations
El_{ant} is the antenna elevation angle in decimal degrees
Lat_{site} is antenna site latitude in decimal degrees
$Long_{site}$ is the antenna site longitude in decimal degrees
$Long_{sat}$ is the satellite longitude in decimal degrees

Example:
A C-band earth station antenna is located at a north latitude (Lat_{site}) of 37.635587 decimal degrees and a west longitude ($Long_{site}$) of –76.104050 decimal degrees. The satellite of interest is in an orbital position located at –135 decimal degrees west longitude ($Long_{sat}$). Determine the elevation angle of the earth station antenna.

Solution:
First, calculate the value of Y:

$Y = cos^{-1}[\cos(Long_{sat} - Long_{site}) * \cos(Lat_{site})]$

$Y = cos^{-1}[\cos(-135 - (-76.104050)) * \cos(37.635587)]$

$Y = cos^{-1}[\cos(-135 + 76.104050) * \cos(37.635587)]$

$Y = cos^{-1}[\cos(-58.89595) * \cos(37.635587)]$

$Y = cos^{-1}[(0.51659385) * (0.79191052)]$

$Y = cos^{-1}(0.40909610)$

$Y = 65.85193414$

Next, calculate the elevation angle:

$$El_{ant} = tan^{-1}\left[\frac{\cos(Y) - 0.15116}{\sin(Y)}\right]$$

$$El_{ant} = tan^{-1}\left[\frac{\cos(65.85193414) - 0.15116)}{\sin(65.85193414)}\right]$$

$$El_{ant} = tan^{-1}\left[\frac{0.40909611 - 0.15116}{0.91249130}\right]$$

$$El_{ant} = tan^{-1}\left[\frac{0.25793611}{0.91249130}\right]$$

$El_{ant} = tan^{-1}(0.28267240)$

$El_{ant} = 15.78$

Answer: The elevation angle of the earth station antenna is 15.78 decimal degrees.

24. Terrestrial Microwave Communications Formulas

The cable industry has for many decades used terrestrial analog microwave communications for point-to-point and point-to-multipoint signal transport. Depending on implementation, analog microwave communications technology used by cable operators has been based on either frequency modulation or amplitude modulation techniques, commonly in the 12.7 GHz to 13.2 GHz CARS band.[50] One example of an application using amplitude modulation was the Hughes AML low-power and high-power multichannel microwave technology. While optical fiber transport and distribution have to a large extent replaced cable-specific terrestrial microwave links, some terrestrial microwave technology remains in use by the cable industry.

Microwave path engineering is a complex subject, and an in-depth treatment of all of the involved mathematics is beyond the scope of this Operational Practice. Some of the more common mathematical formulas used in terrestrial microwave applications are included in this section, but others, such as path reliability, are not. For more information on terrestrial microwave communications and path engineering, see, for example, [19], [20], and [23].

Refer to Figure 40 for some of the formulas and examples included in this section.

[50] In the United States, Part 78 of the FCC Rules governs Cable Television Relay Service (CARS) stations, which operate in the 12.7 GHz to 13.2 GHz band and 17.7 GHz to 19.7 GHz band. A CARS station is defined in Part 78 as "A fixed or mobile station used for the transmission of television and related audio signals, signals of standard and FM broadcast stations, signals of instructional television fixed stations, and cablecasting from the point of reception to a terminal point from the point of reception to a terminal point from which the signals are distributed to the public."

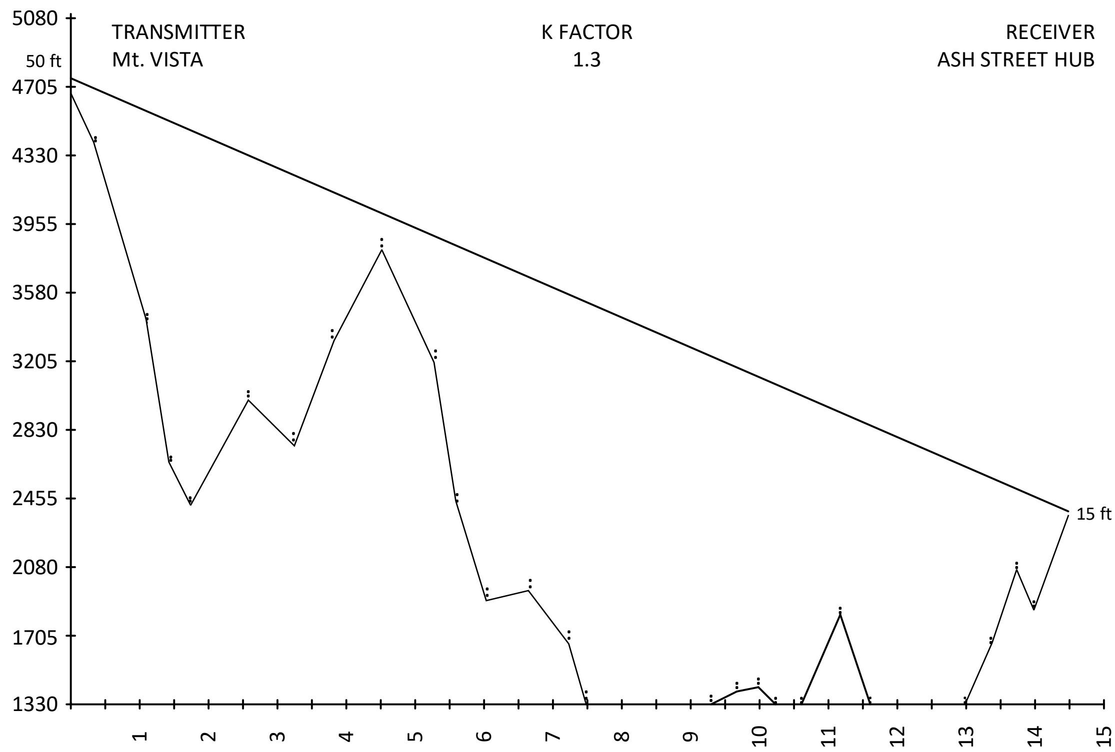


Figure 40 - Sample point-to-point terrestrial microwave path profile.

In Figure 40, the diagonal black line sloping downward from left to right represents the microwave communications path, which in this example is 14.4 miles long. The profile of the terrain under the path has been plotted with elevation in feet above mean sea level (AMSL) in the vertical axis and distance in statute miles in the horizontal axis. This particular example uses a flat earth surface with a flat microwave path.[51]

The small black dots above various points in the terrain profile represent effective earth curvature, 0.6 Fresnel zone radius, tree height and tree growth allowance (where applicable), and survey error (typ. 10 feet) – all in feet – added to the original terrain elevation. The goal is to ensure that the terrain profile and dots remain below the microwave path. Otherwise, antenna heights at one or both ends of the path must be revised.

24.1. Calculate path attenuation

Path attenuation is the amount of signal lost between a terrestrial microwave transmit antenna and a receive antenna (see Figure 40). The path attenuation can be approximated using free space path loss, assuming that there are no obstructions between the antennas (including within the first Fresnel zone), and both antennas are operating in the far field region. Note that the actual signal loss seldom equals the calculated free space path loss, because of the constructive and/or destructive effects of signal reflection(s) and diffraction, weather effects, and so forth.

[51] Some path profiles use a curved earth representation with flat path, and some use a flat earth representation with the microwave path curved downward to represent the equivalent of a curved earth.

The path attenuation in decibels, assuming free space path loss, between a terrestrial microwave transmit antenna and receive antenna can be calculated using the following formula:

$$FSPL = 20\, log_{10}(f) + 20\, log_{10}(D) + 96.58$$

where
$FSPL$ is the free space loss from the transmit antenna to the receive antenna in decibels
log_{10} is base 10 logarithm
f is the frequency in gigahertz
D is the distance from transmit antenna to the receive antenna in statute miles

Example:
The distance between a Cable Television Relay Service (CARS) transmit antenna and receive antenna is 14.4 miles (about 23.17 kilometers). Calculate the free space path loss at an operating frequency of 12.7035 GHz.

Solution:
$FSPL = 20\, log_{10}(f) + 20\, log_{10}(D) + 96.58$

$FSPL = [20 * \ log_{10}(12.7035)] + [20 * \ log_{10}(14.4)] + 96.58$

$FSPL = [20 * (1.103923)] + [20 * (1.158362)] + 96.58$

$FSPL = 22.07846 + 23.16724 + 96.58$

$FSPL = 141.83$

Answer: The path attenuation, assuming free space path loss, between the CARS transmit antenna and the receive antenna is 141.83 dB.

24.2. Calculate Fresnel zone radius

Kizer [19] describes Fresnel zones as follows:

> Microwave radio transmit antennas do not just send a thin beam to the receive antenna. They actually illuminate a wide area along the microwave radio path. The effect of terrain in reflecting the transmitted energy toward the receive antenna can significantly influence the received signal. Analysis of terrain reflections is done on the basis of Fresnel zones. A Fresnel zone is described as the locus of points above or below the direct path from the transmitter to the receiver where the distance from one end of the path to the point and then to the other end of the path is an integer number of ½ wavelengths longer than the direct path. The first Fresnel zone, F_1, has a total additional path length of ½ wavelength. The second Fresnel zone, F_2, has 2 x ½ wavelengths, the third Fresnel zone, F_3, has 3 x ½, and so on.
>
> A Fresnel zone radius, F_n, is the distance perpendicular to the path from a location of interest to a point on the Fresnel zone.

Figure 41 illustrates the Fresnel zone concept. The vertical scale is exaggerated to help clarify the concept.

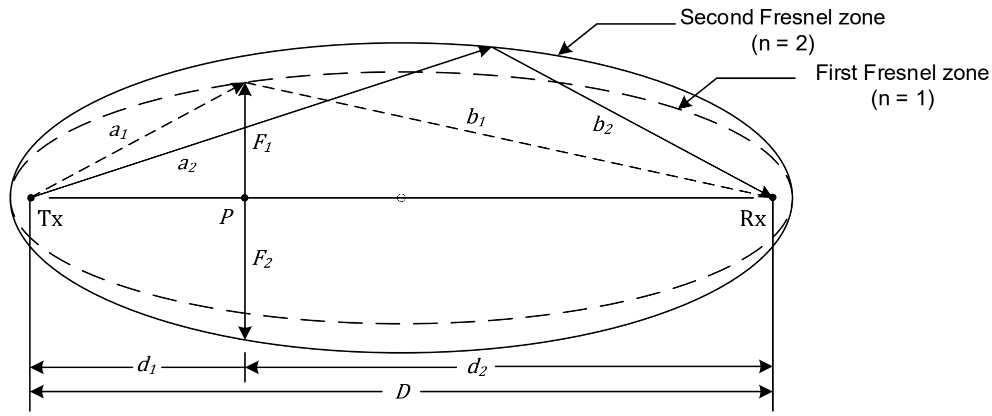


Figure 41 - Concept of Fresnel zones in terrestrial microwave paths; not to scale. Adapted from [19].

For Figure 41:
$F_1 = 72.1\ [(d_1 * d_2)\ /\ (f * D)]^{1/2}$
$a_n + b_n = D + n(\lambda/2)$
$F_n = F_1\ [n]^{1/2}$
d_1 and d_2 are the distances from Tx and Rx in miles
F_1 is the first Fresnel zone radius in feet at point P
D is the path length in miles
f is the operating frequency in gigahertz
n is the nth Fresnel zone
F_n is the radius of the nth Fresnel zone in feet
λ is the operating frequency wavelength

The Fresnel zone radius can be calculated using the following formula:

$$F_n = 72.1 * \sqrt{\left[\frac{n * d_1 * d_2}{f * D}\right]}$$

where
F_n is the nth Fresnel zone radius in feet
n is the Fresnel zone number (a positive integer)
d_1 is the distance from the transmit antenna to a point on the Fresnel zone in statute miles
d_2 is the distance from the receive antenna to a point on the Fresnel zone in statute miles
f is the frequency in gigahertz
D is the total path distance in statute miles = $d_1 + d_2$

Example 1:
Refer to Figure 40. A CARS system is operating at a frequency of 12.7035 GHz. The path distance between the transmit antenna and receive antenna is 14.4 miles (about 23.17 kilometers). Calculate the radius of the first Fresnel zone at the path's midpoint.

Solution 1:

$$F_n = 72.1 * \sqrt{\left[\frac{n * d_1 * d_2}{f * D}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{1 * 7.2 * 7.2}{12.7035 * 14.4}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{1 * 51.84}{12.7035 * 14.4}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{1 * 51.84}{12.7035 * 14.4}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{51.84}{182.9304}\right]}$$

$$F_n = 72.1 * \sqrt{0.283386}$$

$$F_n = 72.1 * 0.532340$$

$$F_n = 38.38$$

Answer: The radius of the first Fresnel zone at the path's midpoint is 38.38 feet.

Example 2:
Referring to the system described in Example 1, a potential obstruction located 4.5 miles (about 7.24 kilometers) from the transmitter has been observed. Calculate the radius of the first Fresnel zone at this point in the path.

Solution 2:

$$F_n = 72.1 * \sqrt{\left[\frac{n * d_1 * d_2}{f * D}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{1 * 4.5 * 9.9}{12.7035 * 14.4}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{1 * 44.55}{12.7035 * 14.4}\right]}$$

$$F_n = 72.1 * \sqrt{\left[\frac{44.55}{182.9304}\right]}$$

$F_n = 72.1 * \sqrt{0.243535}$

$F_n = 72.1 * 0.493493$

$F_n = 35.58$

Answer: The radius of the first Fresnel at a point located 4.5 miles from the transmitter is 35.58 feet.

Example 3:
In most path profiles, a minimum of 60% (0.6) of the first Fresnel zone radius must be clear of obstructions or reflections. Referring to Figure 40, what is the 0.6 Fresnel zone radius at the path midpoint?

Solution 3:
$F_{0.6} = 0.6 * F_1$

$F_{0.6} = 0.6 * 38.38$

$F_{0.6} = 23.03$

Answer: The 0.6 Fresnel zone radius at the path midpoint is 23.03 feet.

Example 4:
In most path profiles, a minimum of 0.6 of the first Fresnel zone radius must be clear of obstructions or reflections. Referring to Figure 40, what is the 0.6 Fresnel zone radius at the potential obstruction 4.5 miles from the transmit antenna?

Solution 4:
$F_{0.6} = 0.6 * F_1$

$F_{0.6} = 0.6 * 35.58$

$F_{0.6} = 21.35$

Answer: The 0.6 Fresnel zone radius at the point 4.5 miles from the transmit antenna is 21.35 feet.

24.3. Earth curvature

When preparing microwave path profiles, effective earth curvature must be taken into account. From [23]: "Although the surface of the earth is curved, a beam of microwave energy tends to travel in a straight line. However, the beam is normally bent downward a slight amount by atmospheric refraction. The amount of bending varies with atmospheric conditions. The degree and direction of bending can be conveniently defined by an equivalent earth radius factor, K. This factor, K, multiplied by the actual earth radius, R, is the radius of a fictitious earth curve."

K factor can be determined from a sea level refractivity profile chart for the area of interest. If that information is not available, one can plot a path profile using various K factors, the most common being K = 4/3, K = 1, and K = 2/3. The following formulas can be used to calculate effective earth curvature height in feet for different K factors:

$$h = \frac{d_1 * d_2}{1.5 * K}$$

$$h_{(K=4/3)} = \frac{d_1 * d_2}{2}$$

$$h_{(K=1)} = 0.67 * (d_1 * d_2)$$

$$h_{(K=2/3)} = d_1 * d_2$$

where
h is height in feet
K is equivalent earth radius factor
$h_{(k=4/3)}$ is height in feet at a K factor of 4/3
$h_{(k=1)}$ is height in feet at a K factor of 1
$h_{(k=2/3)}$ is height in feet at a K factor of 2/3
d_1 is the distance from one end of the microwave path to an obstruction point in statute miles
d_2 is the distance from the same obstruction point to the other end of the microwave path in statute miles

Example 1:
Referring to Figure 40, what is the effective earth curvature in feet at the path midpoint for K = 4/3?

Solution 1:

$$h_{(K=4/3)} = \frac{d_1 * d_2}{2}$$

$$h_{(K=4/3)} = \frac{7.2 * 7.2}{2}$$

$$h_{(K=4/3)} = \frac{51.84}{2}$$

$$h_{(K=4/3)} = 25.92$$

Answer: The effective earth curvature height is 25.92 feet.

Example 2:
Referring to Figure 40, what is the effective earth curvature in feet at the obstruction point 4.5 miles from the transmit antenna for K = 4/3?

Solution 2:

$$h_{(K=4/3)} = \frac{d_1 * d_2}{2}$$

$$h_{(K=4/3)} = \frac{4.5 * 9.9}{2}$$

$$h_{(K=4/3)} = \frac{44.55}{2}$$

$$h_{(K=4/3)} = 22.28$$

Answer: The effective earth curvature height is 22.28 feet.

24.4. Calculate path distance and bearing

The following formulas can be used to determine the path distance and bearing (azimuth) between the microwave transmit antenna and the receive antenna.

Note: North latitude and east longitude are expressed as positive numbers, and south latitude and west longitude are expressed as negative numbers when calculating distance and bearing. The formula to convert latitude or longitude in degrees, minutes, seconds to decimal degrees can be found in Section 31.4.

$$D = 69.047 * cos^{-1}[sin(Lat_{Tx}) * sin(Lat_{Rx}) + cos(Lat_{Tx}) * \cos(Lat_{Rx}) * cos(Long_{Tx} - Long_{Rx})]$$

$$\varphi = cos^{-1}\left[\frac{sin(Lat_{Rx}) - sin(Lat_{Tx}) * cos\left(\frac{D}{69.047}\right)}{sin\left(\frac{D}{69.047}\right) * cos(Lat_{Tx})}\right]$$

If $Long_{Rx}$ is less than $Long_{Tx}$, then the transmit antenna bearing = 360 – φ. If $Long_{Rx}$ is greater than $Long_{Tx}$, then the transmit antenna bearing = φ

where
D is the total path distance in statute miles
φ is the bearing (azimuth) from the transmit antenna to the receive antenna relative to true north in decimal degrees
cos is cosine
cos^{-1} is arccosine or inverse cosine
sin is sine
Lat_{Tx} is the transmit antenna site latitude in decimal degrees
Lat_{Rx} is the receive antenna site latitude in decimal degrees
$Long_{Tx}$ is the transmit antenna site longitude in decimal degrees
$Long_{Rx}$ is the receive antenna site longitude in decimal degrees

Example:
A CARS transmit antenna has a north latitude (Lat_{Tx}) of 37.738841 decimal degrees and a west longitude ($Long_{Tx}$) of –76.199829 decimal degrees. The receive antenna has a north latitude (Lat_{Rx}) of 37.582465 decimal degrees and a west longitude ($Long_{Rx}$) of –76.025857 decimal degrees. Calculate the path distance and bearing between the transmit antenna and the receive antenna.

Solution:
First, calculate the path distance:

$$D = 69.047 * cos^{-1}[sin(Lat_{Tx}) * sin(Lat_{Rx}) + cos(Lat_{Tx}) * \cos(Lat_{Rx}) * cos(Long_{Tx} - Long_{Rx})]$$

$$D = 69.047 * cos^{-1}[sin(37.738841) * sin(37.582465) + cos(37.738841) * \cos(37.582465) * cos(-76.199829 - (-76.025857))]$$

$$D = 69.047 * cos^{-1}[(0.61206327 * 0.60990266) + (0.79080879 * 0.79247634 * cos(-0.173972))]$$

$$D = 69.047 * cos^{-1}[(0.61206327 * 0.60990266) + (0.79080879 * 0.79247634 * 0.99999539)]$$

$$D = 69.047 * cos^{-1}[(0.61206327 * 0.60990266) + (0.79080879 * 0.79247634 * 0.99999539)]$$

$$D = 69.047 * cos^{-1}[0.99999338]$$

$$D = 69.047 * 0.20848118$$

$$D = 14.39500004$$

Answer: The path distance between the transmit antenna and the receive antenna is about 14.4 statute miles (about 23.17 kilometers).

Next, calculate the bearing:

$$\varphi = cos^{-1}\left[\frac{sin(Lat_{Rx}) - sin(Lat_{Tx}) * cos\left(\frac{D}{69.047}\right)}{sin\left(\frac{D}{69.047}\right) * cos(Lat_{Tx})}\right]$$

$$\varphi = cos^{-1}\left[\frac{sin(37.582465) - sin(37.738841) * cos\left(\frac{14.39500004}{69.047}\right)}{sin\left(\frac{14.39500004}{69.047}\right) * cos(37.738841)}\right]$$

$$\varphi = cos^{-1}\left[\frac{0.60990266 - 0.61206327 * cos(0.20848118)}{sin(0.20848118) * cos(37.738841)}\right]$$

$$\varphi = cos^{-1}\left[\frac{0.60990266 - 0.61206327 * 0.99999338}{0.00363868 * 0.79080879}\right]$$

$$\varphi = cos^{-1}\left[\frac{-0.00215656}{0.0028775}\right]$$

$$\varphi = cos^{-1}[-0.74945613]$$

$$\varphi = 138.54$$

If $Long_{Rx}$ is less than $Long_{Tx}$, then the transmit antenna bearing = 360 – φ. If $Long_{Rx}$ is greater than $Long_{Tx}$, then the transmit antenna bearing = φ.

Since –76.025857 – (–76.199829) = 0.173972, $Long_{Rx}$ is greater than $Long_{Tx}$, the transmit antenna bearing = φ

Answer: The bearing (azimuth) between the transmit antenna and the receive antenna relative to true north is 138.54 decimal degrees.

Note that the bearing for the receive antenna is the opposite of that at the transmit antenna; that is, for φ_{Tx} between 0 and 180 degrees, $\varphi_{Rx} = \varphi_{Tx} + 180$. For φ_{Tx} between 180 and 360 degrees, $\varphi_{Rx} = \varphi_{Tx} - 180$.

24.5. Sample path analysis

An important part of terrestrial microwave path analysis is calculation of input power at the microwave receiver. The process is relatively straightforward, involving little more than starting with transmitter output power, then subtracting various transmit site passive device, waveguide, and other losses; adding transmit and receive antenna gains; subtracting free space path loss; and subtracting various receive site waveguide and passive device losses. The following formula can be used to calculate microwave receiver input power:

$$P_{RX} = P_{TX} - L_{TX\,passive} - L_{TX\,wg} + G_{TX} - L_{path} + G_{RX} - L_{RX\,wg} - L_{passive} - F$$

where
P_{RX} is transmitter output power in decibel milliwatt (typ. power per channel)
P_{TX} is transmitter output power in decibel milliwatt (typ. power per channel)
$L_{TX\,passive}$ is the total insertion of loss of transmit site passive components (directional couplers, magic tees, circulators, attenuators, etc.) in decibels
$L_{TX\,wg}$ is the total attenuation of transmit site waveguide (rectangular, elliptical, circular) in decibels
G_{TX} is the transmit antenna gain in decibel isotropic
L_{path} is the free space path loss between the transmit and receive antennas in decibels (see Section 24.1)
G_{RX} is the receive antenna gain in decibel isotropic
$L_{RX\,wg}$is the total attenuation of receive site waveguide (rectangular, elliptical, circular) in decibels
$L_{passive}$ is the total insertion loss of receive site passive components (filters, circulators, etc.) in decibels
F is field factor in decibels (typ. 3 dB) to account for antenna misalignment, polarization misalignment, etc.

Example:
The following example is for a hypothetical 12.7 GHz to 13.2 GHz CARS band multi-channel amplitude modulated link path analysis to highlight the basic concepts of calculating predicted path performance. The assumption is 55 analog NTSC channels. Transmitter output power and receiver noise and distortion performance parameters are from original Hughes AML equipment specifications and are used here for illustrative purposes only. Not included are statistical estimates for path reliability.

PATH FROM MT. VISTA TO ASH STREET HUB

TRANSMITTER OUTPUT POWER	55	(DBM/CH)	17.0
6 DB DIRECTIONAL COUPLER			–6.0

LEVEL SET ATTENUATOR			–0.9
TRANSMIT ELLIPTICAL WAVEGUIDE	75	FEET	–2.8
TRANSMIT ANTENNA	8	FEET	47.6
FREE SPACE PATH LOSS	14.4	STATUTE MILES	–141.8
RECEIVE ANTENNA	8	FEET	47.6
RECEIVE ELLIPTICAL WAVEGUIDE	5	FEET	–0.2
FIELD FACTOR			–3.0
RECEIVE CARRIER LEVEL			–42.5

NOISE PER MHZ	–114.0
4 MHZ CORRECTION	6.0
RECEIVER NOISE FIGURE	8.0
RECEIVER THERMAL NOISE	–100.0

RECEIVER CARRIER-TO-NOISE RATIO WITHOUT AGC	57.5
RECEIVER INPUT AGC ATTENUATION	–4.5
RECEIVER CARRIER-TO-NOISE RATIO IN AGC	53.0
RECEIVER COMPOSITE TRIPLE BEAT IN AGC	83.4

25. Fiber Optics

Cable operators have been using optical fiber technology in hybrid fiber/coax (HFC) architectures since the late 1980s. This section includes several formulas and examples applicable to optical fiber and optical links, with a focus on analog links.

25.1. Calculate index of refraction

The speed of an electromagnetic signal is slightly slower when traveling through a physical medium such as air, water or glass than it is in a vacuum. As a general rule the speed of light decreases as the density of the medium increases. The index of refraction is a measure of that reduction in speed, defined as the ratio of the speed of light in a vacuum to the speed of light in the material through which the electromagnetic signal is traveling.

The following formula can be used to calculate index of refraction:

$$n = \frac{c_0}{c}$$

where
n is index of refraction
c_0 is the speed of light in a vacuum (299,792,458 meters per second or 983,571,056.43 feet per second)
c is the speed of light in some other medium (same units as c_0)

Example:
What is the index of refraction of an optical fiber when the speed of light through that fiber is 670,190,144.75 feet per second?

Solution:

$$n = \frac{c_0}{c}$$

$$n = \frac{983{,}571{,}056.43}{670{,}190{,}144.75}$$

$$n = 1.4676$$

Answer: The index of refraction is 1.4676.

25.2. Calculate speed of light in optical fiber

The following formula can be used to calculate the speed of light in optical fiber when the index of refraction is known:

$$c = \frac{c_0}{n}$$

where
c is the speed of light in some other medium (same units as c_0)
c_0 is the speed of light in a vacuum (299,792,458 meters per second or 983,571,056.43 feet per second)
n is index of refraction

Example:
What is the speed of light in meters per second in an optical fiber with an index of refraction of 1.4676?

Solution:

$$c = \frac{c_0}{n}$$

$$c = \frac{299{,}792{,}458}{1.4676}$$

$$c = 204{,}273{,}956.12$$

Answer: The speed of light in the example optical fiber is 204,273,956.12 meters per second.

25.3. Optical fiber velocity factor and velocity of propagation

If one knows the index of refraction for a given optical fiber, the fiber's velocity factor and velocity of propagation can be calculated using the formulas in this section.

25.3.1. Calculate optical fiber velocity factor

The following formula can be used to calculate velocity factor for optical fiber when the fiber's index of refraction is known:

$$VF = \frac{1}{n}$$

where
VF is velocity factor
n is index of refraction

Example:
The published effective group index of refraction for a popular single mode optical fiber is 1.4676 at 1,310 nanometers. What is the fiber's velocity factor at that wavelength?

Solution:
$$VF = \frac{1}{n}$$

$$VF = \frac{1}{1.4676}$$

$$VF = 0.6814$$

Answer: The fiber's VF at 1,310 nm is approximately 0.68.

25.3.2. Calculate optical fiber velocity of propagation

From Section 20.4, velocity of propagation is velocity factor expressed as a percentage:

$$VoP = VF * 100$$

where
VoP is velocity of propagation
VF is the velocity factor

Example:
What is the VoP for the single mode optical fiber in the previous example?

Solution:
$$VoP = VF * 100$$

$$VoP = 0.6814 * 100$$

$$VoP = 68.14$$

Answer: The single mode optical fiber's VoP at 1,310 nm is 68.14%.

The "speed" of coax
Some are surprised to learn that light travels through optical fiber slightly slower than RF travels through coaxial cable. From the previous example, the speed of light (at 1,310 nm) through the fiber is 68.14% of the speed of light in a vacuum, or 983,571,056.43 ∗ 0.6814 = 670,190,144.75 feet per second. The velocity of propagation for some hardline coaxial cables is 87%, which gives a speed for RF traveling through the coax of 983,571,056.43 ∗ 0.87 = 855,706,819.09 feet per second. Looked at another way, light or RF in a vacuum travels 1 ft. in 1/983,571,056.43 = 1.02E–9 second (1.02 ns); light (at 1,310 nm) travels through 1 ft. of single mode optical fiber in 1/670,190,144.75 = 1.49E–9 second (1.49 ns); and RF travels through 1 ft. of coaxial cable in 1/855,706,819.09 = 1.17E–9 second (1.17 ns).

25.4. Calculate optical wavelength

The following formula can be used to calculate optical wavelength in nanometers (nm) when the frequency in terahertz (THz) is known:

$$\lambda_{nm} = \frac{299{,}792.458}{f_{THz}}$$

where
λ_{nm} is wavelength in nanometers
f_{THz} is frequency in terahertz (THz)

Example:
What is the wavelength of an optical signal whose frequency is 193.414 THz?

Solution:

$$\lambda_{nm} = \frac{299{,}792.458}{f_{THz}}$$

$$\lambda_{nm} = \frac{299{,}792.458}{193.414}$$

$$\lambda_{nm} = 1{,}550.004$$

Answer: The wavelength of a 193.414 THz optical signal is 1,550 nm.

It is common to describe optical sources, optical fiber links, and optical signals by wavelength instead of frequency – for instance, 1,550 nm instead of 193.414 THz or 1,310 nm instead of 228.849 THz. An important point: By convention, the optical wavelength is its value in a vacuum (free space), and the formula in this section calculates that free-space value.

Note that the velocity of propagation of an optical signal is *slower* in optical fiber than it is in a vacuum. If one were to measure the wavelength of a 1,310 nm optical signal inside of single mode optical fiber, the measured wavelength value would be *less* than 1,310 nm because of the fiber's velocity of propagation compared to that of a vacuum. Even so, the signal would still be called a 1,310 nm signal.

25.5. Calculate frequency

The following formula can be used to calculate frequency in terahertz (THz) when the optical wavelength in nanometers (nm) is known:

$$f_{THz} = \frac{299{,}792.458}{\lambda_{nm}}$$

where
f_{THz} is frequency in terahertz (THz)
λ_{nm} is wavelength in nanometers

Example:
What is the frequency in terahertz of an optical signal whose wavelength is 1,310 nanometers?

Solution:

$$f_{THz} = \frac{299{,}792.458}{\lambda_{nm}}$$

$$f_{THz} = \frac{299{,}792.458}{1{,}310}$$

$$f_{THz} = 228.849$$

Answer: The frequency of a 1310 nm optical signal is 228.85 THz.

25.6. Optical return loss

Like RF return loss (see Section 21.4) one can think of optical return loss in units of decibels. Optical return loss at the interface between two media with different indices of refraction can be calculated with the following formula:

$$R = -10log_{10}\left[\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]$$

where
R is return loss in decibels
log_{10} is base 10 logarithm
n_1 is the index of refraction for one medium
n_2 is the index of refraction for the other medium

Example:
What is the optical return loss at the interface of a single mode optical fiber with an index of refraction of 1.4676 and a splice with an index of refraction of 1.473?

Solution:

$$R = -10log_{10}\left[\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]$$

$$R = -10 * log_{10}\left[\left(\frac{1.4676 - 1.473}{1.4676 + 1.473}\right)^2\right]$$

$$R = -10 * log_{10}\left[\left(\frac{-0.0054}{2.9406}\right)^2\right]$$

$$R = -10 * log_{10}[0.00000337]$$

$$R = -10 * [-5.47208439]$$

$$R = 54.72$$

Answer: The optical return loss is 54.72 dB.

25.7. Noise in analog optical links

This section includes formulas to calculate CNR in analog optical links; the information here is adapted from [4] and [5]. The contribution of erbium doped fiber amplifiers (EDFAs) is provided to support analysis of systems that use optical amplifiers (note that 1,310 nm links do not use optical amplifiers). Refer to Appendix F for more information on analog intensity modulation, which is used in the majority of optical links in HFC networks.

25.7.1. Laser Noise

One of the contributors to the carrier-to-noise ratio in an analog optical link is laser noise, specifically what is called relative intensity noise (RIN). The RIN produced by a laser is caused by the spontaneous emission of photons, and results in the production of non-coherent light.

The following formula can be used to calculate laser noise CNR contribution as CNR_{RIN} in decibels:

$$CNR_{RIN} = 20log_{10}(m) - 10log_{10}(2B) - RIN$$

where
CNR_{RIN} is the carrier-to-noise ratio in decibels as a result of RIN
log_{10} is base 10 logarithm
m is the per-channel optical modulation index (OMI)
B is the noise measurement bandwidth in hertz
RIN is relative intensity noise

Example:
Assume a loading of 78 analog NTSC channels and 33 SC-QAM signals, resulting in a per-channel OMI of 3.58%. What is the laser noise contribution (CNR_{RIN}) in which the noise measurement bandwidth is 4 MHz (4,000,000 Hz), and the RIN is –160 dB/Hz?

Solution:
$CNR_{RIN} = 20log_{10}(m) - 10log_{10}(2B) - RIN$

$CNR_{RIN} = 20log_{10}(0.0358) - 10log_{10}(2 * 4{,}000{,}000) - (-160)$

$CNR_{RIN} = [20 * log_{10}(0.0358)] - [10 * log_{10}(8{,}000{,}000)] - (-160)$

$CNR_{RIN} = [20 * (-1.4461)] - [10 * (6.9031)] - (-160)$

$CNR_{RIN} = [-28.9223] - [69.0309] - (-160)$

$CNR_{RIN} = 62.0468$

Answer: The CNR_{RIN} is 62.05 dB.

25.7.2. EDFA Noise

If a 1,550 nm optical link includes an erbium doped fiber amplifier, that optical amplifier will contribute to the overall CNR of the optical link. Noise in an optical amplifier is produced by the spontaneous emission of photons, and is known as amplified spontaneous emission (ASE).

The following formula can be used to calculate optical amplifier CNR contribution as CNR_{EDFA} in decibels:

$$CNR_{EDFA} = 86.2 + P_{input} + 20log_{10}(m) - NF_{EDFA}$$

where
CNR_{EDFA} is the EDFA carrier-to-noise ratio per channel in decibels (measured in a 4 MHz bandwidth)
P_{input} is the optical input power to the EDFA, in dBm
log_{10} is base 10 logarithm
m is the per-channel optical modulation index (OMI)
NF_{EDFA} is the noise figure of the optical amplifier, in decibels

Example:
What is the CNR of an EDFA that has a noise figure of 5.5 dB, an optical input power of +5 dBm, and a per-channel OMI of 3.58% (m = 0.0358)?

Solution:
$CNR_{EDFA} = 86.2 + P_{input} + 20log_{10}(m) - NF_{EDFA}$

$CNR_{EDFA} = 86.2 + 5 + [20 * log_{10}(0.0358)] - 5.5$

$CNR_{EDFA} = 86.2 + 5 + [20 * (-1.4461)] - 5.5$

$CNR_{EDFA} = 86.2 + 5 + [-28.9223] - 5.5$

$CNR_{EDFA} = 56.78$

Answer: The CNR_{EDFA} is 56.78 dB.

25.7.3. Receiver shot noise

From [4], "The noise performance of an optical receiver is limited by the noise current of the diode (shot noise due to the statistical variation in arriving photon distribution) …" The following formula can be used to calculate the carrier-to-noise ratio contribution of receiver shot noise:

$$CNR_{SHOT} = P_R + 20log_{10}\left(\frac{m}{\sqrt{2}}\right) + 10log_{10}(R) - 10log_{10}(BW) + 154.94$$

where
CNR_{SHOT} is the carrier-to-noise ratio per channel in decibels due to shot noise in the receiver
P_R is the optical input power to the receiver, in dBm
log_{10} is base 10 logarithm
m is the per-channel optical modulation index (OMI)
R is the responsivity of the photodiode detector in amperes per watt (or milliamperes per milliwatt)
BW is the noise susceptibility bandwidth of the channel in hertz (e.g., 4,000,000 Hz)

Example:
What is the CNR contribution of an optical receiver because of shot noise, when the received optical power is 0 dBm, the per-channel OMI is 3.58% (m = 0.0358), the photodiode responsivity is 1.0 A/W, and the noise susceptibility bandwidth is 4 MHz?

Solution:
$CNR_{SHOT} = P_R + 20log_{10}\left(\frac{m}{\sqrt{2}}\right) + 10log_{10}(R) - 10log_{10}(BW) + 154.94$

$CNR_{SHOT} = 0 + 20log_{10}\left(\frac{0.0358}{\sqrt{2}}\right) + 10log_{10}(1.0) - 10log_{10}(4{,}000{,}000) + 154.94$

$CNR_{SHOT} = 0 + \left[20 * log_{10}\left(\frac{0.0358}{\sqrt{2}}\right)\right] + [10 * log_{10}(1.0)] - [10 * log_{10}(4{,}000{,}000)] + 154.94$

$CNR_{SHOT} = 0 + \left[20 * log_{10}\left(\frac{0.0358}{1.4142}\right)\right] + [10 * (0)] - [10 * (6.6021)] + 154.94$

$CNR_{SHOT} = 0 + [20 * log_{10}(0.0253)] + [0] - [66.0206] + 154.94$

$CNR_{SHOT} = 0 + [20 * (-1.5966)] + [0] - [66.0206] + 154.94$

$CNR_{SHOT} = 0 + [-31.9326] + [0] - [66.0206] + 154.94$

$CNR_{SHOT} = 56.9868$

Answer: The CNR_{SHOT} is 56.99 dB.

25.7.4. Receiver thermal noise

The noise performance of an optical receiver is further limited by thermal noise generated within the post-detection amplifier stage, typically a transimpedance amplifier.

The following formula can be used to calculate the carrier-to-noise ratio contribution of the transimpedance amplifier's thermal noise:

$$CNR_{THERMAL} = 2P_R + 20log_{10}\left(\frac{m}{\sqrt{2}}\right) + 20log_{10}(R) - 10log_{10}(BW) - 20log_{10}(I_r) + 180$$

where
$CNR_{THERMAL}$ is the carrier-to-noise ratio per channel in decibels due to thermal noise in the receiver
P_R is the optical input power to the receiver, in dBm
log_{10} is base 10 logarithm
m is the per-channel optical modulation index (OMI)
R is the responsivity of the photodiode detector in amperes per watt (or milliamperes per milliwatt)
BW is the noise susceptibility bandwidth of the channel in hertz (e.g., 4,000,000 Hz)
I_r is the post-amplifier equivalent noise current density in $pA/\sqrt{Hz}$ (typ. values are 6 to 8)
180 is a factor that results from converting the input power to milliwatts from watts and the amplifier noise from amperes to picoamperes

Example:
What is the CNR contribution of an optical receiver because of thermal noise, when the received optical power is 0 dBm, the per-channel OMI is 3.58% (m = 0.0358), the photodiode responsivity is 1.0 A/W, and noise susceptibility bandwidth is 4 MHz, and the amplifier input noise current density is 7 $pA/\sqrt{Hz}$?

Solution:
$$CNR_{THERMAL} = 2P_R + 20log_{10}\left(\frac{m}{\sqrt{2}}\right) + 20log_{10}(R) - 10log_{10}(BW) - 20log_{10}(I_r) + 180$$

$$CNR_{THERMAL} = 2(0) + \left[20 * log_{10}\left(\frac{0.0358}{\sqrt{2}}\right)\right] + [20 * log_{10}(1.0)] - [10 * log_{10}(4{,}000{,}000)] - [20 * log_{10}(7)] + 180$$

$$CNR_{THERMAL} = 0 + \left[20 * log_{10}\left(\frac{0.0358}{1.4142}\right)\right] + [20 * (0)] - [10 * (6.6021)] - [20 * (0.8451)] + 180$$

$$CNR_{THERMAL} = 0 + [20 * log_{10}(0.0253)] + [0] - [66.0206] - [16.9020] + 180$$

$$CNR_{THERMAL} = 0 + [20 * (-1.5966)] + [0] - [66.0206] - [16.9020] + 180$$

$$CNR_{THERMAL} = 0 + [-31.9326] + [0] - [66.0206] - [16.9020] + 180$$

$$CNR_{THERMAL} = 65.1448$$

Answer: The $CNR_{THERMAL}$ is 65.14 dB.

25.7.5. Combined optical link CNR

The total CNR for an optical link can be calculated using the following formula:

$$CNR_{total} = -10\, log_{10}\left[10^{\frac{-CNR_{RIN}}{10}} + 10^{\frac{-CNR_{EDFA}}{10}} + 10^{\frac{-CNR_{SHOT}}{10}} + 10^{\frac{-CNR_{THERMAL}}{10}}\right]$$

where
CNR_{total} is the combined carrier-to-noise ratio in decibels
log_{10} is base 10 logarithm
CNR_{RIN} is the laser's carrier-to-noise ratio due to RIN
CNR_{EDFA} is the carrier-to-noise ratio in the optical amplifier (not applicable in 1,310 nm links)
CNR_{SHOT} is the carrier-to-noise ratio in the receiver due to shot noise
$CNR_{THERMAL}$ is the carrier-to-noise ratio in the receiver due to thermal noise

Example:
What is the total CNR in an optical link using the calculated CNR values from the examples in the previous sections?
CNR_{RIN} = 62.05 dB
CNR_{EDFA} = 56.78 dB
CNR_{shot} = 56.99 dB
$CNR_{thermal}$ = 65.14 dB

Solution:

$$CNR_{total} = -10\, log_{10}\left[10^{\frac{-CNR_{RIN}}{10}} + 10^{\frac{-CNR_{EDFA}}{10}} + 10^{\frac{-CNR_{SHOT}}{10}} + 10^{\frac{-CNR_{THERMAL}}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}\left[10^{\frac{-62.05}{10}} + 10^{\frac{-56.78}{10}} + 10^{\frac{-56.99}{10}} + 10^{\frac{-65.14}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-6.205} + 10^{-5.678} + 10^{-5.699} + 10^{-6.514}]$$

$CNR_{total} = -10 * log_{10}[0.00000062 + 0.00000210 + 0.00000200 + 0.00000031]$

$CNR_{total} = -10 * log_{10}[0.00000503]$

$CNR_{total} = -10 *[-5.2985]$

$CNR_{total} = 52.9854$

Answer: The total CNR in the example fiber link is 52.99 dB.

Note: When calculating CNR in a multiple-wavelength optical link, the CNR is calculated separately for each wavelength. The optical power referred to in the previous formulas and examples refers to the power of a single wavelength, not the aggregate power.

25.7.6. Calculate combined CNR of headend, optical link, and amplifier cascade

The following formula, adapted from Section 14.3, can be used to calculate the combined CNR of the headend or hub, the optical fiber link, and an amplifier cascade after the node:

$$CNR_{total} = -10\, log_{10}\left[10^{\frac{-CNR_{headend}}{10}} + 10^{\frac{-CNR_{optical}}{10}} + 10^{\frac{-CNR_{cascade}}{10}}\right]$$

where
CNR_{total} is the total carrier-to-noise ratio
log_{10} is base 10 logarithm
$CNR_{headend}$ is the combined carrier-to-noise ratio of all of the headend (or hub) processors, modulators, and other devices
$CNR_{optical}$ is the carrier-to-noise ratio of the optical fiber link
$CNR_{cascade}$ is the carrier-to-noise ratio of the amplifier cascade after the node

Example:
Assume a headend's combined CNR at the input to the downstream laser transmitter is 55 dB. Using the optical fiber link combined CNR from Section 25.7.5 (52.99 dB) and amplifier cascade CNR from Section 14.2 (49.86 dB), what is the total (end-of-line) CNR?

Solution:

$$CNR_{total} = -10\, log_{10}\left[10^{\frac{-CNR_{headend}}{10}} + 10^{\frac{-CNR_{optical}}{10}} + 10^{\frac{-CNR_{cascade}}{10}}\right]$$

$$CNR_{total} = -10\, log_{10}\left[10^{\frac{-55}{10}} + 10^{\frac{-52.99}{10}} + 10^{\frac{-49.86}{10}}\right]$$

$$CNR_{total} = -10 * log_{10}[10^{-5.500} + 10^{-5.299} + 10^{-4.986}]$$

$$CNR_{total} = -10 * log_{10}[0.00000316 + 0.00000502 + 0.00001033]$$

$$CNR_{total} = -10 * log_{10}[0.00001851]$$

$$CNR_{total} = -10 * [-4.73251575]$$

$$CNR_{total} = 47.33$$

Answer: The total (end-of-line) CNR is 47.33 dB.

25.8. Calculate optical link budget

Calculation of an optical link budget is relatively straightforward, involving little more than addition and/or subtraction of gains and losses (in decibels) in the path between the output of an optical transmitter and the input to an optical receiver. One can use either the manufacturer's published loss specifications or actual measured values.

25.8.1. Calculate total optical fiber loss

Optical fiber loss is generally specified in decibels per kilometer (dB/km) at the desired operating wavelength. The following table summarizes some typical values.

Table 17 - Typical optical fiber attenuation values.

Wavelength	Maximum loss (dB/km)
1,310 nm	0.33 to 0.35
1,550 nm	0.19 to 0.20
1,625 nm	0.20 to 0.23

The following formula can be used to calculate the total attenuation in a span or length of fiber when the loss per kilometer is known:

$$L_{fiber} = L_{dB/km} * D_{km}$$

where
L_{fiber} is the total insertion loss of the optical fiber at the wavelength in use, in decibels
$L_{dB/km}$ is the optical fiber attenuation in decibels per kilometer at the wavelength of interest
D_{km} is the total length of the fiber span in kilometers

Example:
What is the total attenuation of a 9.25 km span of fiber at 1,310 nm? Use the maximum value in Table 17.

Solution:
$L_{fiber} = L_{dB/km} * D_{km}$

$L_{fiber} = 0.35 * 9.25$

$L_{fiber} = 3.24$

Answer: The total attenuation of a 9.25 km span of fiber at 1,310 nm is 3.24 dB.

25.8.2. Calculate optical coupler ideal insertion loss

Optical couplers are typically specified by the output power as a function of input power measured in percent (%). For example, a 1 x 2 optical coupler that divides the input optical power equally between the two outputs is called a 50/50 coupler (50% of the input power is present at each of the two outputs). Actual insertion loss is somewhat higher than the calculated ideal value.

The following formula can be used to convert coupled power in percent to decibels:

$$L_{ideal} = -10log_{10}\left(P_{coupled}\right)$$

where
L_{ideal} is coupler ideal insertion loss in decibels
log_{10} is base 10 logarithm
$P_{coupled}$ is the coupled power percentage in decimal form

Example 1:
What is the ideal insertion loss of a 50/50 optical coupler?

Solution 1:
$L_{ideal} = -10log_{10}\left(P_{coupled}\right)$

$L_{ideal} = -10 * log_{10}(0.50)$

$L_{ideal} = -10 * (-0.3010)$

$L_{ideal} = 3.0103$

Answer: The ideal insertion loss of a 50/50 optical coupler is 3.01 dB (per output port).

Example 2:
What is the ideal insertion loss of a 90/10 coupler?

Solution 2:
First calculate the ideal insertion loss of the 90% coupled output port.

$L_{ideal} = -10log_{10}(P_{coupled})$

$L_{ideal} = -10 * log_{10}(0.90)$

$L_{ideal} = -10 * (-0.0458)$

$L_{ideal} = 0.4576$

Answer: The ideal insertion loss of the 90% coupled port is 0.46 dB.

Next, calculate the insertion loss of the 10% coupled port.
$L_{ideal} = -10log_{10}(P_{coupled})$

$L_{ideal} = -10 * log_{10}(0.10)$

$L_{ideal} = -10 * (-1.0000)$

$L_{ideal} = 10.00$

Answer: The ideal insertion loss of the 10% coupled port is 10 dB.

Note: Typical insertion loss values for the examples in this section would be 3.40 dB/3.40 dB for the 50/50 coupler, and 0.60 dB/10.80 dB for the 90/10 coupler. Refer to the coupler manufacturer's specifications for actual insertion loss values in decibels.

25.8.3. Calculate total loss between optical transmitter and receiver

The following formula can be used to calculate the total loss in decibels between an optical transmitter and optical receiver:

$$L_{total} = L_{connector} + L_{splice} + L_{coupler} + L_{fiber} + L_{misc}$$

where
L_{total} is the total combined insertion loss of all components, devices, and optical fiber between an optical transmitter and receiver, in decibels
$L_{connector}$ is the total loss of all mechanical connectors, in decibels
L_{splice} is the total loss of all splices, in decibels
$L_{coupler}$ is the total insertion loss of optical couplers, in decibels
L_{fiber} is the total insertion loss of the optical fiber at the wavelength in use, in decibels
L_{misc} is the total loss of other devices or components (attenuators, multiplexers, etc.), in decibels

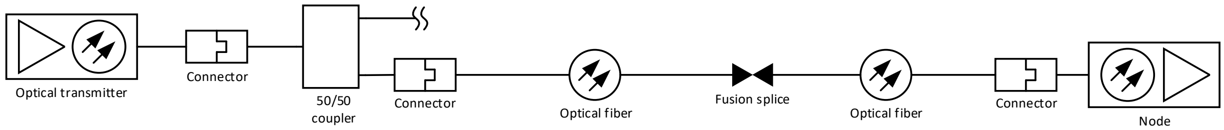


Figure 42 - Sample optical link.

Example:
What is the total insertion loss between the optical transmitter and optical receiver for the sample optical link in Figure 42? Assume the following loss values.
Connector: 0.2 dB
50/50 coupler: 3.4 dB
Fusion splice: 0.05 dB
Optical fiber: 9.25 km, 0.35 dB/km at 1,310 nm = 3.24 dB

Solution:
$L_{total} = L_{connector} + L_{splice} + L_{coupler} + L_{fiber} + L_{misc}$

$L_{total} = (3 * 0.2) + 0.05 + 3.4 + 3.24 + 0$

$L_{total} = 0.6 + 0.05 + 3.4 + 3.24 + 0$

$L_{total} = 7.29$

Answer: The total insertion loss between the optical transmitter and receiver is 7.29 dB.

25.8.4. Calculate optical receiver input power

The following formula can be used to calculate optical receiver input power:

$$P_R = P_T - L_{total}$$

where
P_R is optical receiver input power, in decibel milliwatt (dBm)
P_T is the optical transmitter output power, in dBm
L_{total} is the total combined insertion loss of all components, devices, and optical fiber between an optical transmitter and receiver, in decibels

Example:
What is the input power (in dBm) to the optical receiver in Figure 42 if the optical transmitter output is +6 dBm and the total insertion loss between the transmitter and receiver is 7.29 dB?

Solution:
$P_R = P_T - L_{total}$

$P_R = 6 - 7.29$

$P_R = -1.29$

Answer: The input power to the optical receiver is –1.29 dBm.

26. Ohm's Law

Ohm's Law states that electric current is proportional to voltage, and inversely proportional to resistance.

Current can be thought of as the flow of charged particles per unit of time. An analogy is the volume of water flowing through a garden hose, for instance, 1 gallon of water per second. Ampere is a measure of electric current, where 1 ampere equals 1 coulomb of charge flowing past a given point in 1 second. Coulomb is a unit of measure of electrically charged particles, where 1 coulomb = $6.242 * 10^{18}$ electrons. Electromotive force (EMF) is the force of electrical attraction between two points of different charge potential. EMF is more commonly known as voltage (technically speaking, the volt is a measure of electromotive force), and is analogous to water pressure in a garden hose. 1 volt is the potential difference between two points on a wire carrying 1 ampere of current when the power dissipated between the points is 1 watt.

Resistance (R) is an opposition to the flow of current. Ohm is a unit of resistance, where 1 ohm is defined as the resistance that allows 1 ampere of current to flow between two points that have a potential difference of 1 volt.

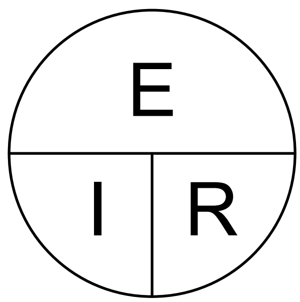


Figure 43 - This graphic can help one remember the three mathematical expressions of Ohm's Law.

The following three formulas are expressions of Ohm's Law, which can be remembered using the graphic in Figure 43:

$$I = \frac{E}{R}$$

$$R = \frac{E}{I}$$

$$E = I * R$$

where
I is current in amperes
E is voltage in volts
R is resistance in ohms

Example 1:
Referring to Figure 44, what is the current in the circuit when the voltage is 12 volts and the resistance is 75 ohms?

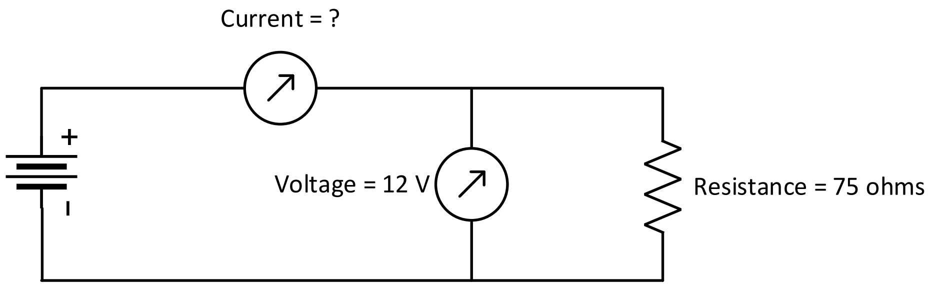


Figure 44 - What is the current with the voltage and resistance values shown?

Solution 1:
$I = \frac{12}{75}$

$I = 0.16$

Answer: The current is 0.16 ampere.

Example 2:
Referring to Figure 45, what is the voltage in the circuit when the current in 0.16 ampere and the resistance is 75 ohms?

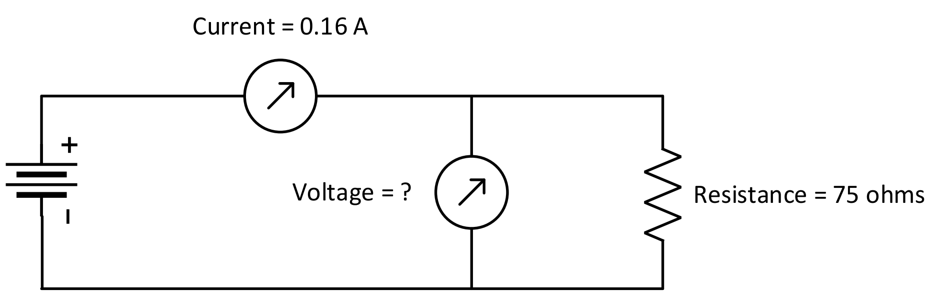


Figure 45 - What is the voltage with the current and resistance values shown?

Solution 2:
$E = 0.16 * 75$

$E = 12$

Answer: The voltage is 12 volts.

Example 3:
Referring to Figure 46, what is the resistance when the current is 0.16 ampere and the voltage is 12 volts?

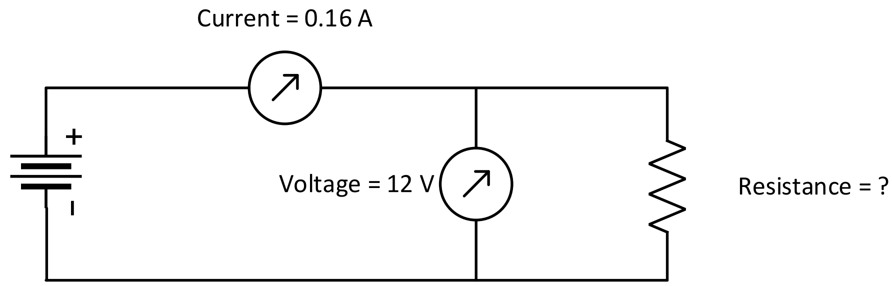


Figure 46 - What is the resistance with the current and voltage values shown?

Solution 3:

$$R = \frac{12}{0.16}$$

$$R = 75$$

Answer: The resistance is 75 ohms.

27. Power in Direct Current Circuits

Power is the rate at which work is done, or energy per unit of time – that is, power can be described as the rate at which energy is consumed in a circuit. 1 watt of power is equal to 1 volt causing a current of 1 ampere. Watt (W) is the power required to do work at a rate of 1 joule per second (J/s). That is, a joule of work per second is 1 watt. One joule is the work done by a force of 1 newton acting over a distance of 1 meter. The joule is a measure of a quantity of energy and equals 1 watt-second.

If you think about it for a moment, 1 watt is simply the use or generation of 1 joule of energy per second. Other electrical units are in fact derived from the watt. For instance, 1 volt is 1 watt per ampere.

27.1. Basic expressions of power in direct current circuits

Another definition of 1 watt is 1 volt of potential (EMF) "pushing" 1 ampere of current through a resistance, or $P = E * I$.

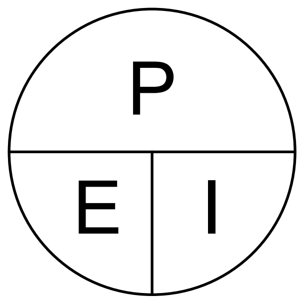


Figure 47 - This graphic can help one remember the basic relationships among power, voltage, and current.

The following three formulas show the basic relationship among power, voltage, and current, and can be remembered using the graphic in Figure 47.

$$P = E * I$$

$$E = \frac{P}{I}$$

$$I = \frac{P}{E}$$

where
P is power in watts
E is voltage in volts
I is current in amperes

Example 1:
Refer to Figure 48. What is the power dissipated in the resistor when the voltage is 12 volts and the current is 0.16 ampere (160 mA)?

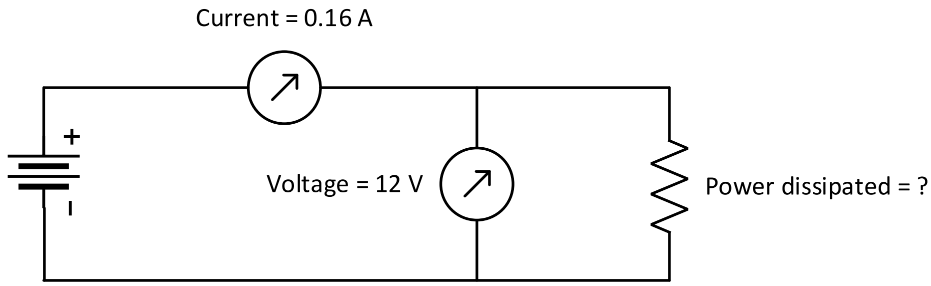


Figure 48 - What is the power dissipated in the resistor with the current and voltage values shown?

Solution 1:

$P = E * I$

$P = 12 * 0.16$

$P = 1.92$

Answer: The power dissipated in the resistor is 1.92 watts.

Example 2:
Refer to Figure 49. What is the voltage when the current is 1.16 ampere and the power dissipated in the resistor is 1.92 watts?

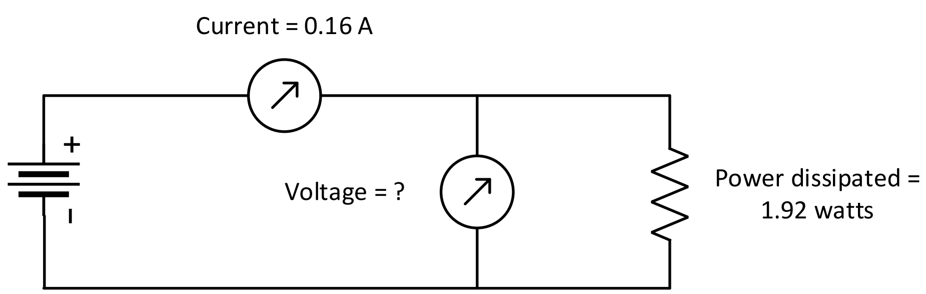


Figure 49 - What is the voltage with the current and power values shown?

Solution 2:

$E = \frac{P}{I}$

$E = \frac{1.92}{0.16}$

$E = 12$

Answer: The voltage is 12 volts.

Example 3:
Refer to Figure 50. What is the current if the voltage is 12 volts and the power dissipated in the resistor is 1.92 watts?

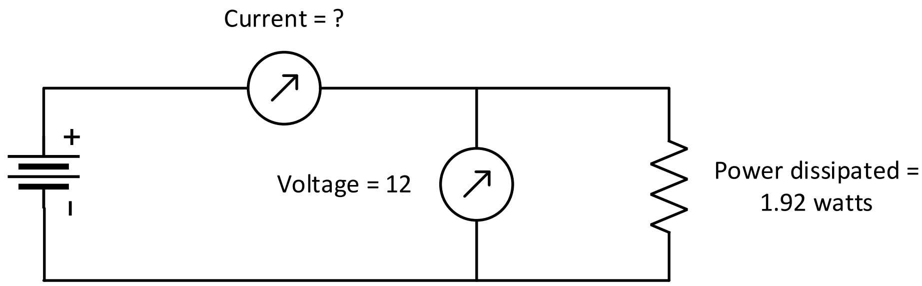


Figure 50 - What is the current with the voltage and power values shown?

Solution 3:

$$I = \frac{P}{E}$$

$$I = \frac{1.92}{12}$$

$$I = 0.16$$

Answer: The current is 0.16 ampere.

27.2. Additional formulas for power in direct current circuits

Using a scientific calculator and some basic algebra, substitute the Ohm's Law equivalents for E and I into the formula $P = E * I$, and you'll get the following two expressions of power:[52]

$$P = \frac{E^2}{R}$$

$$P = I^2 * R$$

where
P is power in watts
E is voltage in volts
R is resistance in ohms
I is current in amperes

[52] $P = I^2R$ (and the alternate form $P = V^2/R$) is sometimes called Joule's Law, which describes the rate at which resistance in a conductor or circuit converts electric energy into heat energy. More precisely, Joule's Law states that $Q = I^2RT$, where Q is the amount of heat, I is current, R is resistance, and T is time.

Example 1:
Refer to Figure 51. What is the power dissipated by the resistor, assuming its resistance is 75 ohms and the applied voltage is 12 volts?

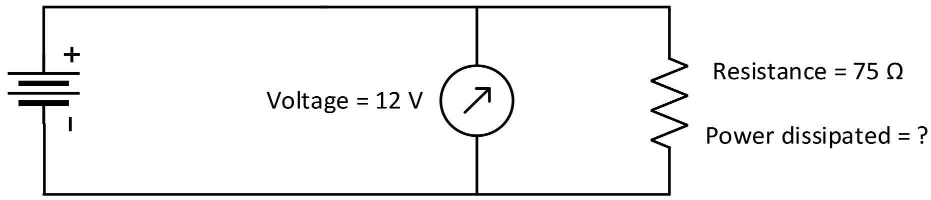


Figure 51 - What is the power dissipated by the resistor for the voltage and resistance values shown?

Solution 1:

$$P = \frac{E^2}{R}$$

$$P = \frac{12^2}{75}$$

$$P = \frac{144}{75}$$

$$P = 1.92$$

Answer: The power dissipated by the 75 ohms resistor is 1.92 watts.

Example 2:
Refer to Figure 52. What is the power dissipated by the resistor assuming its resistance is 75 ohms and the current in the circuit is 0.160 ampere?

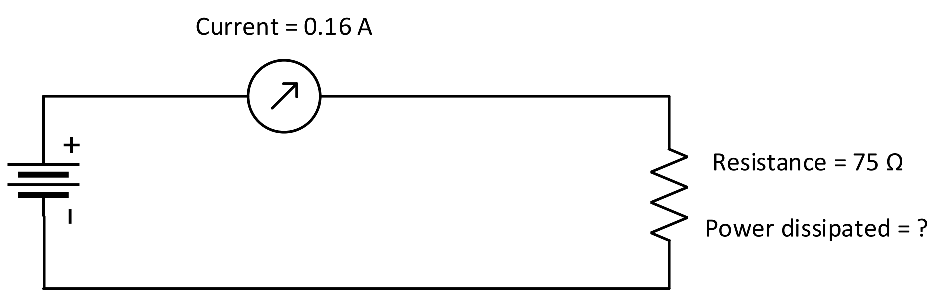


Figure 52 - What is the power dissipated by the resistor for the current and resistance values shown?

Solution 2:

$$P = I^2 * R$$

$$P = 0.16^2 * 75$$

$$P = 0.0256 * 75$$

$P = 1.92$

Answer: The power dissipated by the 75 ohms resistor is 1.92 watts.

27.2.1. *Other power-related formulas*

The following formulas also can be used in power-related calculations:

$$E = \sqrt{P * R}$$

$$I = \sqrt{\frac{P}{R}}$$

$$R = \frac{E^2}{P}$$

$$R = \frac{P}{I^2}$$

where
P is power in watts
E is voltage in volts
I is current in amperes
R is resistance in ohms

Example 1:
What is the voltage in the circuit in Figure 53 when the power dissipated by the 75 ohms resistor is 15 mW (0.015 watt)?

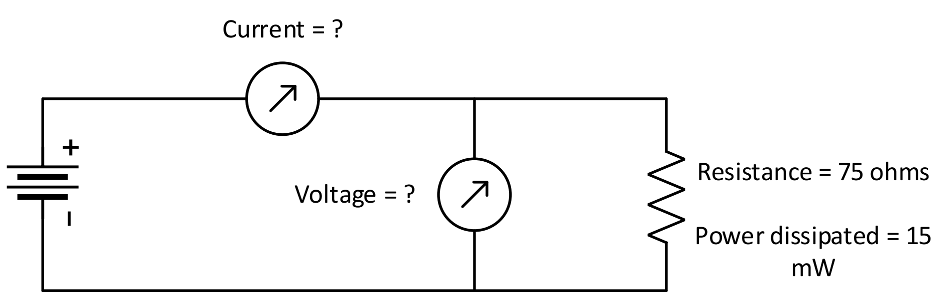


Figure 53 - What are the voltage and current in this circuit?

Solution 1:
$E = \sqrt{P * R}$

$E = \sqrt{0.015 * 75}$

$E = \sqrt{1.1250}$

$E = 1.06$

Answer: The voltage is 1.06 volts.

Example 2:
What is the current in the circuit in Figure 53 when the power dissipated by the 75 ohms resistor is 15 mW?

Solution 2:

$$I = \sqrt{\frac{P}{R}}$$

$$I = \sqrt{\frac{0.015}{75}}$$

$I = \sqrt{0.0002}$

$I = 0.0141$

Answer: The current is 0.0141 ampere or 14.1 mA.

28. Power in Alternating Current Circuits

Power calculations and measurements in direct current circuits and applications are relatively straightforward, as seen in the previous section. For example, as shown in the DC circuit in Figure 54, the power dissipated by a 75 ohms resistor with an applied voltage of 12 volts is 1.92 watts. That is, $P = E^2/R = 12^2/75 = 1.92$ W.

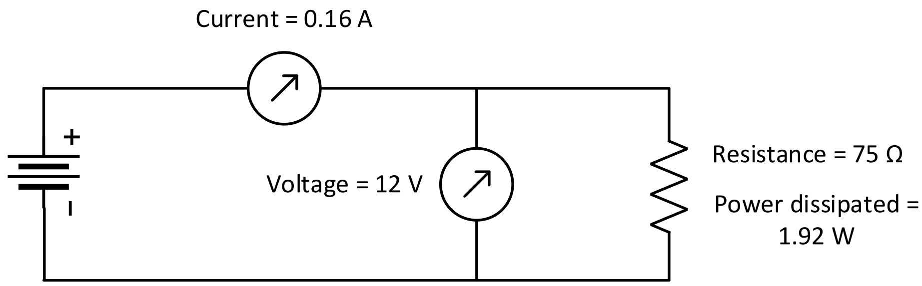


Figure 54 - Simple DC circuit showing the power dissipated by a 75 ohms resistor with an applied voltage of 12 volts.

Because the previous example is a DC circuit, the voltage is a constant 12 V, the current is a constant 0.16 A, and the dissipated power is 1.92 W. As long as the values in that circuit remain constant, it's easy to calculate dissipated power.

Alternating current (AC) circuits and applications are much more complicated because the instantaneous voltage and current are not constant. In order to equate the varying AC waveform to a DC equivalent component, one must work in the world of root mean square (RMS)[53] voltage and current.

In an AC circuit, the instantaneous values of voltage and current are varying continuously over time. How can we define useful values for these varying quantities? RMS gives us effective quantities equivalent to DC values.

For instance, 12 volts RMS AC voltage causes the same average power dissipation in a resistor as does 12 volts DC voltage. Likewise, 0.160 ampere RMS alternating current has the same heating effect as 0.160 ampere direct current. See Figure 55.

[53] RMS, or root mean square, in the context as used here is based upon equating the values of AC and DC power to heat a resistive element to exactly the same degree. An RMS value is found by squaring the individual values of all the instantaneous values of voltage or current in a single AC cycle. Take the average of those squares and find the square root of the average.

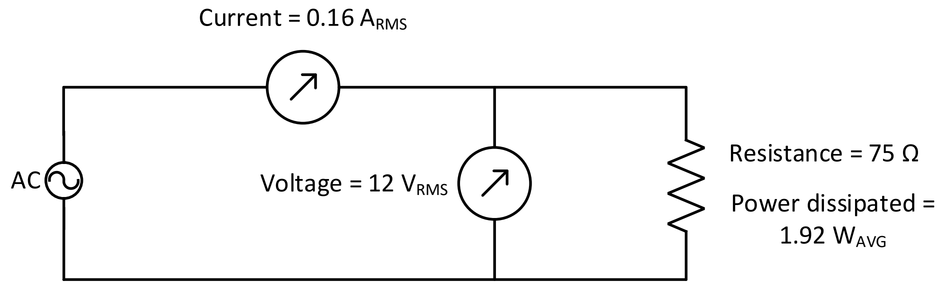


Figure 55 - Simple AC circuit showing the average power dissipated by a 75 ohms resistor with the RMS values of current and voltage indicated.

28.1. Calculate average power in a simple AC circuit with only resistance

RMS values simplify AC calculations by making the product of RMS voltage and RMS current equal to average power:

$$P_{AVG} = E_{RMS} * I_{RMS}$$

where
P_{AVG} is average power in watts
E_{RMS} is root mean square voltage in volts
I_{RMS} is root mean square current in amperes

Example:
Refer to Figure 55. What is the average power dissipated by the resistor if the current is 0.160 ampere RMS and the voltage is 12 volts RMS?

Solution:
$P_{AVG} = E_{RMS} * I_{RMS}$

$P_{AVG} = 12 * 0.160$

$P_{AVG} = 1.92$

Answer: The average power dissipated by the resistor is 1.92 watts.

Why Average Power?
Consider an unmodulated RF carrier, which really is nothing more than a sinusoidal AC waveform. AC power measurement can be a bit tricky, though, because the product of voltage and current varies during the AC cycle by twice the frequency of the sine wave. In other words, the output of a signal source such as an RF signal generator might be a sinusoidal current at the desired frequency, but the *product* of the carrier's voltage and current has what amounts to an equivalent average DC component along with a component at twice the original frequency. In most cases of RF power measurement, "power" refers to the equivalent average DC component of the voltage and current product.

If you connect a thermocouple power meter to the output of an RF signal source, the power meter's power sensor will respond to the RF carrier's DC component by averaging. Of course, this averaging usually is done over many cycles, which, at RF, still can be a relatively short period of time.

Otherwise, if the power meter simply measured an instantaneous point of the sine wave, then measured that sine wave at another instantaneous point, the result would vary according to the instantaneous product of the voltage and current at each measurement point.

This is the primary reason why most RF carrier power measurements are expressed in terms of average power.

28.2. Calculate power in an AC circuit with reactance

For a simple AC circuit as shown in Figure 55 with just an applied voltage and resistance, the voltage and current are in phase and the power calculation is straightforward. But if the circuit also includes reactance, there will be a phase difference between the current and voltage that must be taken into account.

Consider the AC circuit shown in Figure 56. The applied voltage is still 12 volts RMS and the resistance is 75 ohms, but now there is an inductor with an inductive reactance of 100 ohms in series with the resistor.[54] Since the circuit includes an inductor, the phase of the voltage will lead the phase of the current by 90 degrees.[55]

As mentioned in the introduction to Section 27, power is the rate at which energy is consumed in a circuit. However, energy is not consumed in a purely reactive circuit. That means in a circuit with resistance and reactance such as that shown in Figure 56, the resistor will dissipate power but the reactance will not. When calculating power in an AC circuit, the power actually dissipated is known as real power. The product of voltage and current in a circuit such as this gives apparent power, which is not the same thing as the real power. The ratio of real power to apparent power is called power factor.

[54] For this example, assume the alternating current has a frequency of 100 MHz and the inductor has a value of 0.1592 microhenry (μH). Using the formula for inductive reactance, $X_L = 2\pi f L$, (where X_L is inductive reactance in ohms, f is frequency in hertz, and L is inductance in henrys), the calculated inductive reactance is about 100 ohms.
[55] An easy way to remember the phase relationship between voltage and current in the AC circuit in Figure 56 is the saying "ELI the ICE man." Here, "ELI" applies to an inductor, since "L" is an abbreviation used for inductance, "E" is voltage, and "I" is current. In "ELI" the E occurs before the I, which reminds us that voltage (E) leads current (I). "ICE" applies to a capacitor, since "C" is an abbreviation for capacitance, and as before "E" is voltage and "I" is current. In "ICE" the "I" occurs before "E," which reminds us that current (I) leads voltage (E).

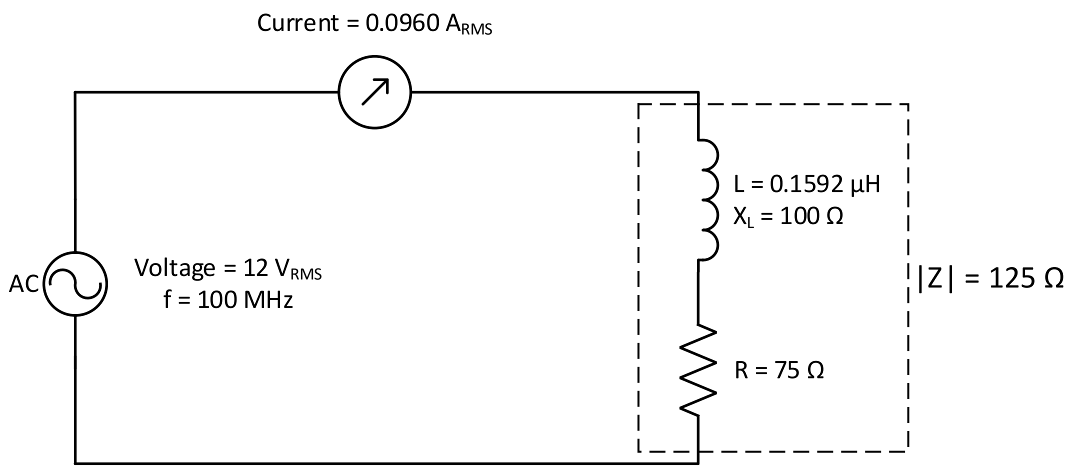


Figure 56 - AC circuit with resistance and reactance.

Several steps must be taken to calculate parameters such as real power, apparent power, and power factor. First, calculate the magnitude of the impedance of the series reactance and resistance, |Z|. Note that in general impedance, Z, is a complex value and includes both resistance (the real part of complex impedance) and reactance (the imaginary part of complex impedance) – that is, both magnitude and phase. This is discussed in detail in Section 30.8. However, many useful calculations can be done using only the magnitude of the impedance, |Z|:

$$|Z| = \sqrt{R^2 + {X_L}^2}$$

where
$|Z|$ is the magnitude of the impedance in ohms
R is resistance in ohms
X_L is inductive reactance in ohms

Example:
What is the magnitude of the impedance of the series resistance (75 ohms) and inductive reactance (100 ohms) in Figure 56?

Solution:

$$|Z| = \sqrt{R^2 + {X_L}^2}$$

$$|Z| = \sqrt{75^2 + 100^2}$$

$$|Z| = \sqrt{5625 + 10{,}000}$$

$$|Z| = \sqrt{15{,}625}$$

$$|Z| = 125$$

Answer: The magnitude of the impedance of the series circuit in Figure 56 is 125 ohms.

Next, calculate the current in the circuit using the following formula (a variation of Ohm's Law, where "$|Z|$" replaces "R"):

$$I = \frac{E}{|Z|}$$

where
I is current in amperes
E is voltage in volts
$|Z|$ is the magnitude of the impedance in ohms

Example:
What is the current in the circuit shown in Figure 56 assuming an applied voltage of 12 volts RMS and the magnitude of the impedance is 125 ohms?

Solution:
$$I = \frac{E}{|Z|}$$

$$I = \frac{12}{125}$$

$$I = 0.096$$

Answer: The current in the circuit in Figure 56 is 0.096 ampere RMS.

28.2.1. Calculate apparent power

Apparent power can be calculated using the following formula:

$$P_{apparent} = E * I$$

where
$P_{apparent}$ is apparent power in volt-ampere (VA)
E is RMS voltage in volts
I is RMS current in amperes

Example:
What is the apparent power for the circuit in Figure 56 assuming an applied voltage of 12 volts RMS and a current of 0.096 amperes RMS?

Solution:
$P_{apparent} = E * I$

$P_{apparent} = 12 * 0.096$

$P_{apparent} = 1.152$

Answer: The apparent power is 1.152 VA.

28.2.2. Calculate real power

Real power can be calculated using the following formula:

$$P_{real} = I^2 * R$$

where
P_{real} is real power in watts
I is RMS current in amperes
R is resistance in ohms

Example:
What is the real power dissipated in the circuit in Figure 56 assuming a current of 0.096 ampere RMS and a resistance of 75 ohms?

Solution:
$P_{real} = I^2 * R$

$P_{real} = 0.096^2 * 75$

$P_{real} = 0.009216 * 75$

$P_{real} = 0.6912$

Answer: The real power dissipated in the circuit in Figure 56 is 0.6912 watt.

28.2.3. Calculate power factor

Power factor, the ratio of real power to apparent power, can be calculated using the following formula:

$$PF = \frac{P_{real}}{P_{apparent}}$$

where
PF is power factor
P_{real} is real power in watts
$P_{apparent}$ is apparent power in volt-ampere (VA)

Example:
What is the power factor for the circuit in Figure 56 assuming the real power is 0.6912 watt and the apparent power is 1.152 watts?

Solution:

$$PF = \frac{P_{real}}{P_{apparent}}$$

$$PF = \frac{0.6912}{1.152}$$

$$PF = 0.6$$

Answer: The power factor for the circuit in Figure 56 is 0.6 or 60%.

28.3. Cable network powering

Cable operators for decades have powered network active devices using the distribution plant's hardline coaxial cables. Line power supplies are connected to the electric utility's 120 VAC/240 VAC service. The line power supplies use ferroresonant transformers to convert the electric utility's sinusoidal AC to a quasi-squarewave AC, typically 60 volts or 90 volts (early line power supplies provided 30 volts). The output of each line power supply is connected to the hardline coaxial cable using a line power inserter, which multiplexes the 60 volts or 90 volts into the coax along with the RF signals. Power supply modules or circuits (sometimes referred to as "power packs") inside of each active device convert the 60 volts or 90 volts to suitable DC voltages to power the various electronic circuits inside of each active device housing.

Given that most active devices used in modern cable networks are constant power loads, calculation of cable network powering has become more complicated and somewhat of an iterative process. A helpful overview of cable network plant powering, written by H. Mark Bowers, can be found in the Summer 2018 and Fall 2018 issues of *Broadband Library*:

"Basic HFC AC Design (Part One)"
https://broadbandlibrary.com/basic-hfc-ac-design-part-one/

"Basic HFC AC Design (Part Two)"
https://broadbandlibrary.com/basic-hfc-ac-design-part-one-2/

28.4. Peak envelope power

Peak envelope power (PEP) is the average power in watts during one cycle at the crest of the modulation envelope. Many of the RF signal level measurements made in cable are PEP. For example, when one measures the signal level of an analog NTSC visual carrier, that measurement is of the carrier's PEP.

Peak envelope power can be calculated using the following formula:

$$PEP = \frac{(PEV * 0.707)^2}{R}$$

where
PEP is peak envelope power in watts
PEV is peak envelope voltage in volts
R is resistance (or impedance)

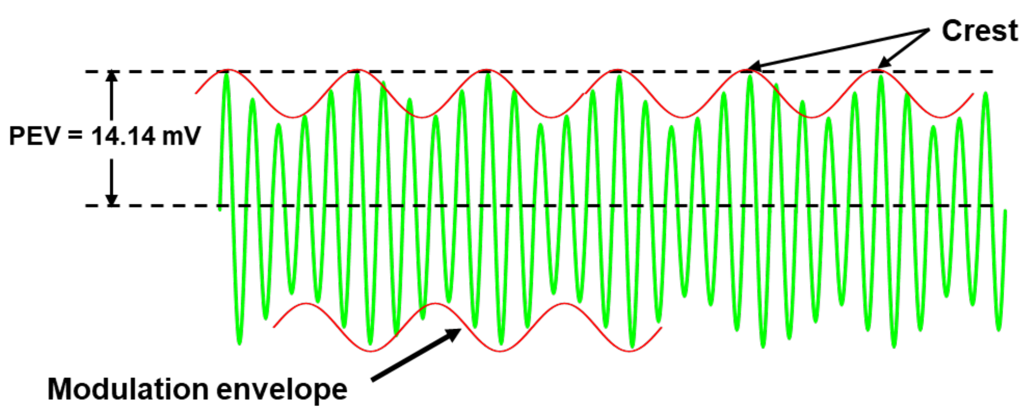


Figure 57 - Example used for PEP calculation.

Example:
Refer to Figure 57: If the peak envelope voltage is 14.14 millivolts (0.01414 volt) in a 75 ohms impedance, what is the peak envelope power?

Solution:

$$PEP = \frac{(PEV * 0.707)^2}{R}$$

$$PEP = \frac{(0.01414 * 0.707)^2}{75}$$

$$PEP = \frac{(0.009997)^2}{75}$$

$$PEP = \frac{0.000100}{75}$$

$$PEP = 0.00000133$$

Answer: The peak envelope power is 0.00000133 watt or 1.33 µW.

28.5. Energy: watt-hour and kilowatt-hour

Contrary to some misconceptions, electric utility customers pay for electrical energy, not power. A common unit for the measurement of electrical energy is the watt-hour (Wh), which is one watt of power sustained for one hour. Note: 1 wH equals $3.6 * 10^3$ joules.

28.5.1. Energy in units of watt-hour

The following formula can be used to calculate energy in units of watt-hour.

$$Wh = P * T$$

where
Wh is energy in units of watt-hour
P is power in watts
T is time in hours

Example:
What is the energy in watt-hours when the power is 300 watts for two hours?

Solution:
$Wh = P * T$

$Wh = 300 * 2$

$Wh = 600$

Answer: The energy is 600 watt-hours.

28.5.2. Energy in units of kilowatt-hour

Because the watt-hour is a fairly small energy unit, it is more common to use larger units such as kilowatt-hour (kWh). Note: 1 kWh equals $3.6 * 10^6$ joules. The following formula can be used to calculate energy in units of kilowatt-hour.

$$kWh = P_{kW} * T$$

where
kWh is energy in units of kilowatt-hour
P_{kW} is power in kilowatts
T is time in hours

Example:
What is the energy in kilowatt-hours when the power is 30,000 watts (30 kW) for 48 hours?

Solution:
$kWh = P_{kW} * T$

$kWh = 30 * 48$

$kWh = 1{,}440$

Answer: The energy is 1,440 kilowatt-hours.

29. Data Communications

This section includes some of the more common data communications-related formulas used by cable operators.

29.1. Calculate SC-QAM channel bandwidth

The following formula can be used to calculate single carrier quadrature amplitude modulation (SC-QAM) channel bandwidth:

$$BW = r_{mod} * (1 + \alpha)$$

where
BW is the bandwidth of the channel in kilohertz
r_{mod} is the modulation rate in kilohertz (or symbol rate in kilosymbols per second)
α is the alpha (roll-off factor)

Example 1:
What is the bandwidth of an upstream SC-QAM channel that has a modulation rate of 5,120 kHz and an alpha of 0.25?

Solution 1:
$BW = r_{mod} * (1 + \alpha)$

$BW = 5{,}120 * (1 + 0.25)$

$BW = 5{,}120 * (1.25)$

$BW = 6{,}400$

Answer: The calculated channel width is 6,400 kHz or 6.4 MHz.

Example 2:
What is the bandwidth of a downstream SC-QAM channel that has a symbol rate of 5.360537 megasymbols per second (5,360.537 ksym/s) and an alpha of 0.12?

Solution 2:
$BW = r_{mod} * (1 + \alpha)$

$BW = 5{,}360.537 * (1 + 0.12)$

$BW = 5{,}360.537 * (1.12)$

$BW = 5{,}360.537 * (1.12)$

$BW = 6{,}003.80$

Answer: The calculated channel width is 6,003.80 kHz or about 6 MHz.

29.2. What is a symbol?

There are two concepts of *symbol* in data communications.[56] The first is defined in [7] as "A collection of some number of bits of data that are transmitted together." For example, in Reed Solomon forward error correction (FEC) coding used in ITU-T Rec. J.83 Annex B (see [33]) SC-QAM signals, Reed Solomon symbols comprise groups of seven bits, and 128 of the seven-bit Reed Solomon symbols comprise a Reed Solomon codeword.

In the second concept, a symbol is assigned and carries (from transmitter to a receiver) a single complex value in digital communications. The complex value is actually two values, and referred to as the real and imaginary components, or sometimes in-phase and quadrature components (I and Q). Also note that the complex value has an amplitude and phase; a complex value is described by a pair of real numbers. An illustrative example of a symbol having a single complex value is a sinusoid of a given frequency, which has an amplitude and a phase. Further, the sinusoid has "quadrature components": a sinusoid and its 90-degree-shifted partner. Multiple similar descriptions and explanations exist for the complex-valued symbols in communications. A symbol can be described as "complex-valued" or as carrying a pair of real values, equivalently.

Another way to look at the second concept of a symbol is to think of the condition of a carrier wave, in which that condition – defined by the carrier's amplitude (magnitude) and phase – represents a group of bits being transmitted. Consider a DOCSIS 2.0 or later upstream 16-QAM signal. Looking at the data constellation for a 16-QAM signal (see Figure 58), each "point" on the constellation represents a four-bit symbol. Each constellation point also represents a specific amplitude (distance from the center of the constellation to the point) and phase (angle relative to the constellation's horizontal, or real axis). To help simplify the concept, imagine that the four outer corner points of a 16-QAM constellation are each normalized to a magnitude of 1. Next, consider the constellation's upper right corner symbol point (the red dot in Figure 58). The length of the blue dashed arrow illustrates the magnitude of the red dot, and the angle of the dashed line shows a phase of 45° relative to the I axis. The position of the red dot on the constellation can be described as having a complex value in terms of a real value (I or in-phase) of 0.707 and an imaginary value (Q or quadrature) of 0.707. Some simple math using Pythagorean's Theorem lets us calculate its magnitude: $\sqrt{0.707^2 + 0.707^2} = 1$. That upper right constellation point or symbol represents the four bits 1111 for both Gray-coded mapping and differential encoding in DOCSIS 2.0 and later.

[56] For baseband communications a symbol is real-valued (not complex), such as non-return to zero (NRZ) or pulse amplitude modulation (PAM).

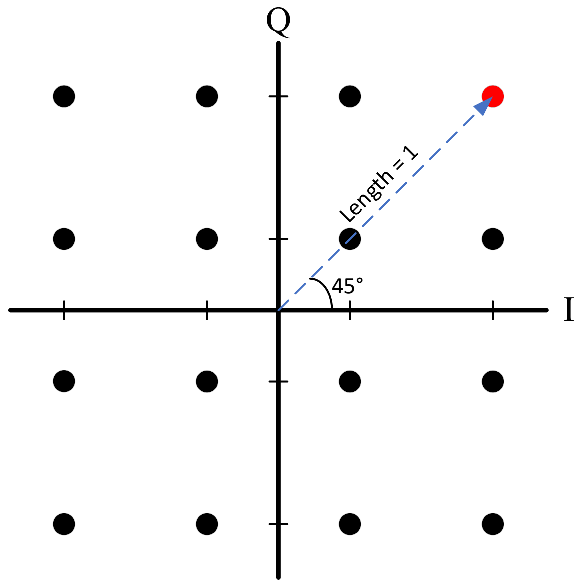


Figure 58 - Example 16-QAM constellation.

Two related terms are complex dimensions and real dimensions. Similar to symbols, in a channel bandwidth of B Hz and a time duration of T seconds, there are B * T "complex dimensions" or 2 * B * T real dimensions.

29.3. Calculate the number of bits per symbol

One can calculate the number of bits per symbol if the number of points per constellation[57] is known (e.g., the "64" in 64-QAM), using the following formula:

$$N = log_2 n$$

where
N is the number of bits per symbol
log_2 is base 2 logarithm (see Appendix A for a discussion about how to calculate base 2 logarithms)
n is the number of points per constellation (i.e., n-point signal constellation)

Example 1:
What is the number of bits per symbol for 64-QAM?

Solution 1:
$N = log_2 n$

[57] In what is commonly called M-ary encoding (where "M-ary" is derived from "binary"), M refers to the number of *conditions* such as amplitudes, phases, frequencies, etc. For example, a 256-QAM digital signal has 256 combinations of amplitudes and phases that represent 256 different symbols, and can be described as a 256-point signal constellation. Each of a 256-QAM signal's symbols is a unique group of 1s and 0s. In the case of 256-QAM, each symbol comprises eight bits.

$N = log_2 64$

$N = 6$

Answer: The number of bits per symbol for 64-QAM is 6.

Example 2:
What is the number of bits per symbol for 1024-QAM?

Solution 2:
$N = log_2 n$

$N = log_2 1024$

$N = 10$

Answer: The number of bits per symbol for 1024-QAM is 10.

29.4. Calculate the number of points per constellation for SC-QAM

One can calculate the number of points per constellation for an SC-QAM signal if the number of bits per symbol is known, using the following formula:

$$n = 2^N$$

where
n is the number of points per constellation (i.e., n-point signal constellation)
N is the number of bits per symbol

Example:
If the number of bits per symbol is 6 for an SC-QAM signal, what is the number of points per constellation?

Solution:
$n = 2^N$

$n = 2^6$

$n = 2 * 2 * 2 * 2 * 2 * 2$

$n = 64$

Answer: The number of points per constellation is 64, or in this case 64-QAM.

29.5. Calculate SC-QAM gross (channel) bit rate

One can calculate the gross (channel) bit rate (i.e., the bit rate including payload and overhead, sometimes called the PHY rate) of SC-QAM signals when the symbol rate[58] and number of bits per symbol are known, using the following formula:

$$bps_{gross} = r_{sym} * s_b$$

where
bps_{gross} is gross (channel) bit rate in bits per second
r_{sym} is symbol rate in symbols per second
s_b is the number of bits per symbol

For the following examples, refer to Table 18 and Table 19 for SC-QAM symbol rates and bits per symbol information.
Example 1:
What is the gross (channel) bit rate for a downstream 6 MHz-wide 256-QAM signal (ITU-T J.83 Annex B)?

Solution 1:
From Table 18, the number of bits per symbol for downstream ITU-T J.83 Annex B 256-QAM is 8, and the symbol rate is 5.360537 Msym/s (5,360,537 symbols per second).

$bps_{gross} = r_{sym} * s_b$

$bps_{gross} = 5{,}360{,}537 * 8$

$bps_{gross} = 42{,}884{,}296$

Answer: The gross (channel) bit rate is 42.88 Mbps.

Example 2:
What is the gross (channel) bit rate for an upstream 6.4 MHz-wide 64-QAM signal?

Solution 2:
From Table 19, the number of bits per symbol for upstream 64-QAM is 6, and the symbol rate is 5,120 ksym/s (5,120,000 symbols per second).

$bps_{gross} = r_{sym} * s_b$

$bps_{gross} = 5{,}120{,}000 * 6$

$bps_{gross} = 30{,}720{,}000$

Answer: The gross (channel) bit rate is 30,720,000 bits per second (30.72 Mbps).

[58] In DOCSIS 2.0 the term “modulation rate” was introduced, defined as “The signaling rate of the upstream modulator (1280 to 5120 kHz). In S-CDMA, the chip rate. In TDMA, the channel symbol rate.”

Table 18 - Downstream SC-QAM signal symbol rates and bits per symbol.

Modulation order	Bits per symbol	Channel bandwidth	Symbol rate (Msym/s)	Alpha (roll-off factor)
64-QAM (ITU-T J.83 Annex B)	6	6 MHz	5.056941	0.18
256-QAM (ITU-T J.83 Annex B)	8	6 MHz	5.360537	0.12
64-QAM (ITU-T J.83 Annex A)	6	8 MHz	6.952	0.15
256-QAM (ITU-T J.83 Annex A)	8	8 MHz	6.952	0.15

Table 19 - Upstream SC-QAM modulation rates and bits per symbol.

Modulation order	Bits per symbol	Channel bandwidth	Modulation rate (kHz), or symbol rate (ksym/s)	Alpha (roll-off factor)
QPSK	2	1.6 MHz	1,280	0.25
QPSK	2	3.2 MHz	2,560	0.25
QPSK	2	6.4 MHz	5,120	0.25
8-QAM	3	1.6 MHz	1,280	0.25
8-QAM	3	3.2 MHz	2,560	0.25
8-QAM	3	6.4 MHz	5,120	0.25
16-QAM	4	1.6 MHz	1,280	0.25
16-QAM	4	3.2 MHz	2,560	0.25
16-QAM	4	6.4 MHz	5,120	0.25
32-QAM	5	1.6 MHz	1,280	0.25
32-QAM	5	3.2 MHz	2,560	0.25
32-QAM	5	6.4 MHz	5,120	0.25
64-QAM	6	1.6 MHz	1,280	0.25
64-QAM	6	3.2 MHz	2,560	0.25
64-QAM	6	6.4 MHz	5,120	0.25

29.6. Calculate SC-QAM symbol rate or modulation rate

The symbol rate or modulation rate can be calculated using the following formula when the gross (channel) bit rate and number of bits per symbol are known:

$$r_{sym} = \frac{bps_{gross}}{s_b}$$

where
r_{sym} is symbol rate in symbols per second (or modulation rate in kilohertz)
bps_{gross} is the gross (channel) bit rate in bits per second
s_b is the number of bits per symbol

Example:
What is the symbol rate of an upstream SC-QAM signal with 6 bits per symbol and whose gross (channel) bit rate is 30.72 Mbps (30,720,000 bits per second)?

Solution:

$$r_{sym} = \frac{bps_{gross}}{s_b}$$

$$r_{sym} = \frac{30{,}720{,}000}{6}$$

$$r_{sym} = 5{,}120{,}000$$

Answer: The symbol rate is 5,120,000 symbols per second (5.12 Msym/s or 5,120 kHz modulation rate).

29.7. Bit error ratio

The abbreviation BER has long been spelled out as “bit error rate” or “bit error ratio,” with both terms used more or less interchangeably. For example, [26] defines *bit error rate* as

> A measurement of error rate stated as a ratio of the number of bits with an error to the total number of bits passing a given point on the ring. A BER of 10^{-6} indicates that an average of one bit per million bits is in error.

[26] defines *bit error ratio* as

> The ratio of the number of bits received in error to the total number of bits received.

According to [12]:

> “The BER of a digital communication system can be defined as the estimated probability that any bit transmitted through the system will be received in error, e.g., a transmitted ‘one’ will be received as a zero and vice versa. In practical tests, the BER is measured by transmitting a finite number of bits through the system and counting the number of bit errors received. The ratio of the number of bits received erroneously to the total number of bits transmitted is the BER.”

Expressed mathematically, BER is equal to the ratio of the number of errored bits b_e to the total number of bits b_t transmitted, received, or processed:

$$BER = \frac{b_e}{b_t}$$

where
BER is the bit error ratio
b_e is the number of errored bits
b_t is the total number of bits transmitted, received, or processed

Example:
Let's say that 1,000,000 bits are transmitted, and 3 bits out of the 1,000,000 bits received are errored because of some kind of interference between the transmitter and receiver. What is the BER?

Solution:

$$BER = \frac{3}{1{,}000{,}000}$$

$$BER = 0.000003$$

Answer: The BER is 0.000003, but most BER measurements are expressed in scientific notation format, so 0.000003 = $3 * 10^{-6}$. The latter also can be written as 3 ∗ 10^–6 or 3.0E–06.

Note: Most cable test instruments that report BER actually report a BER estimate derived from the instrument's forward error correction, as discussed in the next section.

29.7.1. How is BER measured?

Bit error ratio is commonly measured using a BER test set. A data source transmits a bit pattern – usually a pseudo-random binary sequence – through the device, system, or network being tested (see Figure 59). The error detector has to either reproduce the original bit pattern or directly receive it from the bit pattern generator. The error detector compares bit-by-bit the original bit pattern with the one received from the device, system, or network being tested. The method just described is usually an out-of-service test, making it impractical to perform where service disruptions are not acceptable. How, then, are BER measurements performed in operating cable networks?

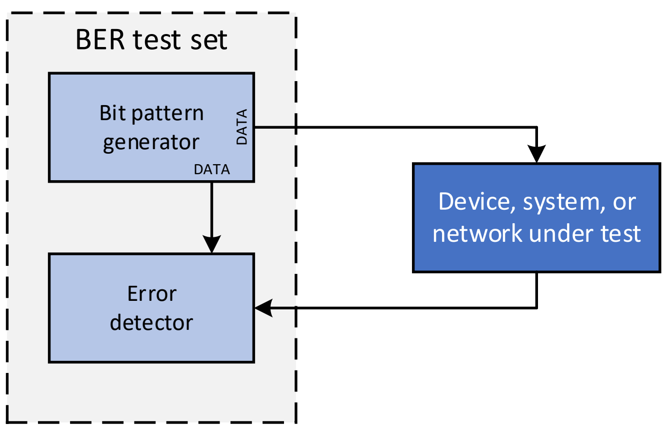


Figure 59 - Simplified block diagram of BER measurement.

Field meters used by the cable industry don't perform BER measurements the same way that a BER test set does. Instead, field meters perform an in-service measurement using an internal algorithm to derive a BER estimate based upon what the forward error correction is doing. The terms pre-FEC BER and post-FEC BER are widely used by cable test equipment companies and cable operators, with some variations in the terminology (e.g., "pre-FEC," "pre BER," or just "pre"). Many understand the terminology to generally mean the BER before and after FEC decoding fixes errors. However, some clarification is in order, since this assumption isn't quite correct.

In a typical field meter being used to measure an ITU-T J.83 Annex B SC-QAM signal (e.g., DOCSIS 3.0 or earlier), what is called pre-FEC BER is estimated at the input to the Reed Solomon (RS) decoder but after the Trellis decoder, descrambler (de-randomizer), and de-interleaver. Post-FEC BER is estimated after Reed Solomon decoding. The estimates of BER made by the in-service decoders for ITU-T J.83 Annex B downstream SC-QAM signals are imperfect approximations (but are adequate for routine maintenance and troubleshooting in cable networks). In the BER test sets used for out-of-service testing, received bit errors are actually counted.

Several factors can affect BER measurements. The type of data sent during a BER measurement is important; for instance, a long string of the same bits – say, all 0s – will generally yield different BER numbers than when doing the same measurement with a pseudo random binary sequence (PRBS). A long string of identical bits can result in something called deterministic jitter (and other distortions) which could affect the integrity of the BER measurement, so a PRBS bit pattern is usually preferred. Other factors include the number of bits transmitted during the measurement, the duration of the measurement, and whether the bit errors are independent and identically distributed (IID). Appendix E discusses these latter factors and how they affect the confidence level of the BER test results.

29.8. Spectral efficiency

Spectral efficiency describes the information bit rate that is supported in a given RF bandwidth. The following examples have been excerpted and adapted from [29], with permission of the authors (some of the assumptions and values have been changed). The reader is urged to review the referenced paper and Appendix L of this Operational Practice for more information on spectral efficiency in DOCSIS 3.1 systems, including downstream and upstream SC-QAM, OFDM, and OFDMA signals.

29.8.1. Calculate spectral efficiency

Spectral efficiency is the net bit rate (that is, the bit rate excluding overhead) in bits per second divided by the channel bandwidth in hertz, and is stated in units of bits per second per hertz (b/s/Hz or bps/Hz).[59] The following formula can be used to calculate spectral efficiency:

$$SE = \frac{bps_{net}}{BW_{Hz}}$$

where
SE is the spectral efficiency in bits per second per hertz (b/s/Hz)
bps_{net} is the net bit rate in bits per second
BW_{Hz} is the communications channel bandwidth in hertz

[59] The spectral efficiency in bits per second per hertz (b/s/Hz) is also the same as spectral efficiency in bits per symbol (b/sym). Since the channel BW = 1/T for a symbol duration of T seconds per symbol (the Nyquist rate), BW = 1/T symbols per second and spectral efficiency = b/s/BW = b/s/(1/T symbols/s) = b/s/(BW symbols/s) = b/symbol. Note that the Nyquist rate may occupy slightly more bandwidth due to the use of a roll-off factor which lowers the spectral efficiency in the occupied channel bandwidth.

Example:
What is the spectral efficiency of a 6 MHz-wide ITU-T J.83 Annex B SC-QAM signal that has a gross (channel) bit rate of 42.88 Mbps and net bit rate (after excluding overhead[60]) of 38.8107 Mbps?

Solution:

$$SE = \frac{bps_{net}}{BW_{Hz}}$$

$$SE = \frac{38,810,700}{6,000,000}$$

$$SE = 6.4685$$

Answer: The spectral efficiency is 6.4685 b/s/Hz.

29.8.2. Calculate QAM-independent system efficiency for SC-QAM

QAM-independent system efficiency is spectral efficiency in b/s/Hz divided by the number of bits per symbol, and is stated in units of symbols per second per hertz (sym/s/Hz). The following formula can be used to calculate QAM-independent system efficiency:

$$S_{eff} = \frac{SE}{s_b}$$

where
S_{eff} is the QAM-independent system efficiency in symbols per second per hertz (sym/s/Hz)
SE is the spectral efficiency in bits per second per hertz (b/s/Hz)
s_b is the number of bits per symbol for the modulation order in use

Example:
What is the QAM-independent system efficiency for the previous example, in which the spectral efficiency is 6.4685 b/s/Hz and the number of bits per symbol for 256-QAM is 8?

Solution:

$$S_{eff} = \frac{SE}{s_b}$$

$$S_{eff} = \frac{6.4685}{8}$$

[60] See Annex A of [30] for a discussion about calculation of the concatenated code rate for SC-QAM signals in cable networks.

$S_{eff} = 0.8086$

Answer: The QAM-independent system efficiency is 0.8086 sym/s/Hz.

29.8.3. *Calculate QAM-independent system efficiency for OFDM*

The following formula can be used to calculate the estimated QAM-independent system efficiency for an asynchronous OFDM signal:

$$S_{eff} = \frac{SC_{CH} - SC_{GB} - P_{cont} - P_{scat} - SC_{PLC} - SC_{NCP}}{SC_{CH}} * \frac{FFT}{(FFT + CP)} * FEC$$

where
S_{eff} is the QAM-independent system efficiency in symbols per second per hertz (sym/s/Hz)
SC_{CH} is the number of subcarriers in the OFDM channel bandwidth
SC_{GB} is the total number of subcarriers in the guard bands (taper regions)
P_{cont} is the number of continuous pilots
P_{scat} is the number of scattered pilots
SC_{PLC} is the number of subcarriers in the PHY link channel (PLC)
SC_{NCP} is the number of subcarriers used for next codeword pointers (NCPs)
FFT is the FFT duration in microseconds
CP is the cyclic prefix duration in microseconds
FEC is the effective forward error correction code rate

Example 1:
What is the estimated QAM-independent system efficiency for a 192 MHz-wide asynchronous OFDM signal with the configuration parameters listed in Table 20?

Solution 1:

$$S_{eff} = \frac{SC_{CH} - SC_{GB} - P_{cont} - P_{scat} - SC_{PLC} - SC_{NCP}}{SC_{CH}} * \frac{FFT}{(FFT + CP)} * FEC$$

$$S_{eff} = \frac{7{,}680 - 80 - 88 - 60 - 16 - 48}{7{,}680} * \frac{40}{(40 + 2.5)} * 0.8775$$

$$S_{eff} = \frac{7{,}388}{7{,}680} * \frac{40}{42.5} * 0.8775$$

$$S_{eff} = 0.9620 * 0.9412 * 0.8775$$

$$S_{eff} = 0.7945$$

Answer: The estimated QAM-independent system efficiency for an asynchronous OFDM signal is 0.7945 sym/s/Hz.

Example 2:
When the OFDM signals are synchronous, the QAM-independent system efficiency can be estimated by leaving the "subcarriers in guard bands" term (SC_{GB}) out of the formula. Using the same assumptions from Example 1 (less SC_{GB}), what is the QAM-independent system efficiency?

Solution 2:

$$S_{eff} = \frac{SC_{CH} - P_{cont} - P_{scat} - SC_{PLC} - SC_{NCP}}{SC_{CH}} * \frac{FFT}{(FFT + CP)} * FEC$$

$$S_{eff} = \frac{7{,}680 - 88 - 60 - 16 - 48}{7{,}680} * \frac{40}{(40 + 2.5)} * 0.8775$$

$$S_{eff} = \frac{7{,}468}{7{,}680} * \frac{40}{42.5} * 0.8775$$

$$S_{eff} = 0.9724 * 0.9412 * 0.8775$$

$$S_{eff} = 0.8031$$

Answer: The estimated QAM-independent system efficiency for a synchronous OFDM signal is 0.8031 sym/s/Hz.

Table 20 - OFDM configuration parameters

Parameter	Assumed value
Channel bandwidth	192 MHz (190 MHz encompassed spectrum with no exclusion bands)
Subcarrier spacing	25 kHz
FFT size	8K (8,192)
FFT duration	40 µs
Subcarriers in channel bandwidth	7,680
Active subcarriers in encompassed spectrum	7,600
Subcarriers in guard bands (2 MHz total)	80
Continuous pilots	88
Scattered pilots	60
PLC subcarriers	16
Cyclic prefix duration	2.5 µs
Subcarriers used for next codeword pointers	48
Effective FEC code rate[61]	0.8775

29.8.4. Calculate OFDM spectral efficiency

Calculation of OFDM spectral efficiency can be somewhat more complicated than for an individual legacy SC-QAM signal. This is especially true given DOCSIS 3.1's support for multiple modulation

[61] Effective FEC code rate = (16,200 codeword size – 16 codeword pointer – 168 BCH parity – 1,800 LDPC parity)/16,200 codeword size = 0.8775

profiles, where a weighted average spectral efficiency is a useful metric for characterizing overall performance.

When the QAM-independent system efficiency for an OFDM signal is known, the spectral efficiency can be calculated with the following formula:

$$SE = S_{eff} * s_b$$

where
SE is the spectral efficiency in bits per second per hertz (b/s/Hz)
S_{eff} is the QAM-independent system efficiency in symbols per second per hertz (sym/s/Hz)
s_b is the number of bits per symbol for the modulation order in use

Example:
What is the spectral efficiency for a synchronous OFDM signal using 512-QAM (9 bits per symbol) and has a QAM-independent system efficiency of 0.8031 symbols per second per hertz?

Solution:
$SE = S_{eff} * s_b$

$SE = 0.8031 * 9$

$SE = 7.2279$

Answer: The spectral efficiency is 7.2279 b/s/Hz.

29.8.5. Calculate weighted average OFDM spectral efficiency

Cable operators often take advantage of DOCSIS 3.1's support for multiple modulation profiles, which means that different modulation orders can be used for different cable modems. Multiple modulation profiles can accommodate the normal variations in network performance in different parts of, for example, a node's service area (e.g., end-of-line versus closer to the node), or different signal quality within the OFDM signal (e.g., part of the OFDM signal overlaps the downstream rolloff). When multiple modulation profiles are used, characterizing performance can be done using a weighted average of OFDM spectral efficiency. Calculating the weighted average OFDM spectral efficiency involves several steps, discussed in this section. For more information, see [29].

Example:
Assume OFDM operation in a cable network in which the following modulation orders can be used: 128-QAM, 256-QAM, 512-QAM, 1024-QAM, 2048-QAM, and 4096-QAM, and the estimated QAM-independent system efficiency is 0.8031 sym/s/Hz. First calculate the spectral efficiency for each modulation order using the formula from the previous section ($SE = S_{eff} * s_b$).

128-QAM (7 bits per symbol):
$SE = 0.8031 * 7$

$SE = 5.6217$

Answer: The spectral efficiency for 128-QAM is 5.6217 b/s/Hz

256-QAM (8 bits per symbol):
$SE = 0.8031 * 8$

$SE = 6.4248$

Answer: The spectral efficiency for 256-QAM is 6.4248 b/s/Hz.

512-QAM (9 bits per symbol):
$SE = 0.8031 * 9$

$SE = 7.2279$

Answer: The spectral efficiency for 512-QAM is 7.2279 b/s/Hz.

1024-QAM (10 bits per symbol):
$SE = 0.8031 * 10$

$SE = 8.0310$

Answer: The spectral efficiency for 1024-QAM is 8.0310 b/s/Hz.

2048-QAM (11 bits per symbol):
$SE = 0.8031 * 11$

$SE = 8.8341$

Answer: The spectral efficiency for 2048-QAM is 8.8341 b/s/Hz.

4096-QAM (12 bits per symbol):
$SE = 0.8031 * 12$

$SE = 9.6372$

Answer: The spectral efficiency for 4096-QAM is 9.6372 b/s/Hz.

Next, assume that variations in signal-to-noise ratio performance throughout a cable network means that some modems can use 4096-QAM, some can use 2048-QAM, some can use 1024-QAM, and so forth. Further assume the distribution of cable modems capable of using each modulation order is as shown in Figure 60. Of particular interest is the percentages of modems that are able to reliably use each modulation profile, which will be used to calculate a weighted average OFDM spectral efficiency.

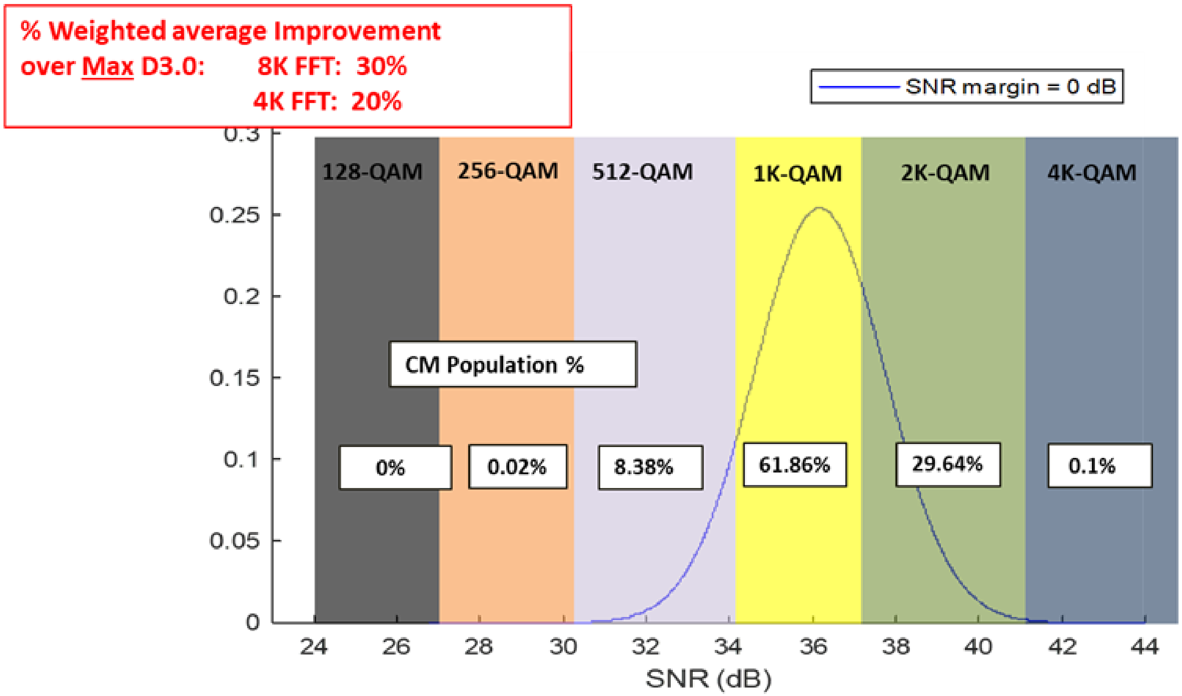


Figure 60 - Distribution of modems versus SNR (courtesy Ayham Al-Banna, CommScope).

Note: A weighted average value is not the same thing as an arithmetic average. For more information, see, for instance, https://www.wikihow.com/Calculate-Weighted-Average. In this example, the values used to calculate the desired weighted average spectral efficiency are summarized in Table 21 - Parameters for calculating weighted average spectral efficiency.

Table 21 - Parameters for calculating weighted average spectral efficiency

Modulation order	SE (b/s/Hz)	Percentage of modems	Weighting factor	SE * weighting factor
128-QAM	5.6217	0%	0	0
256-QAM	6.4248	0.02%	0.0002	0.0013
512-QAM	7.2279	8.38%	0.0838	0.6057
1024-QAM	8.0310	61.86%	0.6186	4.9680
2048-QAM	8.8341	29.64%	0.2964	2.6184
4096-QAM	9.6372	0.1%	0.0010	0.0096

Since the weighting factors in the fourth column of Table 21 total 1, then the weighted average of the spectral efficiency values is simply the sum of the values in the table's fifth column: 0 + 0.0013 + 0.6057 + 4.9680 + 2.6184 + 0.0096 = 8.2030 b/s/Hz, or about 8.2 b/s/Hz.

30. Reference Equations

This section includes a variety of useful formulas and related information, but typically without the "how to" example and solution format used in other parts of this Operational Practice.

30.1. Shannon-Hartley theorem

30.1.1. Introduction

The Shannon-Hartley theorem describes the channel capacity in terms of the maximum amount of data that can be transmitted error-free, in a specified bandwidth with a given signal-to-noise ratio. Expressed mathematically:

$$C = B * log_2\left(1 + \frac{S}{N}\right)$$

where
C is channel capacity in bits per second (bps)
B is bandwidth in hertz (Hz)
log_2 is base 2 logarithm (see Appendix A for information about calculating base 2 logarithms)
S is average signal power within bandwidth B, in units of watts
N is average noise power within bandwidth B, in the same unit of watts as S

Note: The ratio $\frac{S}{N}$ is a linear power ratio, not a logarithmic ratio in decibels. For example, if the logarithmic ratio of $\frac{S}{N} = 40\ dB$, the linear power ratio is $10^{40/10} = 10{,}000$.

Discussion:
The Shannon-Hartley theorem, named after Claude Shannon and Ralph Hartley, provides what is sometimes called the "Shannon limit," the maximum bit rate that can be transmitted without errors in a communications channel of a specified bandwidth in the presence of noise.

The following is excerpted from [31]:

> *Theorem 2:* Let P be the average transmitter power, and suppose the noise is white thermal noise of power N in the band W. By sufficiently complicated encoding systems it is possible to transmit binary digits at a rate
>
> $$C = W \log_2 \frac{P + N}{N}$$
>
> with as small a frequency of errors as desired. *It is not possible by any encoding method to send at a higher rate and have an arbitrarily low frequency of errors.* [emphasis added]

In order to support an increased bit rate, the communications channel's bandwidth and/or the signal-to-noise ratio must be increased. For example, consider an analog voice circuit with a bandwidth of 3,000

Hz. If the signal-to-noise ratio is 20 dB (linear power ratio = $10^{20/10} = 100$), then the maximum bit rate is

$$C = B * log_2\left(1 + \frac{S}{N}\right)$$

$$C = 3{,}000 * log_2(1 + 100)$$

$$C = 3{,}000 * log_2(101)$$

$$C = 3{,}000 * (6.66)$$

$$C = 19{,}974.63\ bps$$

If the bandwidth is doubled to 6,000 Hz but the signal-to-noise ratio is unchanged at 20 dB, the maximum bit rate increases to

$$C = 6{,}000 * log_2(1 + 100)$$

$$C = 6{,}000 * log_2(101)$$

$$C = 6{,}000 * (6.66)$$

$$C = 39{,}949.27\ bps$$

Alternatively, maintaining the original 3,000 Hz bandwidth but increasing the signal-to-noise ratio from 20 dB to 30 dB (linear power ratio = $10^{30/10} = 1{,}000$), then the maximum bit rate is

$$C = 3{,}000 * log_2(1 + 1{,}000)$$

$$C = 3{,}000 * log_2(1{,}001)$$

$$C = 3{,}000 * (9.97)$$

$$C = 29{,}901.68\ bps$$

30.1.2. Power limited, bandwidth unlimited channel

Clearly, there are tradeoffs with respect to bandwidth and signal-to-noise ratio, and the maximum possible bit rate – the "Shannon limit." Improving the signal-to-noise ratio generally means increasing the power of the transmitted data signal. If the transmit power is constrained, then using the widest possible bandwidth will yield the best performance. An important point: One cannot continually get more capacity with more bandwidth. From [31]:

> As we increase the band, the capacity increases rapidly until the total noise power accepted is about equal to the signal power; after this, the increase is low, and it approaches an asymptotic value $\log_2 e$ times the capacity for W = W_0.

There are three parameters which determine the channel capacity for bits per second (with arbitrarily low errors): transmit power, P watts; channel bandwidth, W Hz; and one-sided noise power spectral density, N_0 watts per Hz. Figure 61 shows the capacity as a function of bandwidth, with transmit power held constant. This tradeoff or view of the capacity theorem, with the transmit power held constant, is sometimes called "power limited operation" of the channel, since the bandwidth is unconstrained, but the transmit power is constrained; the transmit power is held *the same* within each triplet of the defining parameters in this view. Rather than calling this the "power limited view" it may be more appropriate to call it the "bandwidth unlimited view," but that description doesn't seem as commonly used.

In Figure 61 (from [31]), C/W_0 is plotted as a function of W/W_0, where C is channel capacity, W is bandwidth, and W_0 is P/N_0, that is, W_0 is the bandwidth in which noise power is equal to signal power.

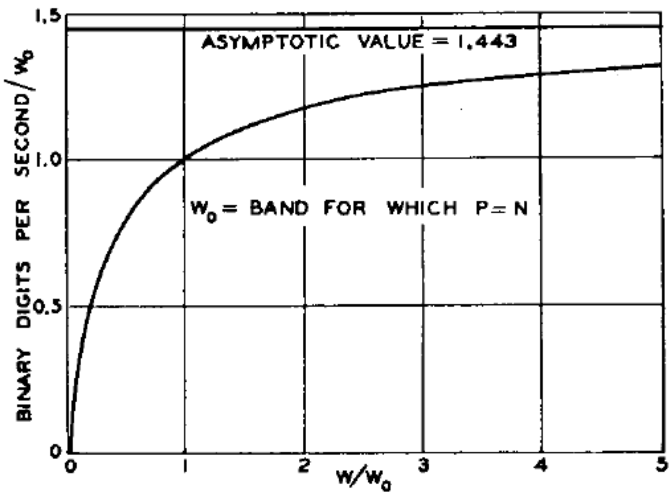


Figure 61 - Graph from the original Figure 7 in [31], Shannon's 1949 paper. See text.

Since $P/(N_0 * W)$ is "1" for W equal W_0, by the definition of W_0, it is apparent that the capacity C for bandwidth W_0 Hz is W_0 bits per second, numerically, where W_0 units are Hz, but in this case the capacity of the channel in bits per seconds equals W_0, too, because the argument of the $\log_2()$ function is "2."

Figure 61 illustrates that the normalized channel capacity, bits per second per W_0 Hz, *does not* "grow without bound" as the channel bandwidth increases without bound, but rather approaches a finite limit for arbitrarily large bandwidth channels. With W_0 numerically expressed in Hz, in the power limited channel, the capacity C for unlimited channel bandwidth approaches

$$log_2(exp(1)) * W_0 \; bits \; per \; second \cong 1.4427 * W_0 \; bits \; per \; second$$

$$\cong 1.4427 * (P/N_0) \; bits \; per \; second$$

To explore an example, let T_b be the time it takes to transmit 1 bit in the channel, and thus the energy per bit divided by the one-sided power spectral density is $E_b/N_0 = P * T_b/N_0$. Consider a case where the E_b/N_0 is 0 dB with a proposed bit rate of 10 Mbps. This pair of values determines the ratio P/N_0.

We will answer the following three questions for this example:

Q1. For this value of P/N_0, what is the channel capacity with unlimited bandwidth?

Q2. What is the channel bandwidth which is required according to the capacity theorem, for the proposed bit rate?

Q3. If the channel bandwidth is unlimited and the bit rate is operated at the channel capacity for this same value of P/N_0, what is the resulting value of E_b/N_0?

Answer 1. For this value of P/N_0, what is the channel capacity with unlimited bandwidth?

$$(E_b/N_0) * (1/T_b) = P/N_0 = (1) * (10^7),$$

Thus, $C_{unlimited\_bw\_ch} \cong 1.4427 * 10^7 \cong 14.43\ Mbps$

Answer 2. What is the channel bandwidth which is required according to the capacity theorem, for the proposed bit rate?

Since we see that for the value of P/N_0 the unlimited bandwidth channel supports more than 10 Mbps, we can use the capacity theorem to determine what finite bandwidth *is* needed to support 10 Mbps. The solution involves solving the capacity equation for W Hz, where we start by setting the channel capacity to the objective of 10 Mbps, and (P/N_0) is known:

$$C = 10\ Mbps = W * log_2\left(1 + \frac{(P/N_0)}{W}\right) = W * log_2\left(1 + \frac{(10^7)}{W}\right)$$

and an iterative search (trial, adjust, new trial, etc.) shows that W = 10 MHz, which can easily be checked since the argument of the $log_2()$ function is “2” exactly, for this case.

Answer 3. If the channel bandwidth is unlimited and the bit rate is operated at the channel capacity for this same value of P/N_0, what is the resulting value of E_b/N_0?

With the bandwidth unlimited, we found in Answer 1 for the 10^7 value of P/N_0 that the capacity is 14.43 Mbps. So, what is the corresponding E_b/N_0 for this channel when it is operated with the maximum possible rate of 14.43 Mbps?

We see

$$T_b = \frac{1}{(14.43 * 10^6)}\ seconds$$

and

$$E_b/N_0 = (P/N_0) * T_b = \frac{(10^7)}{(14.43 * 10^6)} = 0.693$$

and converting to decibels, we find the E_b/N_0 for this value of P/N_0 when the channel is operated with the unlimited bandwidth at the bit rate corresponding to the unlimited bandwidth capacity, is

$$10log_{10}(E_b/N_0) = 10log_{10}(0.693) = -1.6\ dB$$

Summarizing, for any value of P/N_0, we can find the capacity of the unlimited bandwidth channel, $C_{unlimited\_bw\_ch}$. If the channel is operated at that bit rate, $C_{unlimited\_bw\_ch}$ bits per second, the corresponding E_b/N_0 is –1.6 dB. Operating at a bit rate R_b less than $C_{unlimited\_bw\_ch}$, the unlimited bandwidth channel capacity, a corresponding finite bandwidth can be calculated which has capacity equal to that bit rate, R_b. The E_b/N_0 corresponding to R_b will be greater than –1.6 dB.

30.1.3. Bandwidth limited, power unlimited channel

Another common view of the capacity equation is the "bandwidth constrained operation," where the bandwidth is held constant and the power increases without bound. This is best illustrated after taking two steps to adjust, or "view," the capacity equation.

The first step is to view the capacity of the fixed-bandwidth channel, of W Hz, normalized by the channel bandwidth, W Hz:

$$\frac{C}{W} = log_2\left(1 + \frac{P}{(N_0 * W)}\right) \; bits\ per\ second\ per\ Hz$$

The second step for this view is defining the channel SNR as $[P/(N_0 * W)]$ and expressing SNR in decibels: $SNR_{dB} = 10 * \log_{10}[\, P/(N_0 * W)\,]$. This is facilitated by the relationship:

$$log_2(x) = \frac{log_{10}(x)}{log_{10}(2)}$$

Thus, we see,

$$\frac{C}{W} = log_2\left(1 + \frac{P}{(N_0 * W)}\right)$$

$$= \frac{\left[log_{10}\left(1 + \frac{P}{(N_0 * W)}\right)\right]}{log_{10}(2)}$$

Continuing,

$$\frac{C}{W} = \frac{\left(10log_{10}\left[1 + \frac{P}{(N_0 * W)}\right]\right)}{[10log_{10}(2)]}$$

$$= \frac{\left(10log_{10}\left[1 + \frac{P}{N_0 * W}\right]\right)}{3}$$

So, the capacity in units of bits per second per Hz is given by:

$$\frac{C}{W} = \frac{\left(10log_{10}\left[1 + \frac{P}{N_0 * W}\right]\right)}{3} \; bits\ per\ second\ per\ Hz.$$

And using the definition for SNR,

$$\frac{C}{W} = log_2(1 + SNR)$$

$$= \frac{(10log_{10}[1 + SNR])}{3} \ bits\ per\ second\ per\ Hz.$$

This expression allows a computation of channel capacity in units of bits per second per Hz with the channel bandwidth held constant, and allowing P (and equivalently, SNR) to increase without limit. This is called the "bandwidth limited operation" or bandwidth limited view of the channel capacity theorem. As the complementary view of the power limited operation, in this latter view, the channel capacity is examined as the channel power increases without limit.

The channel capacity for the bandwidth limited channel is illustrated in Figure 62, showing capacity, C bits per second, versus SNR in dB.

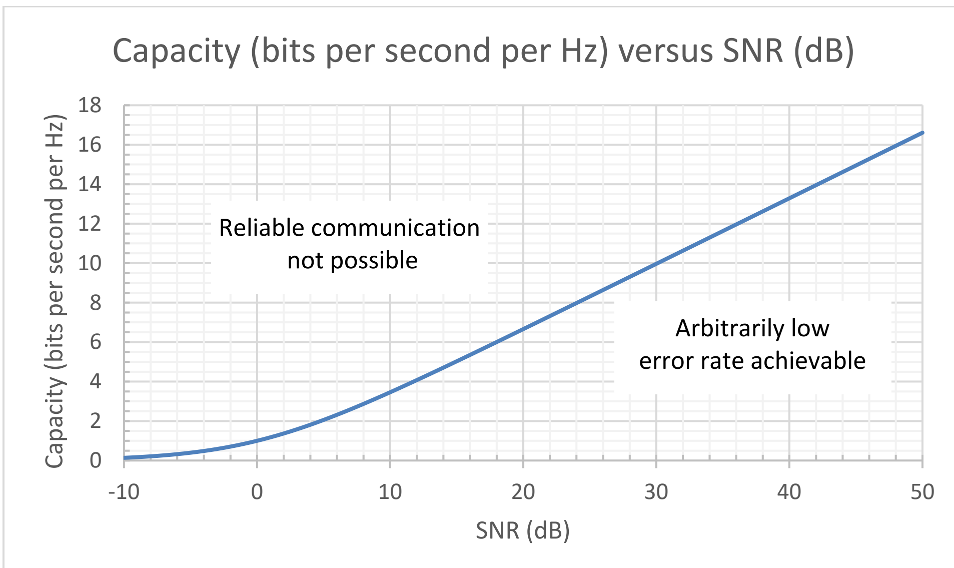


Figure 62. Capacity for a bandwidth limited channel; the diagonal blue curve is often called the "Shannon limit."

30.1.4. Asymptotic behavior of the bandwidth limited channel with large SNR

We have seen,

$$\frac{C}{W} = \frac{(10log_{10}[1 + SNR])}{3} \ bits\ per\ second\ per\ Hz.$$

And for large SNR, for example SNR ≥ 100, the approximation holds well (better than 1 percent):

$$\frac{C}{W} \cong \frac{(10log_{10}[SNR])}{3}\ bits\ per\ second\ per\ Hz.$$

But we see that this is directly using the definition of SNR_{dB}, such that:

$$\frac{C}{W} \cong \frac{SNR_{dB}}{3}\ bits\ per\ second\ per\ Hz.$$

Thus, for [P/(N_0 ∗ W)] equal 1,000, so that SNR is 1,000 and SNR_{dB} is 30 dB, we arrive at,

$$\frac{C}{W} \cong \frac{30}{3} \cong 10\ bits\ per\ second\ per\ Hz.$$

For an additional 3 dB SNR, i.e., SNR = 2,000 or SNR_{dB} = 33 dB, so that [P/(N_0 ∗ W)] is 2,000, and

$$\frac{C}{W} \cong \frac{33}{3} \cong 11\ bits\ per\ second\ per\ Hz.$$

We see that for larger SNR, such as 20 dB and larger, the channel capacity in bits per second per Hz is well approximated by (SNR_{dB})/3 bits per second per Hz. *This gives rise to the expression that there is 1 bit additional capacity for each 3 dB increase in SNR, for channels with large SNR.* Recall that this applies for a fixed channel bandwidth, W Hz, and the capacity increase is in units of bits per second per Hz.

30.1.5. Shannon theory in terms of E_b/N_0

A commonly referenced view of the Shannon capacity theory discusses the theorem and the insights in terms of E_b/N_0, the ratio of the energy per bit to the one-sided noise power spectral density. This is especially relevant for the power limited channel view, as described in Section 30.1.2. We will examine this view of the channel capacity theory, in terms of E_b/N_0, and begin with the expression for channel capacity from Section 30.1.1:

$$C = W * log_2\left(1 + \frac{P}{(N_0 * W)}\right)\ bits\ per\ second.$$

Raising each side to the power of 2:

$$2^C = 2^{\left[W*log_2\left(1+\frac{P}{(N_0*W)}\right)\right]}$$

$$2^C = 2^{\left[log_2\left\{\left(1+\frac{P}{(N_0*W)}\right)^W\right\}\right]}$$

$$2^C = \left\{\left(1 + \frac{P}{(N_0 * W)}\right)^W\right\}$$

Rearranging a little bit:

$$2^{C} = \left\{ \left(1 + \frac{P}{(N_0 * W)}\right)^{(W*N_0/P)} \right\}^{(P/N_0)}$$

Then, we recognize the form of the expression within brackets {} and take the limit as $W \rightarrow infinity$, where the term within the brackets goes to e in the limit, leaving:

$$2^{C} = e^{(P/N_0)} \; as \; W \rightarrow infinity$$

Taking $\log_2()$ of both sides, returns an expression for channel capacity back in units of bits per second, but now as a limit as the channel bandwidth increases to infinity.

$$C = \left(\frac{P}{N_0}\right) * log_2(e) \; as \; W \rightarrow infinity$$

It is apparent that this is the same result as shown as the limit in Figure 61, with $W_0 = P/N_0$.
Also, recall the units for channel capacity, C, are bits per second.

Multiplying by a generic time, T seconds, provides:

$$C * T \; bits = \left(P * \frac{T}{N_0}\right) * log_2(e) \; as \; W \rightarrow infinity$$

Since C ∗ T is the number of bits at the channel capacity, in a duration of T seconds, the energy for these bits is P ∗ T joules.

Now, solving for the time duration needed in this channel, if it operates at its full capacity, to transmit *one full bit of information*, is found by solving for T so that C ∗ T = 1 bit:

$$C * T = \left(P * \frac{T}{N_0}\right) * log_2(e) = 1,$$

$$when \; T_b = \frac{(N_0/P)}{log_2(e)} \; seconds.$$

So, T_b is the time duration, in seconds, according to the capacity of the channel, to deliver 1 bit, with infinite bandwidth, with P/N_0 for a given signal power-to-noise PSD ratio.

Since T_b is the time duration for one bit capacity with these channel parameters, $P * T_b$ is the energy per bit. We can assign $E_b = P * T_b$, in units of joules.

Further, since for this unlimited channel bandwidth, we solved for T_b specifically so that we would achieve:

$$C * T_b = \left(P * \frac{T_b}{N_0}\right) * log_2(e) = 1, unlimited \; channel \; bandwidth,$$

and,

$$\left(P * \frac{T_b}{N_0}\right) = E_b/N_0$$

Then, $(E_b/N_0)_{unlimited\_ch\_bw} * log_2(e) = 1$,

So that, $(E_b/N_0)_{unlimited\_ch\_bw} = \frac{1}{log_2(e)}$

This provides, $10log_{10}\left[(E_b/N_0)_{unlimited\_ch\_bw}\right] = -1.6\ dB$

For $E_b/N_0 < -1.6$ dB, reliable communication is impossible.

For $E_b/N_0 > -1.6$ dB, a finite bandwidth exists where the capacity will allow reliable communication.

Consider the following example: If you are given a channel of 6 MHz, and –10 dB SNR in AWGN, and a service provider claims to have technology to support 10 Mbps in that channel, how does this compare to the capacity theorem? We can walk through that.

$\frac{P}{(N_0*W)} = SNR = 10^{-1}$, with W = 6 MHz,

So, $P/N_0 = SNR * W = (10^{-1}) * (6 * 10^6) = 6 * 10^5$.

And 10 Mbps implies $T_b = 10^{-7}$ seconds,

So $E_b/N_0 = P * T_b/(N_0) = (6 * 10^5) * (10^{-7}) = 6 * 10^{-2}$

Computing the value of E_b/N_0 in decibels,

E_b/N_0 dB = 7.77 – 20 = –12.2 dB.

E_b/N_0 dB = –12.2 dB < –1.6 dB.

So, this service provider cannot really deliver on this claim! It is impossible! The claim is on the wrong side of Shannon's capacity theory.

Note that even if the channel SNR were 0 dB instead of –10 dB, or if the bit rate were 1 Mbps instead of 10 Mbps, the result of either change, individually, would correspond to 10 dB higher E_b/N_0, or –2.2 dB, which is STILL on the wrong side of the capacity theorem. However, if the claims were 1 Mbps and channel the SNR is 0 dB, that is 20 dB higher than the originally stated claim, and the E_b/N_0 with the revised parameters is 7.8 dB. This is more than 9 dB above the Shannon limit for E_b/N_0 (i.e., –1.6 dB). In today's (2021) technology, commercial personal communications systems routinely operate within a few dB of the Shannon limit. This was unheard of capability when Shannon developed and published his work!

30.2. Crest factor

Crest factor is a characteristic of a waveform, defined as the ratio of its peak to effective value. Expressed mathematically:

$$C = \frac{|\chi_{peak}|}{\chi_{rms}}$$

where
C is crest factor
χ_{peak} is the peak value of a waveform
χ_{rms} is the effective or root mean square value of a waveform

Expressed in decibels:

$$C_{dB} = 20log_{10}\left(\frac{|\chi_{peak}|}{\chi_{rms}}\right)$$

where
C_{dB} is crest factor in decibels
log_{10} is base 10 logarithm
χ_{peak} is the peak value of a waveform
χ_{rms} is the effective or root mean square value of a waveform

30.2.1. Peak-to-average power ratio

Related to crest factor is peak-to-average power ratio (PAPR), defined as the ratio of peak power (i.e., the square of the peak value) to average power (i.e., the square of the root mean square value). Expressed mathematically:

$$PAPR = \frac{|\chi_{peak}|^2}{{\chi_{rms}}^2}$$

where
$PAPR$ is peak-to-average power ratio
χ_{peak} is the peak value of a waveform
χ_{rms} is the effective or root mean square value of a waveform

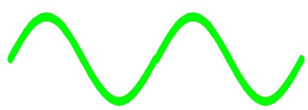
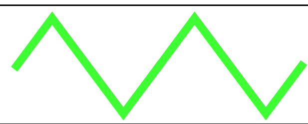
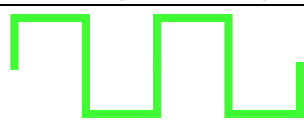
Expressed in decibels,

$$PAPR_{dB} = 10log_{10}\left(\frac{|\chi_{peak}|^2}{\chi_{rms}{}^2}\right)$$

where
$PAPR_{dB}$ is peak-to-average power ratio in decibels
log_{10} is base 10 logarithm
χ_{peak} is the peak value of a waveform
χ_{rms} is the effective or root mean square value of a waveform

The following table summarizes crest factor and PAPR for some common waveforms.

Table 22 - Crest factor and PAPR for various waveforms.

Waveform type	Waveform	RMS value	Crest factor	PAPR (dB)
Direct current		1	1	0.0
Sinusoidal		$\frac{1}{\sqrt{2}} \approx 0.707$	$\sqrt{2} \approx 1.414$	3.01
Triangle		$\frac{1}{\sqrt{3}} \approx 0.577$	$\sqrt{3} \approx 1.732$	4.77
Square		1	1	0.0

30.3. Series and parallel resistance formulas

30.3.1. Series resistance

The following formula can be used to calculate the total resistance of multiple resistors in series.

$$R_T = R_1 + R_2 + \cdots R_n$$

where
R_T is total resistance in ohms
R_1 is the first resistor's resistance in ohms
R_2 is the second resistor's resistance in ohms
R_n is the n^{th} resistor's resistance in ohms

30.3.2. Parallel resistance

The following formula can be used to calculate the total resistance of two resistors in parallel.

$$R_T = \frac{R_1 * R_2}{R_1 + R_2}$$

where
R_T is total resistance in ohms
R_1 is the first resistor's resistance in ohms
R_2 is the second resistor's resistance in ohms

The following formula can be used to calculate the total resistance of multiple resistors in parallel.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$$

where
R_T is total resistance in ohms
R_1 is the first resistor's resistance in ohms
R_2 is the second resistor's resistance in ohms
R_n is the n^{th} resistor's resistance in ohms

30.4. Series and parallel capacitance formulas

30.4.1. Parallel capacitance

The following formula can be used to calculate the total capacitance of multiple capacitors in parallel.

$$C_T = C_1 + C_2 + \cdots C_n$$

where
C_T is total capacitance in units of farads (e.g., farads, microfarads, picofarads, etc.)
C_1 is the first capacitor's capacitance in the same units of farads
C_2 is the second capacitor's capacitance in the same units of farads
C_n is the n^{th} capacitor's capacitance in the same units of farads

30.4.2. *Series capacitance*

The following formula can be used to calculate the total capacitance of two capacitors in series.

$$C_T = \frac{C_1 * C_2}{C_1 + C_2}$$

where
C_T is total capacitance in units of farads (e.g., farads, microfarads, picofarads, etc.)
C_1 is the first capacitor's capacitance in the same units of farads
C_2 is the second capacitor's capacitance in the same units of farads

The following formula can be used to calculate the total capacitance of multiple capacitors in series.

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}}$$

where
C_T is total capacitance in units of farads (e.g., farads, microfarads, picofarads, etc.)
C_1 is the first capacitor's capacitance in the same units of farads
C_2 is the second capacitor's capacitance in the same units of farads
C_n is the n^{th} capacitor's capacitance in the same units of farads

30.5. Capacitive reactance

Capacitive reactance is the opposition to alternating current by a capacitor (or capacitance), and is expressed in ohms. Capacitive reactance can be calculated using the following formula:

$$X_C = \frac{1}{2\pi f C}$$

where
X_C is capacitive reactance in ohms
f is frequency in hertz (Hz)
C is capacitance in farads (F)

30.6. Inductive reactance

Inductive reactance is the opposition to alternating current by an inductor (or inductance), and is expressed in ohms. Inductive reactance can be calculated using the following formula:

$$X_L = 2\pi f L$$

where
X_L is inductive reactance in ohms
f is frequency in hertz (Hz)
L is inductance in henrys (H)

30.7. Resonance

A circuit that contains both inductive reactance (X_L) and capacitive reactance (X_C) is said to be a tuned or resonant circuit. When the inductive reactance and capacitive reactance are the same (that is, $X_L = X_C$), a resonant frequency can be calculated using the following formula:

$$f = \frac{1}{2 * \pi * \sqrt{L * C}}$$

where
f is frequency in hertz (Hz)
L is inductance in henrys (H)
C is capacitance in farads (F)

30.8. Impedance

Impedance is the total opposition to current in a component, device, or circuit, and is expressed in ohms. Impedance is further defined as the frequency domain ratio of voltage to current. Impedance in an AC circuit, including RF, is a complex value and includes both resistance (the real part of complex impedance) and reactance (the imaginary part of complex impedance) – that is, both magnitude and phase. Impedance can be thought of as a way to describe the concept of AC resistance.[62]

The Cartesian or rectangular form of complex impedance is expressed mathematically as:

$$Z = R + jX$$

where
Z is complex impedance in ohms
R is the resistance in ohms (the real part of the complex impedance)
j is the imaginary unit or imaginary number[63]
X is the reactance in ohms (the imaginary part of the complex impedance)

[62] Resistance in a DC circuit also is expressed in ohms but has only magnitude. DC resistance is sometimes called "DC impedance."
[63] In mathematics "*i*" is commonly used for the imaginary unit, but in electronics and related fields "*j*" is used instead because the letter "i" (or "I") is used for current.

The polar form of impedance and phase angle is expressed mathematically as:

$$Z = |Z|\angle\theta$$

where
Z is impedance in ohms
$|Z|$ is the magnitude of the impedance
$\angle\theta$ is the phase angle of the impedance

Discussion:
One can graph the Cartesian or rectangular form of complex impedance as shown in Figure 63. Resistance R is on the horizontal axis, and reactance X is on the vertical axis. From [2] the impedance of a resonant thin half-wave dipole antenna in free space is $Z = 73 + j42.5$ ohms. In this example the reactance is inductive, so the complex impedance expression includes a plus sign ($R + jX$). If the reactance were capacitive, the complex impedance expression would include a minus sign ($R - jX$).

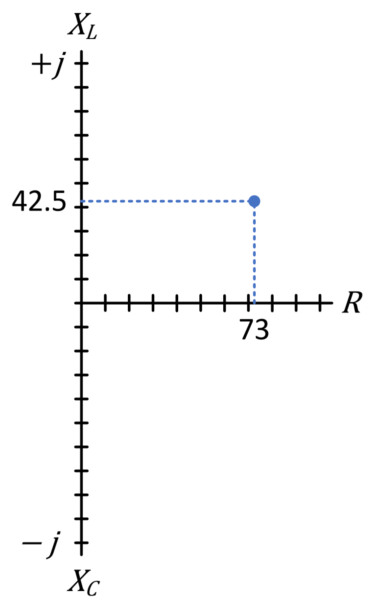


Figure 63 - Graph of Cartesian or rectangular form of the complex impedance of a resonant thin half-wave dipole antenna, *Z* = 73 + *j*42.5 ohms.

Impedance can be graphed in a polar form, too. Figure 64 shows the impedance of a resonant thin half-wave dipole antenna in free space, in polar form: $Z = 84.47\Omega\angle 30.21°$. The magnitude of the dipole's impedance is derived from the rectangular form of its complex impedance, using Pythagorean's Theorem: $|Z| = \sqrt{73^2 + 42.5^2} = 84.47$ ohms. The phase angle of the impedance is $\theta = atan\left(\frac{42.5}{73}\right) =$ 30.21 degrees.

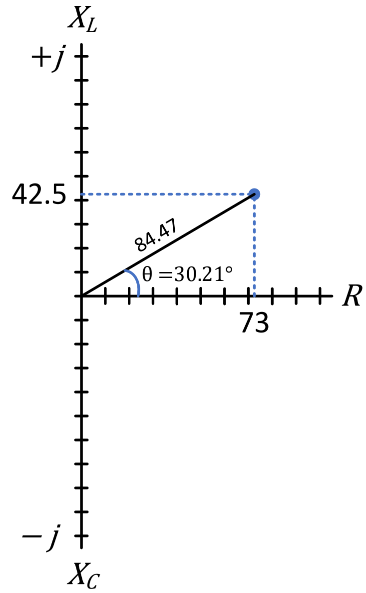


Figure 64 - Graph of polar form of the complex impedance of a half-wave dipole, *Z* = 84.47Ω∠30.21°. The latter is read or spoken as "84.47 ohms at an angle of 30.21 degrees."

Impedance also can be plotted on a Smith Chart; a simplified example is shown in Figure 65. In most cases the values on a Smith Chart are normalized – that is, the actual impedance is divided by the characteristic impedance.[64] The horizontal green line in Figure 65 is the resistance component (R/Z_0), with the center point "r = 1" representing the normalized characteristic impedance. Here, the point "r = 1" is 75 ohms normalized to a value of 1.

The red circles in Figure 65 are normalized resistance circles, centered on the R/Z_0 axis. The curved green lines are normalized reactance circles; the green lines above the R/Z_0 axis are positive reactance circles, and the green lines below the R/Z_0 axis are negative reactance circles. The dashed circles are admittance circles. The normalized complex impedance of a resonant thin halfwave dipole is plotted as the blue dot in Figure 65. The dipole's complex impedance of $Z = 73 + j42.5$ ohms, when normalized, is $Z_{norm} = 0.97 + j0.57$ ohms.

For more information about the Smith Chart, see [32].

[64] Multiplication or division of complex numbers can be done in polar or rectangular form using complex conjugates.

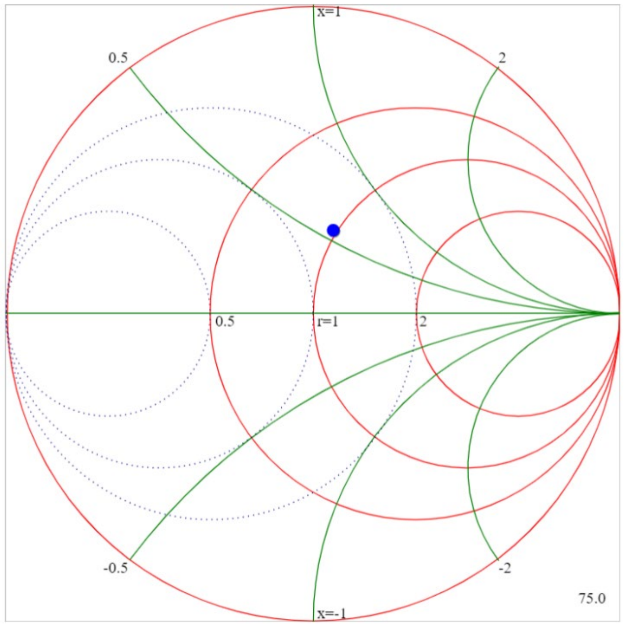


Figure 65 - Normalized complex impedance of a resonant thin half-wave dipole, Z_{norm} = 0.97 + *j*0.57 ohms, plotted on a Smith Chart. Plot created online at https://quicksmith.online

30.9. Group delay distortion

If propagation or transit time through a device is the same at all frequencies, phase is said to be linear with respect to frequency. If phase changes uniformly with frequency, an output signal will be identical to the input signal, except that it will have a time shift because of the uniform delay through the device. If propagation or transit time through a device is different at different frequencies, the result is a delay shift or non-linear phase shift. If phase changes non-linearly with frequency, the output signal will be distorted.

Delay distortion – also known as phase distortion – is usually expressed in units of time: millisecond (ms), microsecond (µs) or nanosecond (ns) relative to a reference frequency. Phase distortion is related to phase delay, and phase distortion is measured using a parameter called envelope delay distortion, or group delay distortion. To limit the amount of group delay distortion in a system, equipment often has a specification on group delay variation (GDV) setting a limit on the amount of variation in the group delay over a given frequency range, in some manner.

According to [27], group delay is "the [negative] derivative of radian phase with respect to radian frequency. It is equal to the phase delay for an ideal non-dispersive delay device, but may differ greatly in actual devices where there is a ripple in the phase versus frequency characteristic."

In its simplest form, group delay can be expressed mathematically as:

$$GD = -\frac{d\varphi}{d\omega}$$

where
GD is group delay
d is derivative

φ is phase in radians
ω is angular frequency in radians per second

In other words, if phase-versus-frequency is non-linear, group delay distortion exists. In a system, network, device or component with no group delay distortion, all frequencies propagate through the system, network, device or component in the same amount of time – that is, with equal time delay. If group delay distortion exists, signals (or parts of signals) at some frequencies propagate faster or slower than signals (or parts of signals) at other frequencies.

30.10. Modulation error ratio

A commonly used metric to characterize the health of digital signals is modulation error ratio (MER), defined as the ratio of average signal constellation power to average constellation error power. Most MER measurements of interest in cable are receive modulation error ratio (RxMER), which is the MER as measured in a digital receiver after demodulation, with or without adaptive equalization. Less commonly used is transmit modulation error ratio (TxMER), the MER produced by a transmitter under test, as measured by an ideal test receiver. Expressed mathematically:

$$MER = 10\log_{10}\left[\frac{\sum_{j=1}^{N}\left(I_j^2 + Q_j^2\right)}{\sum_{j=1}^{N}\left(\delta I_j^2 + \delta Q_j^2\right)}\right]$$

where
MER is modulation error ratio
log_{10} is base 10 logarithm
I and Q are the real (in-phase) and imaginary (quadrature) parts of each sampled ideal target symbol vector
δI and δQ are the real (in-phase) and imaginary (quadrature) parts of each modulation error vector

30.11. Error vector magnitude

The following discussion about error vector magnitude is excerpted from [6]. Used with permission of the authors.

> Another measurement metric that is closely related to MER is error vector magnitude (EVM). As shown … in Figure 13 in [6], EVM is the magnitude of the vector drawn between the ideal (reference or target) symbol position of the constellation, or hard decision, and the measured symbol position, or soft decision. By convention, EVM is reported as a percentage of peak signal level, usually defined by the constellation corner states. Mathematically, EVM follows:

$$EVM = \left(\frac{E_{RMS}}{S_{max}}\right) * 100$$

where
EVM is error vector magnitude
E_{RMS} is the root mean square error magnitude
S_{max} is the maximum symbol magnitude

Error vector magnitude is normally expressed as a linear measurement in percent, and MER is normally expressed as a logarithmic measurement in dB. Why use EVM instead of MER to characterize a data signal? Many data engineers are familiar with EVM, and for some, linear measurements are easier to work with than logarithmic measurements. Error vector magnitude links directly with the constellation display, and there is a linear relationship between EVM and a constellation symbol point “cloud size” or “fuzziness.”

Maximum-to-Average Constellation Power Ratio and EVM/MER Conversion
Because EVM and MER are referenced differently, in order to relate EVM to MER, we must first compute the ratio of the peak constellation symbol power to the average constellation power. The peak constellation power is the squared magnitude of the outermost (corner) QAM symbol. Its formula for a square QAM constellation on an integer grid follows:

$$P_{peak} = 2\left(\sqrt{M} - 1\right)^2$$

where M is the number of points in the constellation (M = 4, 16, 64, 256, etc.) and the points are spaced by 2 on each axis. For example, for 16-QAM, the I and Q coordinates take on values from the set {–3, –1, 1, 3} and the peak power is $2(4 – 1)^2 = 3^2 + 3^2 = 18$. (Use of the integer grid is for illustration purposes only and does not imply any particular power normalization.)

The average constellation power (averaged equally over all symbols in the constellation) follows:[65]

$$P_{av} = \frac{2}{3}(M - 1)$$

For example, for 16-QAM, the average constellation power is (2/3) (16 – 1) = 10. Note that this result happens to equal the power of one of the constellation points; the point (3,1) also has power $3^2 + 1^2 = 10$.

The maximum-to-average constellation power ratio (MTA) is, therefore, the unitless ratio

$$\mathrm{MTA} = \frac{P_{peak}}{P_{av}} = 3\,\frac{\sqrt{\mathrm{M}} - 1}{\sqrt{\mathrm{M}} + 1}$$

[65] Simon, Hinedi, and Lindsey, *Digital Communication Techniques*, equation 10.25, page 628.

which approaches 3, or in decibels, $10\log_{10}(3) = 4.77$ dB, for very-high-order QAM. MTA (converted to dB) is tabulated in Table 23, which contains entries for the standard square constellations as well double-square constellations. A double-square constellation is a subset consisting of half the points of the next-higher square constellation, arranged like the black squares on a checkerboard, and contains the same peak and average values as the next-higher square constellation. DOCSIS uses 64-QAM and 256-QAM square constellations for downstream transmission, and both square and double-square constellations from QPSK to 128-QAM for upstream transmission.

Table 23 - MTA ratio for square and double-square QAM constellations.

Constellation (DS = Double-Square)	MTA Ratio for Constellation Symbols (dB)
QPSK and BPSK	0
16-QAM and 8-QAM-DS	2.55
64-QAM and 32-QAM-DS	3.68
256-QAM and 128-QAM-DS	4.23
1024-QAM and 512-QAM-DS	4.50
Limit for infinite QAM	4.77

We can now convert from MER to EVM using the formula:

$$EVM\_\% = 100 * 10^{-(MER\_dB+MTA\_dB)/20}$$

where
$EVM\_\%$ is error vector magnitude (percent)
MER_dB is modulation error ratio (dB)
MTA_dB is maximum-to-average constellation ratio (dB)

MTA Versus Peak-to-Average Ratio of an RF Signal
It is important not to confuse MTA with the peak-to-average ratio (PAR) of the actual transmitted signal. MTA accounts only for the distribution of the ideal QAM constellation symbols. Because of the subsequent spreading, filtering, and modulation processes that operate on the symbols, the effective PAR of a single modulated RF carrier will typically lie in the range of 6 to 13 dB or more. (The effect of filtering was illustrated previously by the long tails in the distribution in part (c) of Figure 9 of [6].) The actual PAR value depends on the modulation, excess bandwidth, and whether pre-equalization or S-CDMA spreading is in use.[66] The PAR of a combined signal containing multiple carriers (such as the aggregate upstream or downstream signal on the cable plant) can become very large. Fortunately, the peaks occur very seldom, and the aggregate signal can often be treated like a random Gaussian signal.

[66] HEYS Professional Services' Francis Edgington has measured practical PAR values in the 6.3- to 7.3-dB range for 64-QAM signals and 6.5- to 7.5-dB range for 256-QAM signals.

30.12. Understanding CMTS OFDM power configuration

The material in this section is excerpted and adapted from *SCTE 258 2020 DOCSIS 3.1 Downstream OFDM Power Definition, Calculation, and Measurement Techniques*. Refer to that document for more information.[67]

The addition of OFDM signals to the cable network signal lineup will normally require the CMTS output power level configuration be consistent with existing headend combining networks. Many operators have elected to migrate their headend systems to converged cable access platform (CCAP) equipment which generates a full line-up of SC-QAM and OFDM signals for digital video, DOCSIS 3.0 and DOCSIS 3.1 services. The CCAP's RF output levels may need to accommodate analog optical transport systems to serve hybrid fiber/coax (HFC) nodes in the field. The CCAP output levels may be flat or tilted as required to satisfy the HFC design rules established by each operator.

Independent of this deployment practice, the DOCSIS 3.1 specification authors needed to describe how the CMTS output fidelity should be defined. They documented their agreement about this in ***Section 7.5.9.1 CMTS Output Electrical Requirements*** in the DOCSIS 3.1 Physical Layer Specification. This section of the DOCSIS 3.1 Specification is not intended to define how the CMTS should be configured for any network deployment. It assumes that the power for each legacy SC-QAM channel in the lineup is set to the same value. Any OFDM signal generated by the CMTS is treated as the equivalent of a sum of SC-QAM channels which would occupy the total spectrum of the OFDM signal. Section 7.5.9.1 says that the total power (the average power) of an OFDM signal is the sum of the power of all these virtual legacy digital channels which were "generated" in place of the OFDM signal.

Here's the relevant wording from the spec: "CMTS power is configured by power per CEA channel and number of occupied CEA channels for each OFDM channel." The spec goes on to say "For each OFDM channel, the total power is Power per CEA channel + $10\log_{10}$(Number of occupied CEA channels) for that OFDM channel."[68]

The aforementioned text seems straightforward enough, but there is potential confusion surrounding how CMTS output power is defined in the spec's ***Table 42 - CMTS Output Power***. As mentioned previously, an OFDM signal's total power is measured (or calculated) over its occupied bandwidth, while Table 42 describes the required power capabilities for the modulator in terms of the power per 6 MHz. Further confusing things is a description in ***Table 44 - CMTS OFDM Channel Characteristic*** that describes certain characteristics of an OFDM channel (emphasis on "channel").

When discussing an OFDM signal, it is important to be clear what is meant by the following: an OFDM channel (sometimes called OFDM block), an OFDM signal (often intended to be the same thing as an OFDM channel), the OFDM signal's total power, and its per 6 MHz power (aka OFDM channel power). A suggestion for the logical progression for configuring the downstream lineup power is outlined in the following paragraph, and then detailed examples are provided.

The first step is to plan the downstream spectrum that is to be modulated, accounting for all the CTA channel slots which are occupied by OFDM signals and legacy SC-QAM signals. Count the number of occupied CTA channels in this lineup, and denote this number of CTA channels for the entire

[67] The referenced document is available on SCTE's standards download page at https://www.scte.org/

[68] The CEA-542-D standard was renamed CTA-542-D when the Consumer Electronics Association changed its name to the Consumer Technology Association. The latest version of the standard is "CTA-542-D R-2018 Cable Television Channel Identification Plan," so the correct designation for 6 MHz channels on cable networks is "CTA channels" rather than "CEA channels" (or the older "EIA channels").

downstream lineup as $N_{occupied}$. Use the DOCSIS 3.1 PHY spec's Table 42 to determine the range of admissible values of modulated power per 6 MHz using $N_{occupied}$ (in place of N_{eq}') and the modulator capability N_{eq}, and select a value from within this range (if the modulator has more capability than the ranges in Table 42, the operator may select such value for which the modulator is capable and is desired). The next step is to determine the OFDM channel power for any OFDM signal, based on the occupied bandwidth of the OFDM signal.

The next step for the modulator is to determine the power settings for each subcarrier, given the OFDM channel power, but this is beyond the scope of this document.

[Also adding potential confusion, the spec intentionally uses a different reference in the spurious emissions requirements for signal power, referenced as "0 dBc" in 6 MHz, than is used to set up the total power for the OFDM channel. The spurious emissions requirements are referenced to the "0 dBc" signal power, which derives from a) the amount of modulated spectrum, and b) the highest power 6 MHz within the OFDM signal (the 6 MHz containing the PHY link channel (PLC) also contains no inactive subcarriers, and more boosted subcarriers than the remaining 6 MHz spans of spectrum in the OFDM signal). On the other hand, the OFDM channel power is based on the occupied bandwidth. The amount of modulated spectrum and the occupied bandwidth are different values. (This document does not delve into why the different definitions are applied in the spec, but the definitions were reasoned.)]

An excerpt from Table 42 of the DOCSIS 3.1 PHY spec is included here for reference:

$$\text{for } N^* \equiv \begin{cases} \textit{minimum}[4N_{eq}', \text{ceiling}\left[\frac{N_{eq}}{4}\right]], & N_{eq}' < N_{eq}/4 \\ N_{eq}', & N_{eq}' \geq N_{eq}/4 \end{cases}, \text{ Adjusted Number of Active Channels Combined per RF Port}$$

Parameter	Value
Required power per channel for N_{eq}' channels combined onto a single RF port:	Required power in dBmV per channel 60 – ceil [3.6*$\log_2(N^*)$] dBmV
Range of commanded transmit power per channel	≥ 8 dB below required power level specified below maintaining full fidelity over the 8 dB range
Range of commanded power per channel; adjusted on a per channel basis	CMTS MUST: 0 dBc to -2 dBc relative to the highest commanded transmit power per channel, within an 8 dB absolute window below the highest commanded power. CMTS MAY: *required power* (in table below) to *required power - 8 dB*, independently on each channel.

The terms N*, N_{eq}, N_{eq}', "ceil," "ceiling," and "minimum" in the table may benefit from clarification. The first usage of some of the terms can be found in the DOCSIS 3.1 PHY spec's section **7.5.9 Fidelity Requirements**: "CMTSs capable of generating *N*-channels of legacy DOCSIS plus N_{OFDM}-channels of OFDM per RF port, for purposes of the DRFI output electrical requirements, the device is said to be capable of generating N_{eq}-channels per RF port, where $N_{eq} = N + 32 * N_{OFDM}$ "equivalent legacy DOCSIS channels.""

Let's look at the terms "ceil" and "minimum" first.

- ceil – An abbreviation for ceiling, which, according to the DOCSIS 3.1 PHY spec's glossary is "A mathematical function that returns the lowest-valued integer that is greater than or equal to a given value." For example, solve the following equation:

  ceil(13/3) = ?

By itself, (13/3) = 4.33, but when the ceiling function is applied, 4.33 is "rounded up" to the nearest integer. So, ceil(13/3) = 5. If the value to which the ceiling function is applied is an integer – for instance, ceil(12) – the returned value is ceil(12) = 12.

- minimum – The smallest value in a set. Consider the set of numbers [10, 2, 15, 8]. In this case, minimum[10, 2, 15, 8] = 2. If the set contains formulas, the minimum is the smallest valued answer or solution to the formulas.

The following is an overview of the various "N" terms (refer to the Appendix of [28] for more information).

- N – The number of legacy DOCSIS channels per RF port that a CMTS is *capable* of generating (According to **6.4.2 Downstream Electrical Input to the CM** "A CMTS MUST support at least 32 active downstream channels").

- N_{OFDM} – The number of downstream OFDM channels per RF port the CMTS is *capable* of generating. (According to **7.2.2 Downstream CMTS Spectrum** "The CMTS MUST support a minimum of two independently configurable OFDM channels each occupying a spectrum of up to 192 MHz in the downstream.")

- N_{eq} – The number of *equivalent* legacy DOCSIS channels per RF port the CMTS is capable of generating, defined by the formula

  $N_{eq} = N + 32 * N_{OFDM}$

  For example, assume the CMTS is capable of 32 SC-QAM channels and two 192 MHz-wide OFDM channels per RF port. In this example, N = 32 and $N_{OFDM} = 2$. Solving for N_{eq}, we get

  $N_{eq} = 32 + 32 * 2$
  $N_{eq} = 32 + 64$
  $N_{eq} = 96$

  In other words, the CMTS is capable of generating the *equivalent* of 96 legacy DOCSIS SC-QAM channels per RF port.

- N_{eq}' – The number of *active* equivalent legacy DOCSIS channels per RF port. According to the DOCSIS 3.1 PHY spec, the number of equivalent active legacy DOCSIS channels in the OFDM channel N_{eq}' is the ceiling function applied to the modulated spectrum[69] divided by 6 MHz.

  Here's an example of calculating N_{eq}'. Assume that the CMTS is configured to generate just one 96 MHz-wide OFDM channel per port. For a 96 MHz-wide downstream OFDM channel with no exclusion bands and with the minimum 1 MHz-wide taper region on each end of the channel, the modulated spectrum is 94 MHz, so N_{eq}' = ceil[94 MHz/6 MHz] = 16 active equivalent legacy SC-QAM DOCSIS channels.

- N* – The *adjusted* number of active channels combined per RF port. The DOCSIS 3.1 PHY spec says "For an N_{eq}-channel per RF port device, the applicable maximum power per channel and

[69] See Section 10 of [28] for more information about *modulated spectrum*, *encompassed spectrum*, and *occupied bandwidth*.

> spurious emissions requirements are defined using a value of N^* = minimum($4N_{eq}'$, ceiling[N_{eq}/4]) for $N_{eq}' < N_{eq}/4$, and $N^* = N_{eq}'$ otherwise."
>
> That is, if $N_{eq}' < N_{eq}/4$, then N^* = minimum[$4N_{eq}'$, ceil(N_{eq}/4)]. Or, if $N_{eq}' \geq N_{eq}/4$, then $N^* = N_{eq}'$.

The importance of N* is that in Table 42 of the DOCSIS 3.1 PHY spec, the required power per channel for N_{eq}' channels combined onto a single RF port is stated as "Required power in dBmV per channel 60 – ceil[3.6 ∗ $\log_2$(N*)] dBmV." Note that the ceiling function is used in this equation. Also note that the logarithm function is base-2, not the more common base-10.[70]

30.12.1. Power Calculation Example

The following example illustrates how to calculate a DOCSIS 3.1 CMTS's downstream OFDM transmit power.[71] For the example, assume the CMTS is capable of generating 32 legacy SC-QAM channels (N = 32) and two 192 MHz-wide OFDM channels (N_{OFDM} = 2) per RF port. For this hypothetical CMTS, then, N_{eq} = 96. Further assume that the intended full downstream lineup will have $N_{occupied}$ as given in the following example. Note that $N_{occupied}$ is the number of CTA channels occupied by the entire downstream lineup, but if there is only one channel in the downstream lineup, then that channel alone determines $N_{occupied}$.

30.12.1.1. OFDM power per channel example

In order to calculate the power per channel (that is, power per 6 MHz), it's necessary to first determine the value of N*. To do that, we need the values for N_{eq} and N_{eq}', to use Table 42 from the DOCSIS 3.1 PHY spec, but where the latter is $N_{eq}' = N_{occupied}$ for the first step in determining output power.

Assume the CMTS is configured for a single 192 MHz-wide OFDM channel per RF port, with the OFDM channel's modulated spectrum equal to 190 MHz and a taper region of 1 MHz on each side, and centered within the CTA channel boundaries so that $N_{occupied}$ = 32. From the previously stated assumptions for these examples, we know that N_{eq} = 96.

$N_{eq}' = N_{occupied}$
N_{eq}' = 32 active equivalent legacy SC-QAM DOCSIS channels

Next, solve for N_{eq}/4, which in this example is 96/4 = 24. Here, $N_{eq}' > N_{eq}/4$ (that is, 32 > 24), so N* = N_{eq}', or N* = 32. Now we can calculate the per-channel power:

60 – ceil[3.6 ∗ $\log_2$(N*)]
60 – ceil[3.6 ∗ $\log_2$(32)]
60 – ceil[3.6 ∗ 5]
60 – ceil[18]
60 – 18 = 42

[70] Refer to Appendix A of this document for information on how to calculate base-2 logarithms.
[71] See the Data-Over-Cable Service Interface Specifications Downstream RF Interface Specification for information on characterization and calculation of downstream DOCSIS SC-QAM per-channel power.

The per-channel power in this example is 42 dBmV.[72]

30.12.2. OFDM total power

What is the total power for the previous example? We know the per-channel power (remember, DOCSIS 3.1 downstream transmit power for the CMTS is configured by *power per CTA channel* or *power per 6 MHz*). Recall that the DOCSIS 3.1 spec says "For each OFDM channel, the total power is Power per CEA channel + $10\log_{10}$(Number of occupied CEA channels) for that OFDM channel."

Knowing that, let's calculate the total power for the previous example.

30.12.2.1. OFDM total power example

The per-channel power was calculated to be 42 dBmV. The occupied bandwidth of the single 192 MHz-wide OFDM channel is 32 CTA channels, so the total power is

$P_{total} = 42 + 10\log_{10}(32)$
$P_{total} = 42 + 10 * \log_{10}(32)$
$P_{total} = 42 + 10 * 1.51$
$P_{total} = 42 + 15.05$
$P_{total} = 57.05$ dBmV

30.13. Phase noise

The characterization of a signal source such as an oscillator includes a variety of parameters. Some examples of those parameters are output power capability, frequency stability, amplitude stability, harmonics and spurious signals, and phase noise. The latter can be thought of as an undesired spreading of the signal spectrum in the frequency domain caused by phase fluctuations in the signal source, and is equivalent to jitter in the time domain. From Wikipedia,[73] "…IEEE defines phase noise as $\mathscr{L}(f) = S_\varphi(f)/2$ where the 'phase instability' $S_\varphi(f)$ is the one-sided spectral density of a signal's phase deviation." All signal sources exhibit some phase noise. See Figure 66.

Measurement of phase noise is usually a single-sideband (SSB) measurement in a 1 Hz bandwidth, at a specified frequency offset from the carrier – for example, 1 kHz, 10 kHz, etc. For more information about phase noise, see Appendix K.

[72] Note: If the 192 MHz of OFDM channel bandwidth were used instead for legacy SC-QAM channels, and no other legacy digital channels were transmitted, then (192 MHz)/(6 MHz per channel) = 32 SC-QAM channels would be modulated and the same per-channel power of 42 dBmV would result for each of those legacy SC-QAM channels.
[73] See https://en.wikipedia.org/wiki/Phase_noise

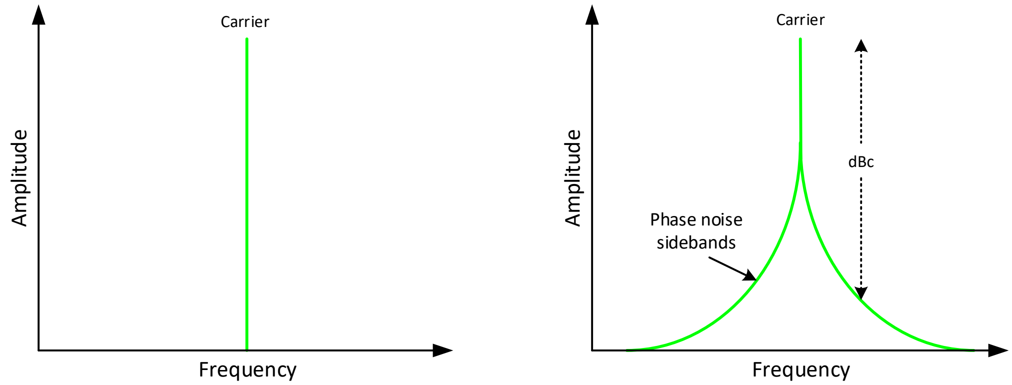


Figure 66 - Frequency domain representation of a carrier from an ideal signal source (left), and a carrier from a signal source with phase noise (right).

The following formula is known as Leeson's equation,[74] an empirical expression that describes an oscillator's phase noise spectrum. The formula gives SSB phase noise in decibel carrier per hertz (dBc/Hz), augmented for flicker noise.

$$L(f_m) = 10log_{10}\left[\frac{1}{2}\left(\left(\frac{f_0}{2Q_l f_m}\right)^2 + 1\right)\left(\frac{f_c}{f_m} + 1\right)\left(\frac{FkT}{P_s}\right)\right]$$

where
$L(f_m)$ is single-sideband phase noise in dBc/Hz
log_{10} is base 10 logarithm
f_0 is the output frequency
Q_l is the loaded quality factor
f_m is the offset from the output frequency hertz (Hz)
f_c is the 1/f corner frequency
F is the noise factor of the amplifier
k is Boltzmann's constant in joules/kelvin
T is absolute temperature in kelvin
P_s is the available power at the sustaining amplifier input

30.14. Total composite power

Signal levels in cable networks are often stated as per-channel values. For example, §76.605(b)(1) of the FCC Rules[75] says

> "The visual signal level, across a terminating impedance which correctly matches the internal impedance of the cable system as viewed from the subscriber terminal, shall not be less than 1 millivolt across an internal impedance of 75 ohms (0 dBmV)."

[74] See https://en.wikipedia.org/wiki/Leeson%27s_equation
[75] As of February 2021

The 1 millivolt (0 dBmV) value in the above text is the *per-channel* peak envelope power of an analog TV signal's visual carrier.

Sometimes it is necessary to know the total composite power (TCP, aka total power) at the input to a device such as a set-top box or cable modem, or at the output of a node or amplifier. Total (composite) power is the combined power of all signals and/or signal components in a defined bandwidth, such as the downstream spectrum.[76]

One reason is that total composite power is important is because of its impact on the performance of an active device such as a STB or amplifier. If the total power at the input to a STB or modem is too high, it can overload the front-end circuitry of the device and result in distortions being generated inside of that device. Likewise, for an active device such as a node or amplifier, the RF stages have limitations on the output total composite power that they can support. As operational total composite power approaches the maximum supported capability of a device, signal compression and clipping can occur.

Consider the RF input to a cable modem. Assume that the downstream spectrum is from 54 MHz to 1,002 MHz, and that there are 154 active 6 MHz-wide channels. Further assume that all 154 channels have the same per-channel power of 0 dBmV. The total power at the input to the modem is $P_{total} = 0\ dBmV + 10log_{10}(154) = 21.88\ dBmV$. As long as the per-channel power is the same value for all channels, calculation of total power is relatively straightforward.

Where things become complicated is when the RF spectrum is not flat, but has tilt, such as at the output of a node or amplifier. The formulas in the next section can be used to calculate total composite power when linear and cable equalization come into play. Refer to Appendix J for a mathematical derivation of the formulas used here.

30.14.1. Amplifier equalizer total composite power formulas

Richard S. Prodan, Ph.D.
Comcast Cable

Linear Equalizer:

$P_{dB}(f) = mf + b\ \ (dBmV/MHz)\ ;\ m \equiv slope,\ b \equiv intercept$

is the linear function of power per MHz (i.e., power spectral density PSD per MHz) in decibel millivolt as a function of frequency in MHz with slope m and intercept b. The total composite power TCP in decibel millivolt (dBmV) from the equalizer start frequency f_{start} to the equalizer stop frequency f_{stop} is given by the logarithm of the integral of $P_{dB}(f)$ as:

$$TCP = 10\ log_{10}\left(\int_{f_{start}}^{f_{stop}} 10^{(mf+b)/10}\,df\right) = 10\ log_{10}\left(\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_{start}}^{f_{stop}}\right)(dBmV)$$

where

$$Equalizer\ Tilt\ (dB) = Level\ @\ Equalizer\ Stop\ Frequency - \ Level\ @\ Equalizer\ Start\ Frequency$$

[76] See Section Appendix I for more information on total power.

$$m = \frac{Equalizer\ Tilt}{Equalizer\ Stop\ Frequency - Equalizer\ Start\ Frequency}$$

$$b = Transmit\ Level\ @\ Equalizer\ Start\ Frequency - \ Equalizer\ Slope * Equalizer\ Start\ Frequency - 10log_{10}(6)$$

and Transmit Level @ Equalizer Start Frequency = PSD per 6 MHz (dBmV/6 MHz) at that initial frequency.

The notation $\left.\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\right|_{f_{start}}^{f_{stop}} = \frac{10^{(mf_{stop}+b)/10}}{\frac{m}{10}\ln(10)} - \frac{10^{(mf_{start}+b)/10}}{\frac{m}{10}\ln(10)}$.

Cable Equalizer:

$$P_{dB}(f) = m\sqrt{f} + b \ \ (dBmV/MHz)\ ;\ m \equiv slope,\ b \equiv intercept$$

is the linear function of power per $\sqrt{MHz}$ (i.e., power spectral density PSD per $\sqrt{MHz}$) in decibel millivolt as a function of the square root of frequency in $\sqrt{MHz}$ with slope m and intercept b. The total composite power TCP in decibel millivolt (dBmV) from the equalizer start frequency f_{start} to the equalizer stop frequency f_{stop} is given by the logarithm of the integral of $P_{dB}(f)$ as:

$$TCP = 10\ log_{10}\left(\int_{f_{start}}^{f_{stop}} 10^{(m\sqrt{f}+b)/10}\,df\right) = 10\ log_{10}\left(\left.\frac{2 \cdot 10^{(m\sqrt{f}+b)/10}}{\left(\frac{m}{10}\ln(10)\right)^2}\left(\frac{m}{10}\ln(10)\sqrt{f} - 1\right)\right|_{f_{start}}^{f_{stop}}\right)\ (dBmV)$$

where

$$m = \frac{Equalizer\ Tilt}{\sqrt{Equalizer\ Stop\ Frequency} - \sqrt{Equalizer\ Start\ Frequency}}$$

$$b = Transmit\ Level\ @\ Equalizer\ Start\ Frequency + Equalizer\ Tilt - m\sqrt{Equalizer\ Stop\ Frequency} - 10\ log_{10}(6)$$

Note**:** Cable equalizers are designed to offset both the tilt and the shape associated with coaxial cable loss. A cable equalizer dB value equals the attenuation at the equalizer stop frequency approximately given by:

$$Cable\ EQ\ (dB) = \frac{Equalizer\ Tilt}{1 - \sqrt{Equalizer\ Start\ Frequency/Equalizer\ Stop\ Frequency}}$$

Example:
The previous formulas can be embedded in a spreadsheet to facilitate calculation of TCP and plots of equalizer response and node or amplifier output, as shown in the following examples.

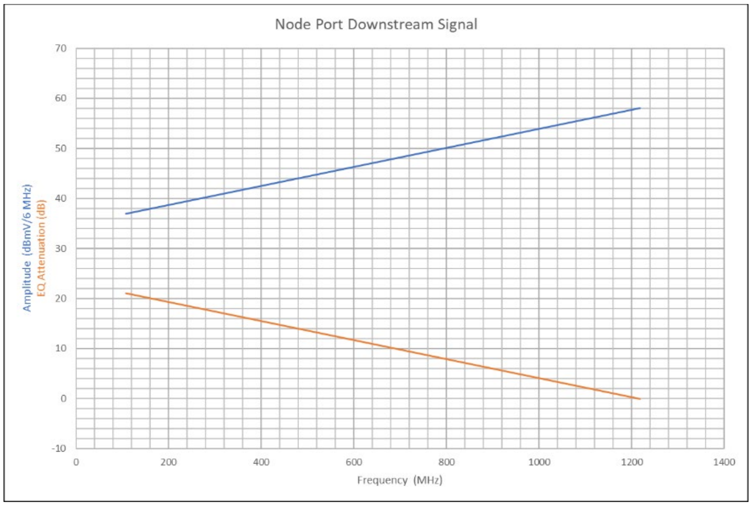
	Node	L.E.
Transmit Level (dBmV/6 MHz):	37	32

Equalizer Direction (Node):	Downstream
Equalizer Type (Cable,Linear,Flat,Tilt,Sawtooth):	Linear
Equalizer Low Frequency (MHz):	111
Equalizer High Frequency (MHz):	1215
Equalizer Tilt (dB/1104 MHz):	21

Total Composite Power = 73.8 (dBmV)

Equalizer Direction (Line Extender):	Downstream
Equalizer Type (Cable,Linear):	Cable
Equalizer Low Frequency (MHz):	111
Equalizer High Frequency (MHz):	1215
Equalizer Tilt (dB/1104 MHz):	21

Total Composite Power = 70.0 (dBmV)



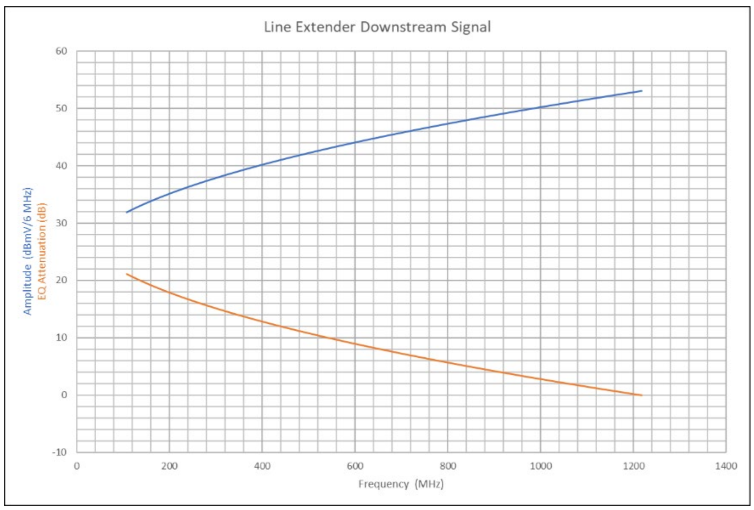


Figure 67 - Graphs of an example node's downstream output signal with a 21 dB Linear Equalizer (top) and an example line extender amplifier's downstream output signal with a 30 dB Cable Equalizer (bottom). The blue trace in each graph is amplitude in dBmV/6 MHz, and the red trace is the equalizer attenuation in dB. For the top graph, TCP = 73.8 dBmV, and for the bottom graph TCP = 70.0 dBmV.

Of particular importance is managing TCP when adjusting levels for improved node signal-to-reflection interference ratio or cable modem downstream receive level in full duplex operation, or managing TCP in extended spectrum operation – for instance, to 1.8 GHz (or higher). Several approaches can be used to manage signal levels and TCP across a wide operating bandwidth, including a suitable constant tilt from the start of the downstream spectrum (f_{start}) to the upper frequency limit (f_{stop}); unused gaps in the spectrum; a reduction ("step" or "stepdown") in the operating levels above a certain frequency; multiple steps in operating levels at higher frequencies; and so forth. See Figure 68.

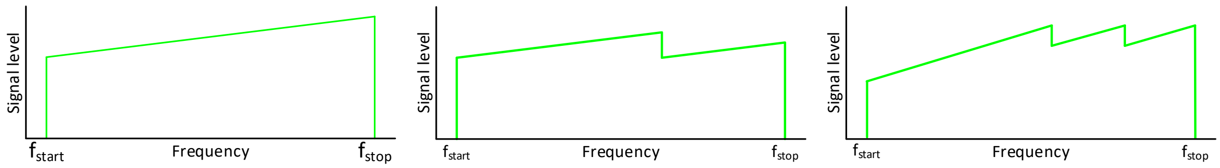


Figure 68 - Examples of ways to manage TCP: constant tilt (left), tilt with single stepdown (center), and tilt with multiple stepdowns (right).

If you have an amplitude step down of D dB above frequency f_D with the same slope, then

$$TCP = 10\,log_{10}\left(\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_{start}}^{f_D} + 10^{-D/10}\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_D}^{f_{stop}}\right)$$

The same basic approach can be used iteratively for piecewise continuous PSD segments with or without gaps and/or stepdowns (e.g., multiple stepdowns producing a “sawtooth” PSD profile).

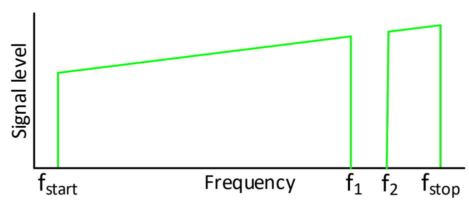


Figure 69 - Example of a gap in the power spectrum lowering TCP.

If the linearly tilted spectrum is continuous between f_{start} and f_{stop}, then

$$TCP = 10\,log_{10}\left(\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_{start}}^{f_{stop}}\right)$$

If there is a gap between f_1 and f_2 (e.g., an exclusion zone), then

$$TCP = 10\,log_{10}\left(\frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_{start}}^{f_1} + \frac{10^{(mf+b)/10}}{\frac{m}{10}\ln(10)}\Bigg|_{f_2}^{f_{stop}}\right)$$

Therefore, a summation of any number of gaps can be accommodated with such additional terms.

30.15. Network reliability and availability

In the 1990s as cable operators were upgrading network architectures to what we now call hybrid fiber/coax (HFC), important metrics for those new architectures included network reliability and availability. At the time, operators were striving to achieve "four nines" (99.99%) availability.[77] This section includes many of the basic formulas related to network reliability and availability, and are from Chapter 20 in [7], where the reader can find more detailed information (the same material also can be found in Chapter 12 in [4]).

30.15.1. Component failure rate

According to [7], "The failure rate of a component or system is defined as the statistical probability of failure of any one of a similar group of components in a given time interval." One way to calculate failure rate is to use actual data from installed devices. Expressed mathematically:

$$\lambda = \frac{k}{n * t}$$

where
λ is annual failure rate
k is the number of observed failures
n is the number of items in the test sample
t is the observation time in years

30.15.2. System failure rate

The following formula is used for a "simple nonredundant connection" and assumes that "n components are connected in such a way that the failure of any one of them will lead to a failure of some identified system or function … and the failures do not have a common cause." The formula is not exact because it is further assumed that the aforementioned failures don't overlap in time.

$$\lambda_{system} = \sum_{i=1}^{n} \lambda_i$$

where
λ_{system} is the net system failure rate
λ_i is the failure rate of component i
n is the number of components (any of whose failure will cause a system failure)

[77] 99.99% availability means that out of 8,760 hours per year, a network or service will be available 99.99% of the time, or about 8,759 hours and 7 minutes. Looked at another way, the network or service will be unavailable 0.01% of the time, or about 53 minutes per year. Five nines (99.999%) availability means that a network, device, or service will be unavailable 0.001% of the time during a year, or just over 5 minutes.

30.15.3. System reliability

System reliability is "the probability that the system will not fail during some specific period." The following formula is for a "simple nonredundant connection":

$$R(t) = e^{-\lambda t}$$

where
λ and t are in consistent units (for example, λ is failure rate per year and t is years, or λ is failure rate per hour and t is hours)

When devices with a known reliability are connected such that the failure of any device causes a system failure, the net system reliability can be calculated using the following formula:

$$R_s(t) = R_1(t)R_2(t) \ldots R_n(t)$$

where
$R_s(t)$ is net system reliability over period t
$R_1, R_2, \ldots, R_n$ is reliabilities of the individual components, measured over period t

30.15.4. Mean time between failures

MTBF is the calculated time between failures and is the inverse of the failure rate. Expressed mathematically:

$$MTBF = \frac{1}{\lambda}$$

where
$MTBF$ and λ are in consistent units

30.15.5. Mean time to restore

From [7], "When a failure does occur, the mean time between failure and restoration of the defined function is the MTTR." Note: MTTR also can be mean time to repair.

30.15.6. Availability

Availability is often confused with reliability. Recall that reliability is the probability that something will not fail during a specified period of time. From [7], availability is the "ratio of time that a service is available for use to the total time." A common availability goal is "four nines" or 99.99% over the course of a year. That means the service is available for 8759.12 hours out of a total of 8760 hours. Availability also can be calculated from MTBF and MTTR, using the following formula:

$$A = \frac{MTBF}{MTBF + MTTR}$$

where
A is availability
$MTBF$ is mean time between failure
$MTTR$ is mean time to restore

30.15.7. Unavailability

Unavailability is the ratio of time that a service is unavailable to total time. Unavailability can be calculated using the following formula:

$$U = 1 - A$$

where
U is unavailability
A is availability

30.15.8. Outage time

From [7], outage time is "the amount of time that the network is unavailable during a defined period." Outage time is commonly stated in minutes per year, calculated using the following formula:

$$T_u(min/yr) = (min/yr)(1 - A) = 525{,}600(1 - A)$$

where
T_u is outage time
min/yr is minutes per year
A is availability

For a discussion about the effects of redundant network connections, the reader is referred to Chapter 20, Section 20.4 in [7].

31. Miscellaneous Formulas

31.1. Amplitude modulation

Amplitude modulation conveys information by varying the amplitude of a carrier wave in proportion to a baseband modulating signal, such as audio, video, or digital data. The visual carriers of analog NTSC television signals are amplitude modulated, as are radio signals in the medium wave AM broadcast band. Some test signals used for cable network leakage monitoring are amplitude modulated.

31.1.1. AM in the frequency domain

Figure 70 illustrates a frequency domain representation of a 139.25 MHz RF carrier that is amplitude modulated by a baseband 1 kHz sine wave, similar to what one would see on a spectrum analyzer. In a simple double-sideband AM signal as shown here, the modulation percentage can be determined by the relationship between the amplitude of the carrier and the amplitude of each of the two sidebands.

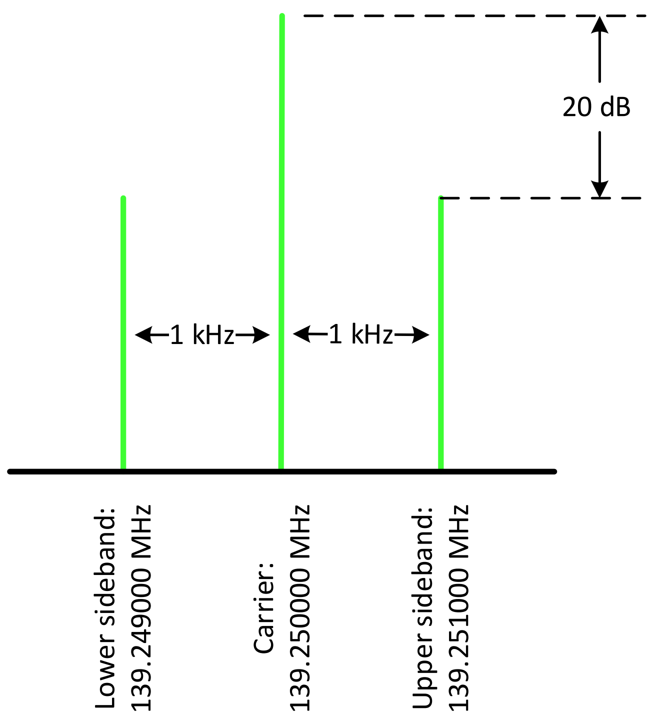


Figure 70 - Frequency domain example of amplitude modulation.

31.1.1.1. Convert modulation percentage to sideband level difference

To convert modulation percentage to sideband level below the carrier, use the following formula:

$$sideband_{dB} = -20log_{10}\left(\frac{M_{\%}}{200}\right)$$

where
$sideband_{dB}$ is the amplitude difference in decibels between the carrier and lower or upper sideband
log_{10} is base 10 logarithm
$M_{\%}$ is the modulation percentage

Example:
What is the difference, in decibels, between the carrier and sidebands for an amplitude modulation percentage of 50%?

Solution:
$$sideband_{dB} = -20log_{10}\left(\frac{M_{\%}}{200}\right)$$

$$sideband_{dB} = -20 * log_{10}\left(\frac{50}{200}\right)$$

$$sideband_{dB} = -20 * log_{10}(0.250)$$

$$sideband_{dB} = -20 * (-0.60206)$$

$$sideband_{dB} = 12.04$$

Answer: The amplitude difference between the carrier and sidebands is 12.04 dB.

31.1.1.2. Convert sideband level difference to modulation percentage

To convert sideband level below the carrier to modulation percentage, use the following formula:

$$M_{\%} = 10^{-sideband_{dB}/20} * 200$$

where
$M_{\%}$ is the modulation percentage
$sideband_{dB}$ is the level of the lower or upper sideband below the carrier in decibels

Example:
What is the modulation percentage if the sidebands are 20 dB lower than the carrier, as shown in Figure 70?

Solution:

$M_{\%} = 10^{-sideband_{dB}/20} * 200$

$M_{\%} = 10^{-20/20} * 200$

$M_{\%} = 10^{-1.00} * 200$

$M_{\%} = 0.10 * 200$

$M_{\%} = 20$

Answer: The modulation percentage is 20%.

31.1.2. AM in the time domain

Figure 71 illustrates a time domain representation of an RF carrier (lower part of figure) amplitude modulated by a sinusoidal baseband signal (upper part of figure), similar to what one would see on an oscilloscope. In an AM signal as shown here, the modulation percentage can be determined by the relationship of the modulated RF carrier envelope's maximum to minimum peak-to-peak voltage. Modulation percentage also is related to the ratio of the peak-to-peak voltage of the modulating signal to the peak-to-peak voltage of the unmodulated RF carrier's envelope.

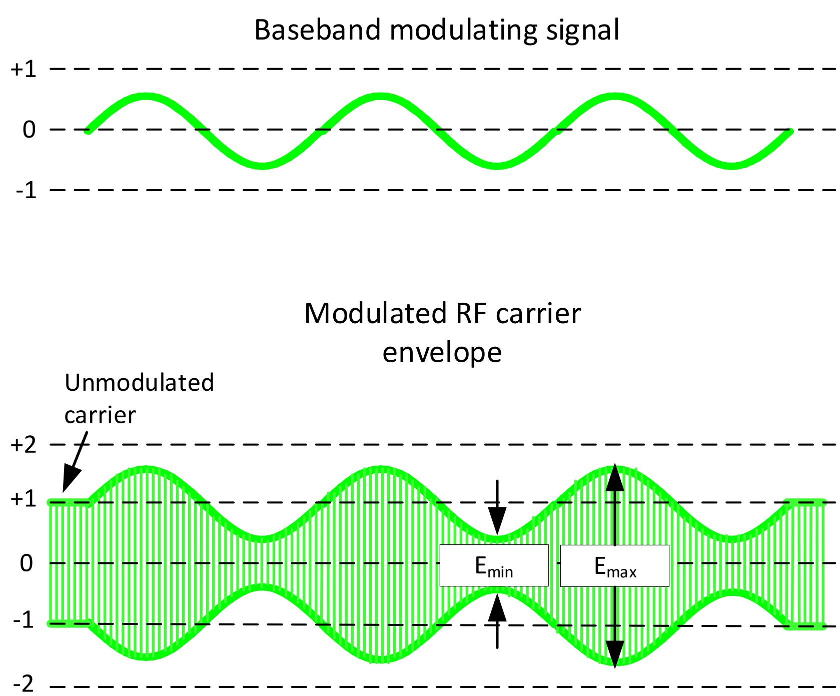


Figure 71 - Time domain representation of baseband modulating signal (top) and amplitude modulated RF carrier (bottom).

31.1.2.1. Calculate modulation percentage from the maximum and minimum peak-to-peak voltages of the modulated carrier's RF envelope

To calculate the modulation percentage from the maximum and minimum peak-to-peak voltages of the modulated carrier's RF envelope use the following formula:

$$M_{\%} = \left(\frac{E_{max} - E_{min}}{E_{max} + E_{min}}\right) * 100$$

where
$M_{\%}$ is the modulation percentage
E_{max} is the maximum peak-to-peak voltage of the modulated carrier's RF envelope
E_{min} is the minimum peak-to-peak voltage of the modulated carrier's RF envelope

Example:
Refer to Figure 71: What is the modulation percentage if the maximum peak-to-peak voltage of the modulated carrier's RF envelope is 3 volts and the minimum peak-to-peak voltage is 1 volt?

Solution:

$$M_{\%} = \left(\frac{E_{max} - E_{min}}{E_{max} + E_{min}}\right) * 100$$

$$M_{\%} = \left(\frac{3 - 1}{3 + 1}\right) * 100$$

$$M_{\%} = \left(\frac{2}{4}\right) * 100$$

$$M_{\%} = (0.50) * 100$$

$$M_{\%} = 50$$

Answer: The modulation percentage is 50%.

31.1.2.2. Calculate modulation percentage from peak-to-peak voltage of unmodulated RF carrier's envelope and peak-to-peak voltage of baseband modulating signal

Consider a simple AM modulator. This modulator is described by the equation

$$E(t) = [cos(2 * \pi * f * t)] * \left[RF_p + S(t)\right]$$

where
$E(t)$ is the modulated AM signal
t is time
f is the frequency of the RF carrier
RF_p is the peak voltage of the unmodulated RF carrier
$S(t)$ the baseband modulating signal

Such a modulator would produce the results shown in Figure 71. To calculate the modulation percentage from the peak-to-peak voltage of the unmodulated RF carrier's envelope and peak-to-peak voltage of the baseband modulating signal, the following formula can be used:

$$M_{\%} = \left(\frac{S_{p-p}}{RF_{p-p}}\right) * 100$$

where
$M_{\%}$ is the modulation percentage
S_{p-p} is the peak-to-peak voltage of the baseband modulating signal
RF_{p-p} is the peak-to-peak voltage of the unmodulated RF carrier's envelope

Example:
Refer to Figure 71: If the peak-to-peak voltage of the baseband modulating signal is 1 volt and the peak-to-peak voltage of the unmodulated RF carrier's envelope is 2 volts, what is the modulation percentage?

Solution:
$M_{\%} = \left(\frac{S_{p-p}}{RF_{p-p}}\right) * 100$

$M_{\%} = \left(\frac{1}{2}\right) * 100$

$M_{\%} = (0.50) * 100$

$M_{\%} = 50$

Answer: The modulation percentage is 50%.

31.2. Frequency modulation

Frequency modulation conveys information by varying the frequency of a carrier wave in proportion to a baseband modulating signal, such as audio or video. The aural carriers of analog NTSC television signals are frequency modulated, as are radio signals in the FM broadcast band.

31.2.1. Modulation index

Modulation index is the ratio of peak frequency deviation to the maximum baseband modulating frequency, and can be calculated with the following formula:

$$h = \frac{\Delta f}{f_m}$$

where
h is the modulation index
Δf is the peak frequency deviation in units of hertz (e.g., kilohertz)
f_m is the highest baseband modulating frequency in the same units as Δf

Example:
What is the modulation index for a monaural NTSC aural carrier, when the peak frequency deviation is 25 kHz and the highest modulating frequency is 15 kHz?

Solution:
$h = \frac{\Delta f}{f_m}$

$h = \frac{25}{15}$

$h = 1.6667$

Answer: The modulation index is 1.67.

31.2.2. Bessel null technique for setting deviation

When frequency modulating a carrier with a sine wave, one can use the Bessel null technique to set peak deviation. The carrier will null to zero and all of the power will be in the FM signal's sidebands when the modulation index is 2.4. The following formula can be used to calculate the frequency of a baseband signal that will result in a modulation index of 2.4.

$$f_m = \frac{\Delta f}{2.4}$$

where
f_m is the baseband modulating frequency in units of hertz
Δf is the peak frequency deviation in the same units as f_m
2.4 is the modulation index used to produce the FM signal's carrier null

Example:
What baseband modulating frequency can be used to create a carrier null when the modulation index is 2.4 and the peak deviation is 25 kHz?

Solution:
$$f_m = \frac{\Delta f}{2.4}$$
$$f_m = \frac{25}{2.4}$$

$$f_m = 10.42$$

Answer: The baseband modulating frequency is 10.42 kHz.

Note: When using the Bessel null technique to set frequency deviation, be sure to take pre-emphasis into account. For monaural NTSC aural carriers, pre-emphasis is used to reduce noise, and at 10.42 kHz adds 14 dB to the low-frequency deviation. This is especially important if using this technique to calibrate a deviation measurement instrument.

31.2.3. Carson's Bandwidth Rule

Also known as Carson's Rule, the following formula can be used to calculate the approximate occupied bandwidth of a modulated FM signal:

$$BW \cong 2 * (\Delta f + f_m)$$

where
BW is the approximate occupied RF bandwidth of a modulated FM signal in units of Hz, kHz, etc.
Δf is the peak frequency deviation in the same units as BW
f_m is the highest baseband modulating frequency in the same units as BW

Example:
What is the approximate occupied RF bandwidth of a monaural NTSC aural carrier when the peak deviation is 25 kHz and the highest baseband modulating frequency is 15 kHz?

Solution:
$BW \cong 2 * (\Delta f + f_m)$

$BW \cong 2 * (25 + 15)$

$BW \cong 2 * (40)$

$BW \cong 80$

Answer: The approximate occupied RF bandwidth is 80 kHz.

31.3. Trigonometry

Certain distances and heights can be determined using trigonometry, the foundation of which is based upon the relationships between the lengths of the sides and the angles of triangles. In a right triangle – that is, a triangle with one 90 degrees angle – such as that shown in Figure 72, the relationship of angle A, called angle θ in the figure, to the lengths of the sides of the triangle can be used for calculating distances and heights.[78]

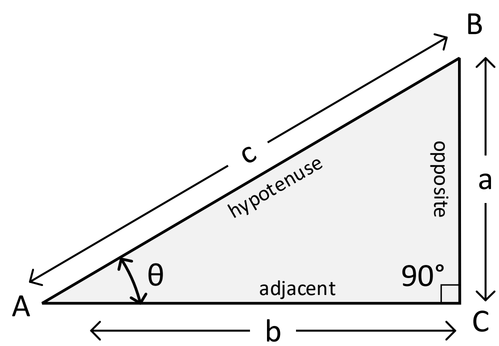


Figure 72 - Right triangle used for the basis of calculations in this section.

The following formulas describe common trigonometric ratios:

[78] The material in this section is adapted from [24] with updates.

$$sin\angle\theta = \frac{opposite}{hypotenuse} = \frac{length\ of\ side\ a}{length\ of\ side\ c}$$

$$cos\angle\theta = \frac{adjacent}{hypotenuse} = \frac{length\ of\ side\ b}{length\ of\ side\ c}$$

$$tan\angle\theta = \frac{opposite}{adjacent} = \frac{length\ of\ side\ a}{length\ of\ side\ b}$$

where
$\angle\theta$ is angle A in Figure 72
sin is the sine function
cos is the cosine function
tan is the tangent function

From the aforementioned trigonometric ratios the formulas in Figure 73 can be derived.

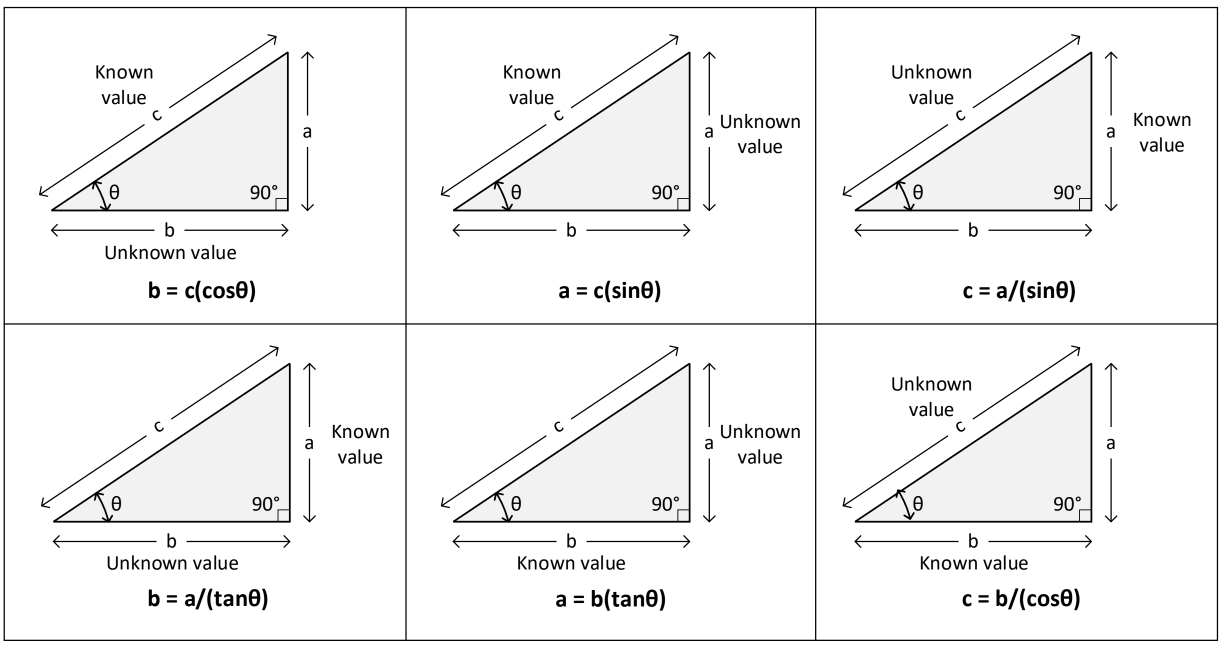


Figure 73 - Formulas and trigonometric relationships for examples in this section.

Example 1:
Refer to Figure 74. What is the height of the utility pole, assuming angle θ (measured with a surveying instrument such as an Abney level) is 20 degrees and the distance from the base of the pole to where the angle is measured is 150 feet?

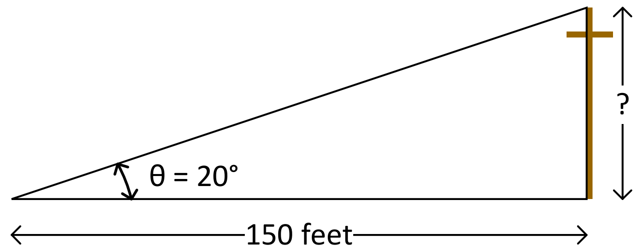


Figure 74 - What is the height of the utility pole if ∠θ = 20° as measured from a distance of 150 feet from the base of the pole?

Solution 1:

$a = b(tan\theta)$

$a = 150 * (tan20°)$

$a = 150 * (0.36)$

$a = 54.6$

Answer: The height of the utility pole is about 55 feet.

Example 2:

Refer to Figure 75. In order to receive signals from a new satellite, a new antenna must be installed at the headend. However, the only available space for the new antenna is directly behind an existing antenna. Assuming the existing antenna is 18 feet tall, the required elevation angle for the new antenna is 32°, and the lower rim of the new antenna will be 3 feet above the ground, what is the minimum horizontal separation between the two antennas in order for the new antenna to “see” over the top of the existing one?

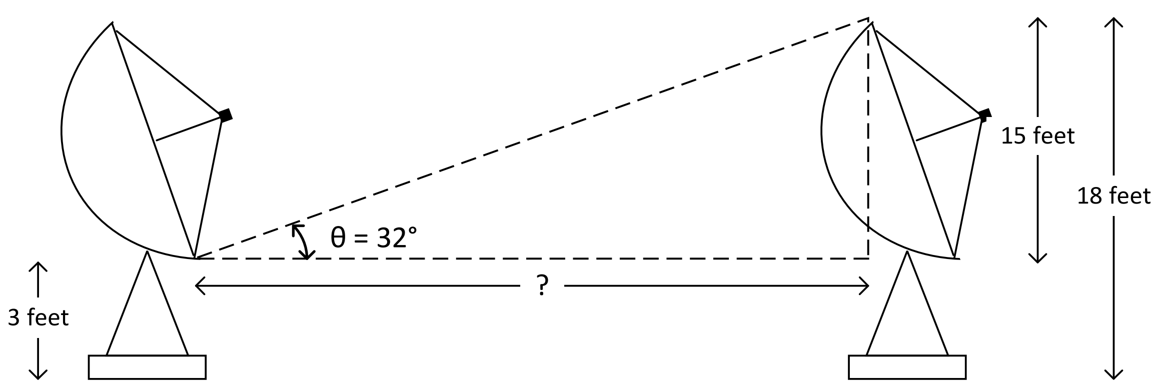


Figure 75 - What is the minimum horizontal separation between the two satellite antennas to ensure a new antenna (left) has an unobstructed view above an existing antenna (right) at the elevation angle shown?

Solution 2:
Subtract the new antenna's lower rim height of 3 feet from the height of the existing antenna (18 – 3 = 15). That gives the length of the opposite side (side a) of the superimposed right triangle.

$$b = \frac{a}{(tan\theta)}$$

$$b = \frac{15}{(tan32°)}$$

$$b = \frac{15}{(0.62)}$$

$$b = 24.01$$

Answer: The minimum horizontal separation is 24 feet.

31.4. Convert degrees, minutes, seconds to decimal degrees

The following formula can be used to convert degrees, minutes, seconds to decimal degrees:

$$degrees_{decimal} = degrees + \frac{minutes}{60} + \frac{seconds}{3{,}600}$$

Example:
What is a latitude value of 39˚31'44" N (39 degrees, 31 minutes, 44 seconds north) in decimal degrees?

Solution:

$$degrees_{decimal} = degrees + \frac{minutes}{60} + \frac{seconds}{3{,}600}$$

$$degrees_{decimal} = 39 + \frac{31}{60} + \frac{44}{3{,}600}$$

$$degrees_{decimal} = 39 + 0.516667 + 0.012222$$

$$degrees_{decimal} = 39.528889$$

Answer: The latitude value in decimal degrees is 39.528889˚.

31.5. Distance to optical horizon

The distance to the optical horizon can be calculated with the following formulas:[79]

$$D_{mi} = 1.22459\sqrt{h_{ft}}$$

where
D_{mi} is the distance to the optical horizon in statute miles
h_{ft} is the height of the observer in feet above mean sea level

$$D_{km} = 3.56972\sqrt{h_m}$$

where
D_{km} is the distance to the optical horizon in kilometers
h_m is the height of the observer in meters above mean sea level

Note: The calculations are based on a smooth, spherical Earth.

Example:
What is the distance to the optical horizon for an observer at sea level, when the height of the observer's eyes is 6 feet above the ground?

Solution:
$D = 1.22459\sqrt{h}$

$D = 1.22459 * \sqrt{6}$

$D = 1.22459 * 2.4495$

$D = 2.9996$

Answer: The distance to the optical horizon is approximately 3 miles.

[79] Derivation of the formulas can be found at https://sites.math.washington.edu/~conroy/m120-general/horizon.pdf

31.6. Calculate worst-case downstream frequency response flatness

In the past, the following formula was widely used to determine the worst-case acceptable frequency response flatness after a cascade of amplifiers. The formula was commonly used with a sweep response factor of x = 1 in all-coax tree-and-branch networks whose highest downstream frequency was about 300 MHz. As network RF bandwidth expanded beyond 300 MHz, x increased, too, to values such as 1.5 and 2. Over time – and as cable network downstream bandwidths became even greater – the formula was no longer satisfactory. The formula is included here for historical reference purposes.

$$dB_{peak\ to\ valley} = \left(\frac{N}{10}\right) + x$$

where
$dB_{peak\ to\ valley}$ is the worst case acceptable peak-to-valley frequency response within a defined frequency range, stated in decibels
N is the number of amplifiers in cascade
x is a sweep response factor, typically provided by the amplifier manufacturer

Example:
What is the targeted end-of-line frequency response flatness for a 220 MHz, 25 amplifier cascade? Assume the sweep response factor is 1.

Solution:

$$dB_{peak\ to\ valley} = \left(\frac{N}{10}\right) + x$$

$$dB_{peak\ to\ valley} = \left(\frac{25}{10}\right) + 1$$

$$dB_{peak\ to\ valley} = (2.50) + 1$$

$$dB_{peak\ to\ valley} = 3.50$$

Answer: The targeted end-of-line sweep frequency response is 3.5 dB peak-to-valley.

31.7. Convert watts to British thermal units per hour

The British thermal unit per hour (Btu/h) is a unit of power used for heating and cooling systems. For instance, one can calculate a rough approximation of cooling requirements for a facility such as a headend in Btu/h when the power consumption in watts is known for the facility's equipment. A specialist should be consulted for final heating, ventilation, and air conditioning (HVAC) system design.

The following formulas can be used to convert power in watts (or kilowatts) to Btu/h:

$$Btu/h = W * 3.412$$

where
Btu/h is British thermal units per hour
W is power in watts

Alternatively,

$$Btu/h = kW * 3{,}412.142$$

where
Btu/h is British thermal units per hour
kW is power in kilowatts

Example:
What is the approximate cooling requirement in Btu/h for a small hub site in which the total power consumption of all of the equipment is 19,500 watts?

Solution:
$Btu/h = W * 3.412$

$Btu/h = 19{,}500 * 3.412$

$Btu/h = 66{,}534$

Answer: The approximate cooling requirement is 66,534 Btu/h.

Note: One ton of cooling, a unit used in air conditioning applications, is equal to 12,000 Btu/h. For the previous example, the approximate required air conditioning in tons is (66,534 Btu/h)/12,000 = 5.54 tons.[80]

[80] Calculation of the heat load for a facility requires taking into consideration several factors beyond the scope of this document. Some examples include the area of the facility in square feet or meters; size and location of any windows (i.e., sun-facing); the number of facility occupants, if applicable; and heat generated by equipment and lighting. As mentioned previously, a specialist should be consulted for HVAC system design.

31.8. Temperature conversions

Conversions between kelvin, degree Celsius, and degree Fahrenheit are done using the following formulas:

$$°C = (°F - 32) * \frac{5}{9}$$

$$°F = \left(°C * \frac{9}{5}\right) + 32$$

$$K = °C + 273.15$$

$$°C = K - 273.15$$

$$°F = \left[(K - 273.15) * \frac{9}{5}\right] + 32$$

$$K = \left[(°F - 32) * \frac{5}{9}\right] + 273.15$$

where
$°C$ is degree Celsius
$°F$ is degree Fahrenheit
K is kelvin

Example 1:
A common temperature range over which some products are specified to operate is –40 °C to +60 °C. What is that temperature range in Fahrenheit?

Solution 1:

$$°F = \left(-40 * \frac{9}{5}\right) + 32$$

$$°F = (-72) + 32$$

$$°F = -40$$

The low temperature is –40 °F.

$$°F = \left(60 * \frac{9}{5}\right) + 32$$

$$°F = (108) + 32$$

$$°F = 140$$

The high temperature is 140 °F.

Answer: The range in Fahrenheit is –40 °F to +140 °F.

Example 2:
A standard temperature used in noise figure calculations and measurements is 290 K. What is that standard temperature in Celsius and Fahrenheit?

Solution 2:
$$°C = 290 - 273.15$$

$$°C = 16.85$$

The first answer is 16.85 °C.

$$°F = \left[(290 - 273.15) * \frac{9}{5}\right] + 32$$

$$°F = \left[(16.85) * \frac{9}{5}\right] + 32$$

$$°F = [30.33] + 32$$

$$°F = 62.33$$

The second answer is 62.33 °F.

31.9. Length/distance/size conversions

While most of the world uses the metric system, the United States for the most part still uses U.S./Imperial length, distance, and size measures. Conversions are relatively straightforward, with some of the more commonly used parameters and applicable conversion formulas summarized in two tables.

31.9.1. Convert metric to U.S./Imperial lengths, distances, and sizes

The following table includes formulas to convert from a variety of commonly used metric to U.S./Imperial length, distance, and size measures:

Table 24 - Metric to U.S./Imperial length, distance, and size conversions.

FROM ↓ TO →	inch (in)	foot (ft)	yard (yd)	mile (mi)
millimeter (mm)	$in = mm * 0.03937$	$ft = mm * 0.0032808$	$yd = mm * 0.0010936$	$mi = mm * 0.0000006213$
centimeter (cm)	$in = cm * 0.3937007$	$ft = cm * 0.0328083$	$yd = cm * 0.0109361$	$mi = cm * 0.0000062138$
meter (m)	$in = m * 39.370078$	$ft = m * 3.2808398$	$yd = m * 1.0936132$	$mi = m * 0.0006213881$
kilometer (km)	$in = km * 39370.078$	$ft = km * 3280.8398$	$yd = km * 1093.6132$	$mi = km * 0.6213881$

Example 1:
A point-to-point microwave path is 35.4 kilometers long. What is that path length in miles?

Solution 1:
$mi = km * 0.6213881$

$mi = 35.4 * 0.6213881$

$mi = 21.9971$

Answer: The path length is about 22 miles.

Example 2:
The span length between two utility poles is 60.96 meters. What is that span length in feet?

Solution 2:
$ft = m * 3.2808398$

$ft = 60.96 * 3.2808398$

$ft = 200$

Answer: The span length is 200 feet.

Example 3:
What is the diameter of 12.7 mm coaxial cable in inches?

Solution 3:
$in = mm * 0.03937$

$in = 12.7 * 0.03937$

$in = 0.500$

Answer: The cable's diameter is 0.500 inch.

31.9.2. Convert U.S./Imperial to metric lengths, distances, and sizes

The following table includes formulas to convert from a variety of commonly used U.S./Imperial to metric length, distance, and size measures:

Table 25 - U.S./Imperial to metric length, distance, and size conversions.

FROM ↓ TO →	millimeter (mm)	centimeter (cm)	meter (m)	kilometer (km)
inch (in)	$mm = in * 25.4$	$cm = in * 2.54$	$m = in * 0.0254$	$km = in * 0.0000254$
foot (ft)	$mm = ft * 304.8$	$cm = ft * 30.48$	$m = ft * 0.3048$	$km = ft * 0.0003048$
yard (yd)	$mm = yd * 914.4$	$cm = yd * 91.44$	$m = yd * 0.9144$	$km = yd * 0.0009144$
mile (mi)	$mm = mi * 1{,}609{,}300$	$cm = mi * 160{,}930$	$m = mi * 1609.3$	$km = mi * 1.6093$

Example 1:
A point-to-point microwave path is 22 miles long. What is that path length in kilometers?

Solution 1:
$km = mi * 1.6093$

$km = 22 * 1.6093$

$km = 35.4046$

Answer: The path length is about 35.4 kilometers.

Example 2:
A span between two utility poles measures 200 feet. What is that span length in meters?

Solution 2:
$m = ft * 0.3048$

$m = 200 * 0.3048$

$m = 60.96$

Answer: The span length is 60.96 meters, or about 61 meters.

Example 3:
What is the diameter of .500 hardline coax in millimeters?

Solution 3:
$mm = in * 25.4$

$mm = 0.500 * 25.4$

$mm = 12.7$

Answer: The cable's diameter is 12.7 millimeters.

31.10. Area calculations

While most of the world uses the metric system, the United States for the most part still uses U.S./Imperial units for area. Conversions are relatively straightforward, with some of the more commonly used parameters and applicable conversion formulas summarized in two tables.

31.10.1. Convert metric to U.S./Imperial area units

The following table includes formulas to convert from a variety of commonly used metric to U.S./Imperial area units:

Table 26 - Metric to U.S./Imperial area conversions.

FROM ↓ TO →	square inch (in^2)	square foot (ft^2)	square yard (yd^2)	square mile (mi^2)
square millimeter (mm^2)	$in^2 = mm^2 * 0.00155$	$ft^2 = mm^2 * 0.000010764$	$yd^2 = mm^2 * 0.000001196$	$mi^2 = mm^2 * 3.861 * 10^{-13}$
square centimeter (cm^2)	$in^2 = cm^2 * 0.155$	$ft^2 = cm^2 * 0.0010764$	$yd^2 = cm^2 * 0.0001196$	$mi^2 = cm^2 * 3.861 * 10^{-11}$
square meter (m^2)	$in^2 = m^2 * 1550$	$ft^2 = m^2 * 10.764$	$yd^2 = m^2 * 1.196$	$mi^2 = m^2 * 3.861 * 10^{-7}$
square kilometer (km^2)	$in^2 = km^2 * 1.55 * 10^9$	$ft^2 = km^2 * 10764000$	$yd^2 = km^2 * 1196000$	$mi^2 = km^2 * 0.3861$

Example 1:
A Wi-Fi access point serves an area of 470 square meters. What is that area in square feet?

Solution 1:
$$ft^2 = m^2 * 10.764$$

$$ft^2 = 470 * 10.764$$

$$ft^2 = 5{,}059.08$$

Answer: The area is about 5,059 square feet.

Example 2:
A service technician has been assigned an area of plant covering 1,030 square kilometers. What is that area in square miles?

Solution 2:
$$mi^2 = km^2 * 0.3861$$

$$mi^2 = 1030 * 0.3861$$

$$mi^2 = 397.68$$

Answer: The area is 397.68 square miles.

31.10.2. Convert U.S./Imperial to metric area units

The following table includes formulas to convert from a variety of commonly used U.S./Imperial to metric area units:

Table 27 - U.S./Imperial to metric area conversions.

FROM ↓ TO →	square millimeter (mm^2)	square centimeter (cm^2)	square meter (m^2)	square kilometer (km^2)
square inch (in^2)	$mm^2 = in^2 * 645.16$	$cm^2 = in^2 * 6.4516$	$m^2 = in^2 * 0.00064516$	$km^2 = in^2 * 6.4516 * 10^{-10}$
square foot (ft^2)	$mm^2 = ft^2 * 92902.27$	$cm^2 = ft^2 * 929.0227$	$m^2 = ft^2 * 0.09290227$	$km^2 = ft^2 * 9.290227 * 10^{-8}$
square yard (yd^2)	$mm^2 = yd^2 * 836120.40$	$cm^2 = yd^2 * 8361.2040$	$m^2 = yd^2 * 0.83612040$	$km^2 = yd^2 * 8.361240 * 10^{-7}$
square mile (mi^2)	$mm^2 = mi^2 * 2.59 * 10^{12}$	$cm^2 = mi^2 * 2.59 * 10^{10}$	$m^2 = mi^2 * 259000$	$km^2 = mi^2 * 2.59$

Example 1:
A Wi-Fi access point serves an area of 5,059 square feet. What is that area in square meters?

Solution 1:
$m^2 = ft^2 * 0.09290227$

$m^2 = 5{,}059 * 0.09290227$

$m^2 = 469.99$

Answer: The area is about 470 square meters.

Example 2:
A service technician has been assigned an area of plant covering 397.68 square miles. What is that area in square kilometers?

Solution 2:
$km^2 = mi^2 * 2.59$

$km^2 = 397.68 * 2.59$

$km^2 = 1029.99$

Answer: The area is about 1,030 square kilometers.

31.11. Weight calculations

This section includes formulas to convert between some common measures of weight.[81]

31.11.1. Convert ounce to gram

The following formula can be used to convert from the common or avoirdupois ounce (oz) to gram (g):

$$g = oz * 28.349523125$$

where
g is gram
oz is the common or avoirdupois ounce

Example:
What is the weight of a 32 oz. bag of F connectors in grams?

Solution:
$g = oz * 28.349523125$

$g = 32 * 28.349523125$

$g = 907.17$

Answer: The 32 oz. bag of F connectors weighs 907.17 g.

31.11.2. Convert gram to ounce

The following formula can be used to convert from gram (g) to the common or avoirdupois ounce (oz):

$$oz = g * 0.0352739619$$

where
oz is the common or avoirdupois ounce
g is gram

[81] The terms "weight" and "mass" are often used interchangeably. Although related, they are in fact different. From the National Institute of Standards and Technology: *The mass of a body is a measure of its inertial property or how much matter it contains. The weight of a body is a measure of the force exerted on it by gravity or the force needed to support it. Gravity on earth gives a body a downward acceleration of about 9.8 m/s^2. In common parlance, weight is often used as a synonym for mass in weights and measures. For instance, the verb "to weigh" means "to determine the mass of" or "to have a mass of." The incorrect use of weight in place of mass should be phased out, and the term mass used when mass is meant. The SI unit of mass is the kilogram (kg). In science and technology, the weight of a body in a particular reference frame is defined as the force that gives the body an acceleration equal to the local acceleration of free fall in that reference frame. Thus, the SI unit of the quantity weight defined in this way (force) is the newton (N).*

Example:
What is the weight of a 7.1 grams F connector in ounces?

Solution:
$oz = g * 0.0352739619$

$oz = 7.1 * 0.0352739619$

$oz = 0.25$

Answer: The F connector weighs 0.25 oz.

31.11.3. Convert pound to kilogram

The following formula can be used to convert from pound (lb) to kilogram (kg):

$$kg = lb * 0.45359237$$

where
kg is kilogram
lb is pound (avoirdupois pound)

Example:
What is the weight of a 6 lb meter in kg?

Solution:
$kg = lb * 0.45359237$

$kg = 6 * 0.45359237$

$kg = 2.72$

Answer: The meter weighs 2.72 kg.

31.11.4. Convert kilogram to pound

The following formula can be used to convert from kilogram (kg) to pound (lb):

$$lb = kg * 2.2046226218$$

where
lb is pound (avoirdupois pound)
kg is kilogram

Example:
What is the weight of a 17.7 kg reel of Series 6 coaxial cable in pounds?

Solution:
$lb = kg * 2.2046226218$

$lb = 17.7 * 2.2046226218$

$lb = 39.02$

Answer: The reel of cable weighs 39.02 kg.

31.11.5. Convert pound to tonne (metric ton)

The following formula can be used to convert from pound (lb) to tonne (t), also called metric ton:[82]

$$t = lb * 0.0004535924$$

where
t is tonne (metric ton)
lb is pound (avoirdupois pound)

Example:
What is the weight of a 185 lb partial reel of .500 hardline coaxial cable in tonnes?

Solution:
$t = lb * 0.0004535924$

$t = 185 * 0.0004535924$

$t = 0.08$

Answer: The partial reel of coaxial cable weighs 0.08 tonne.

31.11.6. Convert tonne (metric ton) to pound

The following formula can be used to convert from tonne (t) or metric ton to pound (lb):

$$lb = t * 2204.6226218488$$

where
lb is pound (avoirdupois pound)
t is tonne (metric ton)

[82] A tonne or metric ton equals 1,000 kilograms.

Example:
What is the weight of a 2 tonne trailer in pounds?

Solution:
$lb = t * 2204.6226218488$

$lb = 2 * 2204.6226218488$

$lb = 4409.25$

Answer: The trailer weighs 4,409.25 lb.

31.11.7. Convert pound to ton (short or common ton)

The following formula can be used to convert from pound (lb) to ton, also called a short or common ton:[83]

$$ton = lb * 0.0005$$

where
ton is short or common ton
lb is pound (avoirdupois pound)

Example:
What is the weight of a 185 lb partial reel of .500 hardline coaxial cable in tons?

Solution:
$ton = lb * 0.0005$

$ton = 185 * 0.0005$

$ton = 0.09$

Answer: The partial reel of coaxial cable weighs 0.09 ton.

31.11.8. Convert ton (short or common ton) to pound

The following formula can be used to convert from ton (short or common ton) to pound (lb):

[83] A short or common ton equals 2,000 pounds, not to be confused with the long ton, which equals 2,240 pounds.

$$lb = ton * 2{,}000$$

where
lb is pound (avoirdupois pound)
ton is short or common ton

Example:
What is the weight of a 2 ton trailer in pounds?

Solution:
$lb = ton * 2{,}000$

$lb = 2 * 2{,}000$

$lb = 4{,}000$

Answer: The trailer weighs 4,000 lb.

31.12. Volume calculations

Volume is the amount of space, measured in cubic units, that an object or substance occupies. While most of the world uses the metric system, the United States for the most part still uses U.S. and some Imperial cubic units for volume. Conversions are relatively straightforward, with some of the more commonly used parameters and applicable conversion formulas summarized in four tables.

31.12.1. Convert metric to U.S./Imperial volume units (object)

The following table includes formulas to convert from a variety of commonly used metric to U.S./Imperial volume units as they relate to an object:

Table 28. Metric to U.S./Imperial volume conversions (object).

FROM ↓ TO →	cubic inch (in^3)	cubic foot (ft^3)	cubic yard (yd^3)
cubic millimeter (mm^3)	$in^3 = mm^3 * 0.000061024$	$ft^3 = mm^3 * 3.5315 * 10^{-8}$	$yd^3 = mm^3 * 1.30795 * 10^{-9}$
cubic centimeter (cm^3)	$in^3 = cm^3 * 0.061024$	$ft^3 = cm^3 * 0.000035315$	$yd^3 = cm^3 * 0.00000130795$
cubic meter (m^3)	$in^3 = m^3 * 61024$	$ft^3 = m^3 * 35.315$	$yd^3 = m^3 * 1.30795$

Example 1:
A new earth station antenna installation requires a foundation concrete volume of 13.6 cubic meters. What is that volume in cubic feet?

Solution 1:
$ft^3 = m^3 * 35.315$

$ft^3 = 13.6 * 35.315$

$ft^3 = 480.28$

Answer: The volume is 480.28 cubic feet.

Example 2:
An underground equipment installation requires a pedestal with a minimum volume of 188,872 cubic centimeters. What is that volume in cubic inches?

Solution 2:
$in^3 = cm^3 * 0.061024$

$in^3 = 188{,}872 * \ 0.061024$

$in^3 = 11{,}525.72$

Answer: The volume is 11,525.72 cubic inches.

31.12.2. Convert U.S./Imperial to metric volume units (object)

The following table includes formulas to convert from a variety of commonly used U.S./Imperial to metric volume units as they relate to an object:

Table 29 - U.S./Imperial to metric volume conversions (object).

FROM ↓ TO →	cubic millimeter (mm^3)	cubic centimeter (cm^3)	cubic meter (m^3)
cubic inch (in^3)	$mm^3 = in^3 * 16386.995$	$cm^3 = in^3 * 16.386995$	$m^3 = in^3 * 1.6386995*10^{-5}$
cubic foot (ft^3)	$mm^3 = ft^3 * 28316579.36$	$cm^3 = ft^3 * 28316.579$	$m^3 = ft^3 * 0.028316597$
cubic yard (yd^3)	$mm^3 = yd^3 * 76455522$	$cm^3 = yd^3 * 764555.22$	$m^3 = yd^3 * 0.76455522$

Example 1:
A new earth station antenna installation requires a foundation concrete volume of 480.28 cubic feet. What is that volume in cubic meters?

Solution 1:
$m^3 = ft^3 * 0.028316597$

$m^3 = 480.28 * \ 0.028316597$

$m^3 = 13.6$

Answer: The volume is 13.6 cubic meters.

Example 2:
An underground equipment installation requires a pedestal with a minimum volume of 11,525.72 cubic inches. What is that volume in cubic centimeters?

Solution 2:
$cm^3 = in^3 * 16.386995$

$cm^3 = 11{,}525.72 * 16.386995$

$cm^3 = 188{,}871.92$

Answer: The volume is about 188,872 cubic centimeters.

31.12.3. Convert metric to U.S. volume units (substance)

The following table includes formulas to convert from a variety of commonly used U.S. to metric volume units as they relate to a substance:

Table 30 - Metric to U.S. volume conversions (substance).

FROM ↓ TO →	fluid ounce (fl. oz.)	pint (pt)	quart (qt)	gallon (gal)
milliliters (mL)	fl. oz. = mL * 0.033814	pt = mL * 0.0021134	qt = mL * 0.0010567	gal = mL * 0.00026417
centiliters (cL)	fl. oz. = cL * 0.33814	pt = cL * 0.021134	qt = cL * 0.010567	gal= cL * 0.0026417
deciliters (dL)	fl. oz. = dL * 3.3814	pt = dL * 0.21134	qt = dL * 0.10567	gal = dL * 0.026417
liter (L)	fl. oz. = L * 33.814	pt = L * 2.1134	qt = L * 1.0567	gal = L * 0.26417

Example 1:
A portable backup generator has a fuel tank that holds 23.85 liters. What is that volume in gallons?

Solution 1:
$gal = L * 0.26417$

$gal = 23.85 * 0.26417$

$gal = 6.3$

Answer: The volume is 6.3 gallons.

Example 2:
A fiber optic cleaning kit includes a bottle containing 8.28 deciliters of isopropyl alcohol. What is that volume in fluid ounces?

Solution 2:
$fl.oz. = dL * 3.3814$

$fl.oz. = 8.28 * 3.3814$

$fl.oz. = 27.99$

Answer: The volume is about 28 fluid ounces.

31.12.4. Convert U.S. volume to metric units (substance)

The following table includes formulas to convert from a variety of commonly used U.S. to metric volume units as they relate to a substance:

Table 31 - U.S. to metric volume conversions (substance).

FROM ↓ TO →	milliliters (mL)	centiliters (cL)	deciliters (dL)	liter (L)
fluid ounce (fl. oz.)	mL = fl. oz. ∗ 29.573549	cL = fl. oz. ∗ 2.957349	dL = fl. oz. ∗ 0.29573549	L= fl. oz. ∗ 0.029573549
pint (pt)	mL = pt ∗ 473.17	cL = pt ∗ 47.317	dL = pt ∗ 4.7317	L= pt ∗ 0.47317
quart (qt)	mL = qt ∗ 946.34	cL = qt ∗ 94.634	dL = qt ∗ 9.4634	L= qt ∗ 0.94634
gallon (gal)	mL = gal ∗ 3785.44	cL = gal ∗ 378.544	dL = gal ∗ 37.8544	L= gal ∗ 3.78544

Example 1:
A portable backup generator has a fuel tank that holds 6.3 gallons. What is that volume in liters?

Solution 1:
$L = gal * 3.78544$

$L = 6.3 * 3.78544$

$L = 23.85$

Answer: The volume is 23.85 liters.

Example 2:
A fiber optic cleaning kit includes a bottle containing 28 fluid ounces of isopropyl alcohol. What is that volume in deciliters?

Solution 2:
$dL = fl.oz. * 0.29573549$

$dL = 28 * 0.29573549$

$dL = 8.28$

Answer: The volume is 8.28 deciliters.

31.13. Torque calculations

A commonly used unit of torque is the pound·foot (lbf·ft or lb·ft)[84], and its SI counterpart is the newton·meter (N·m). The pound·inch (lbf·in or lb·in) is another unit of torque, equal to 1/12 of a pound·foot, used, for example, to describe connector tightening.

---

[84] Not to be confused with foot·pound (energy).

The formula to convert from newton·meter to pound·foot is

$$lb{\cdot}ft = N{\cdot}m * 0.737562$$

where
$lb{\cdot}ft$ is torque in units of pound·foot
$N{\cdot}m$ is torque in units of newton·meter

Example:
What is 300 N·m expressed in lb·ft?

Solution:
$lb{\cdot}ft = 300 * 0.737562$

$lb{\cdot}ft = 221.27$

The answer is 221.27 lb·ft.

The formula to convert from pound·foot to newton·meter is

$$N{\cdot}m = lb{\cdot}ft * 1.355818$$

where
$N{\cdot}m$ is torque in units of newton·meter
$lb{\cdot}ft$ is torque in units of pound·foot

Example:
What is 273 lb·ft expressed in N·m?

Solution:
$N{\cdot}m = 273 * 1.355818$

$N{\cdot}m = 370.14$

The answer is 370.14 N·m.

Appendix A How to Calculate Base-2 Logarithms

Some applications involve calculations using base-2 logarithms ($\log_2$) rather than the more common base-10 logarithms ($\log_{10}$).

Scientific calculators have a base-10 logarithm function. Some scientific calculators also have a natural logarithm (ln) – also called base-*e* or $\log_e$ – function. How can one calculate the base-2 logarithm of a given quantity? The following examples illustrate two ways to do so using base-10 and natural logarithm functions.[85]

A.1 Base 10 logarithm method

To find the base-2 logarithm of a quantity x, use the formula

$\log_2(x) = \log_{10}(x)/\log_{10}(2)$

For example, calculate the base-2 logarithm of the number 24.

$\log_2(24) = \log_{10}(24)/\log_{10}(2)$
$\log_2(24) = 1.380211/0.301030$
$\log_2(24) = 4.584963$

A.2 Natural logarithm method

To find the base-2 logarithm of a quantity x, use the formula

$\log_2(x) = \ln(x)/\ln(2)$

For example, calculate the base-2 logarithm of the number 24.

$\log_2(24) = \ln(24)/\ln(2)$
$\log_2(24) = 3.178054/0.693147$
$\log_2(24) = 4.584963$

[85] Microsoft® Excel® supports calculation of base-2 logarithms, using the formula LOG(**number**, [base]). For example, to calculate the base-2 logarithm of the value in a spreadsheet's cell number B6, the formula for that cell would be =LOG(B6, 2).

Appendix B International System of Units (SI)

The following tables are from National Institute of Standards and Technology publications. Care should be taken to ensure that the correct upper or lower case is used. For example, the SI prefix M = mega, while m = milli; lowercase k is used for kilo, while uppercase K is for kelvin.

Table 32 - SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	µ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

Table 33 - SI base units

Base quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 34. Examples of SI coherent derived units expressed in terms of SI base units.

Derived quantity	Name	Symbol
area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second squared	m/s^2
wave number	reciprocal meter	m^{-1}
density, mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2
magnetic field strength	ampere per meter	A/m
amount-of-substance concentration	mole per cubic meter	mol/m^3
luminance	candela per square meter	cd/m^2

Table 35 - SI coherent derived units with special names and symbols.

	SI coherent derived unit(a)			
	Special Name	**Special Symbol**	**Expression in terms of other SI units**	**Expression in terms of SI base units**
plane angle	radian (b)	rad	1(b)	m/m
solid angle	steradian (b)	sr (c)	1(b)	m^2/m^2
frequency	hertz(d)	Hz	-	s^{-1}
force	newton	N	-	$m \cdot kg \cdot s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1} \cdot kg \cdot s^{-2}$
energy, work, amount of heat	joule	J	N·m	$m^2 \cdot kg \cdot s^{-2}$
power, radiant flux	watt	W	J/s	$m^2 \cdot kg \cdot s^{-3}$
electric charge, amount of electricity	coulomb	C	-	s·A
electric potential difference(e), electromotive force	volt	V	W/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
capacitance	farad	F	C/V	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
electric resistance	ohm	Ω	V/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$
magnetic flux	weber	Wb	V·s	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg \cdot s^{-2} \cdot A^{-1}$
inductance	henry	H	Wb/A	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
Celsius temperature	degree Celsius(f)	°C	-	K
luminous flux	lumen	lm	cd·sr (c)	Cd
illuminance	lux	lx	lm/m^2	$m^{-2} \cdot cd$
activity referred to a radionuclide(g)	becquerel(d)	Bq	-	s^{-1}
absorbed dose, specific energy (imparted), kerma	gray	Gy	J/kg	$m^2 \cdot s^{-2}$
dose equivalent, ambient dose equivalent, directional dose equivalent, personal doze equivalent	sievert	Sv	J/kg	$m^2 \cdot s^{-2}$
catalytic activity	katal	kat	-	$s^{-1} \cdot mol$

(a) The SI prefixes may be used with any of the special names and symbols, but when this is done the resulting unit will no longer be coherent.
(b) The radian and steradian are special names for the number one that may be used to convey information about the quantity concerned. In practice the symbols rad and sr are used where appropriate, but the symbol for the derived unit one is generally omitted in specifying the values of dimensionless quantities.
(c) In photometry, the unit name steradian and the unit symbol sr are usually retained in expressions for derived units.
(d) The hertz is used only for periodic phenomena, and the becquerel is used only for stochastic processes in activity referred to a radionuclide.
(e) Electric potential difference is also called “voltage” in the United States.
(f) The degree Celsius is the special name for the kelvin used to express Celsius temperatures. The degree Celsius and the kelvin are equal in size, so that the numerical value of a temperature difference or temperature interval is the same when expressed in either degrees Celsius or in kelvins.
(g) Activity referred to a radionuclide is sometimes incorrectly called radioactivity.

Table 36. Examples of SI coherent derived units expressed with the aid of SI derived units having special names and symbols

Derived quantity	Name	Symbol
dynamic viscosity	pascal second	Pa·s
moment of force	newton meter	N·m
surface tension	newton per meter	N/m
angular velocity	radian per second	rad/s
angular acceleration	radian per second squared	rad/s^2
heat flux density, irradiance	watt per square meter	W/m^2
heat capacity, entropy	joule per kelvin	J/K
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kg·K)
specific energy	joule per kilogram	J/kg
thermal conductivity	watt per meter kelvin	W/(m·K)
energy density	joule per cubic meter	J/m^3
electric field strength	volt per meter	V/m
electric charge density	coulomb per cubic meter	C/m^3
electric flux density, electric displacement	coulomb per square meter	C/m^2
permittivity	farad per meter	F/m
permeability	henry per meter	H/m
molar energy	joule per mole	J/mol
molar entropy, molar heat capacity	joule per mole kelvin	J/(mol·K)
exposure (χ and γ rays)	coulomb per kilogram	C/kg
absorbed dose rate	gray per second	Gy/s
radiant intensity	watt per steradian	W/sr
radiance	watt per square meter steradian	$W/(m^2 \cdot sr)$
catalytic activity concentration	katal per cubic meter	kat/m^3

Appendix C Micro-Reflections

C.1 Micro-reflections: An introduction

Note: The overview of micro-reflections in this section uses just a single frequency (and single value of coaxial cable attenuation and return loss) with a single reflection to simplify the concept.

A micro-reflection is an echo (reflection) with a relatively short time delay, typically from less than a symbol period to several symbol periods. Consider the example shown in Figure 76, in which two water-damaged taps separated by 100 feet of coaxial cable result in a pair of impedance mismatches in the distribution network. The 100 ft. span of cable in this example is an echo tunnel (or echo cavity).

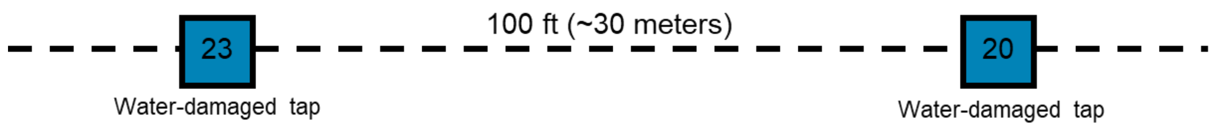


Figure 76. Echo tunnel formed by two water-damaged taps separated by 100 feet of coaxial cable.

Refer to Figure 77 for the following example. Assume that the attenuation in the 100 ft. span of coaxial cable between the taps is 1 dB, the cable's velocity factor is 0.87, and the return loss of each damaged tap is 7 dB. If the signal leaving the output port of the 23 dB tap at time T_0 is +31 dBmV, that signal will be attenuated by 1 dB as it passes through the cable and will arrive at the 20 dB tap at a level of +30 dBmV at time T_1. Because of the 20 dB tap's degraded return loss (7 dB), the amplitude of the signal reflected by the 20 dB tap is 30 dBmV – 7 dB = 23 dBmV. That first reflection will travel back through the span of coax and reach the 23 dB tap at a level of +22 dBmV. Because of the poor return loss of the 23 dB tap (also 7 dB), there will be a reflection from that tap whose amplitude is 22 dBmV – 7 dB = 15 dBmV. That second reflection will travel through the span of coax toward the 20 dB tap, arriving at the 20 dB tap at time T_2 with an amplitude of +14 dBmV.

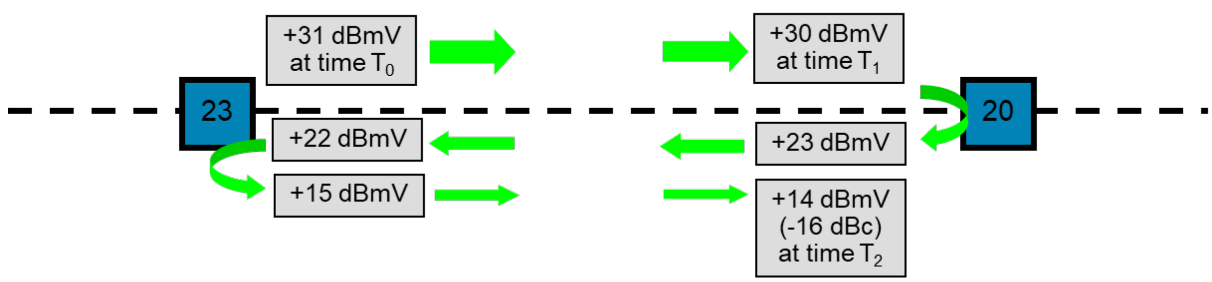


Figure 77. Propagation of incident signal and reflections within echo tunnel.

Figure 78 shows a graphical representation of the +30 dBmV incident signal at the input to the 20 dB tap at time T_1 along with the second reflection (+14 dBmV) at time T_2. The 234 ns time delay is equal to the round trip propagation time through the 100 ft. span of coaxial cable (117 ns each direction).[86] If the

[86] For coaxial cable with a velocity factor of 0.87 (87% velocity of propagation), the propagation or transit time through a foot of cable is 1/(983,571,056.43 x 0.87) = 1.17 nanosecond. The propagation time through 100 feet of that cable is 1.17 ns ∗ 100 = 117 ns. Note: 983,571,056.43 is the speed of light in a vacuum in feet per second.

downstream signals in this example are 6 MHz-wide 256-QAM signals, their symbol period is 1/5360537 symbols per second = $1.87 * 10^{-7}$ second, or 187 ns. The 234 ns time delay in this example is just a bit longer than the symbol period, so the term micro-reflection is certainly applicable.

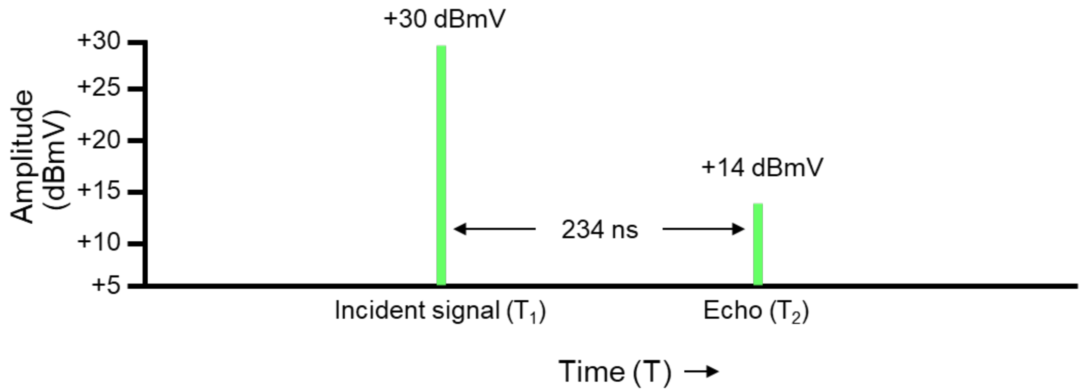


Figure 78. Graphical representation of incident signal and reflection at input to 20 dB tap.

The amplitude ripple produced in this example would be similar to that shown in Figure 79, with peaks at 31.28 dBmV and nulls at 28.5 dBmV. The frequency separation between adjacent peaks (or adjacent nulls) is about 4.27 MHz.

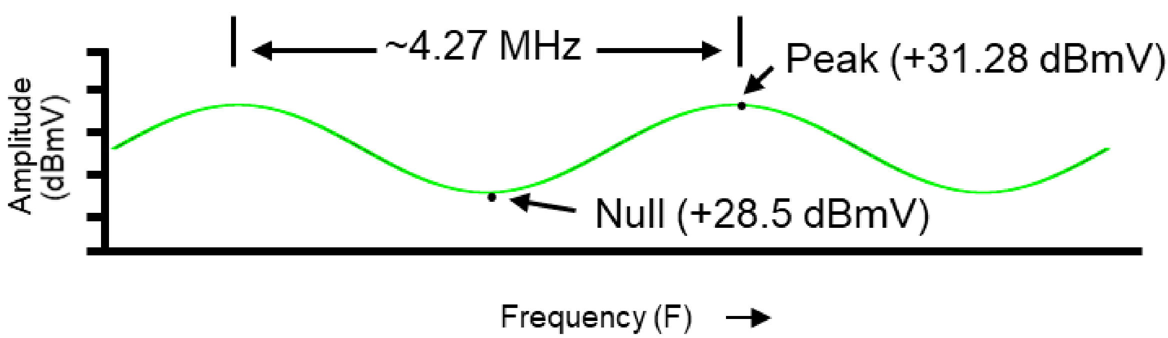


Figure 79. Amplitude ripple produced in example.

Figure 80 illustrates how one can determine the amplitude ripple's appearance. The blue arrow is a vector that represents the magnitude of the incident signal (+30 dBmV = 31.62 mV) normalized to 1, and the shorter green arrow is a vector that represents the echo (+14 dBmV = 5.01 mV), normalized to 0.16. As the shorter green arrow rotates counterclockwise around the end of the longer blue arrow, the vector sum of the two arrows equals the red dashed line. A red dashed line for each rotational position of the green arrow can be plotted as shown in the right side of Figure 80, which produces the shape of the amplitude ripple.

When the blue arrow (equivalent to 31.62 mV) and green arrow (equivalent to 5.01 mV) are lined up to produce a longer overall end-to-end length, the sum of the two arrows is 31.62 mV + 5.01 mV = 36.63 mV, or 31.28 dBmV for the peak on the amplitude ripple. When the green arrow rotates 180 degrees and overlaps the blue arrow, the vector sum is 31.62 mV – 5.01 mV = 26.61 mV, or 28.5 dBmV for the null.

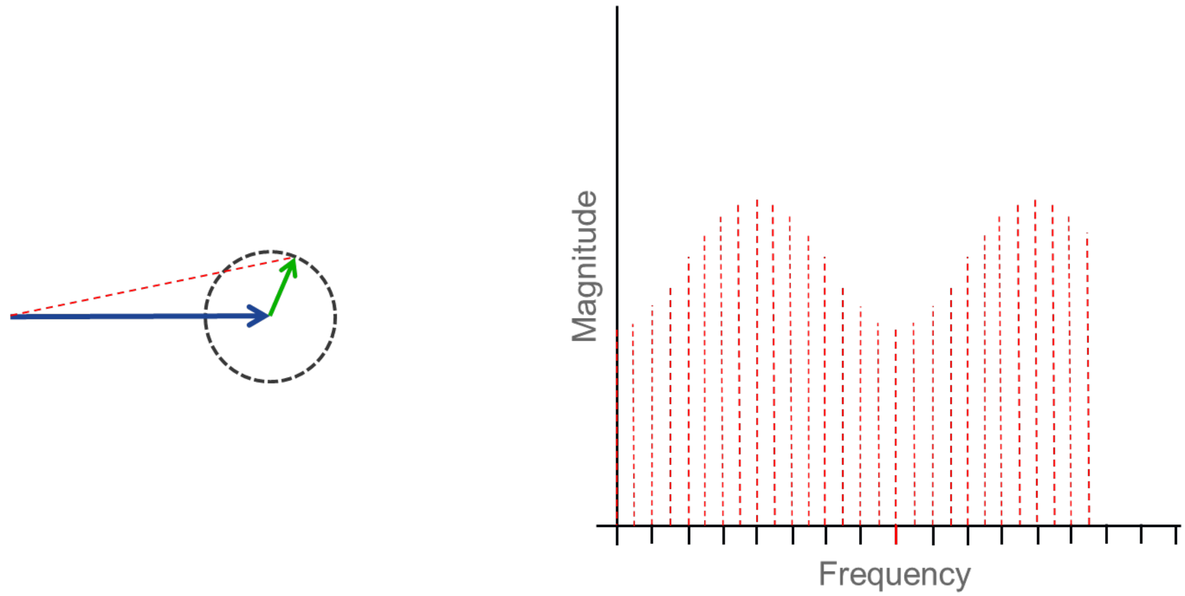


Figure 80. A graphical representation of vectors (left) and how they can be used to determine the shape of amplitude ripple (see text).

The length of the echo tunnel in feet can be confirmed with the formula

$$\mathrm{D} = 492 * \left(\frac{VF}{f_{MHz}}\right)$$

$$\mathrm{D} = 492 * \left(\frac{0.87}{4.27}\right)$$

$$\mathrm{D} = 492 * (0.2037)$$

$$\mathrm{D} = 100.2$$

or about 100 feet between the two water-damaged taps.

The relative amplitude of the echo to the incident signal can be calculated using the following formula:

$$\mathrm{Echo}_{dBc} = 20log_{10}\frac{V_p - V_n}{V_p + V_n}$$

where
$Echo_{dBc}$ is the relative amplitude of the echo to the incident signal in dBc
log_{10} is base 10 logarithm
V_p is the magnitude of the amplitude ripple peak in units of volts (millivolts in this example)
V_n is the magnitude of the amplitude ripple null in the same units of volts as V_p

Example:
What is the relative amplitude of the echo for the amplitude ripple shown in Figure 79?

Solution:
First convert the +31.28 dBmV peak and +28.5 dBmV null values to millivolts.

For V_p:

$$mV = 10^{\frac{dBmV}{20}}$$

$$mV = 10^{\frac{31.28}{20}}$$

$$mV = 10^{1.56}$$

$$mV = 36.64$$

For V_n:

$$mV = 10^{\frac{dBmV}{20}}$$

$$mV = 10^{\frac{28.5}{20}}$$

$$mV = 10^{1.43}$$

$$mV = 26.61$$

Then calculate the echo's relative magnitude in dBc, when V_p = 36.64 mV and V_n = 26.61 mV.

$$\text{Echo}_{dBc} = 20log_{10}\frac{V_p - V_n}{V_p + V_n}$$

$$\text{Echo}_{dBc} = 20 * log_{10}\frac{36.64 - 26.61}{36.64 + 26.61}$$

$$\text{Echo}_{dBc} = 20 * log_{10}\frac{10.03}{63.25}$$

$$\text{Echo}_{dBc} = 20 * log_{10}0.16$$

$$\text{Echo}_{dBc} = 20 * -0.80$$

$$\text{Echo}_{dBc} = -16$$

Answer: The magnitude of the echo is –16 dBc relative to the incident signal.

C.2 Micro-reflections: transmission and echo (reflection) transfer functions

Note: The overview of micro-reflections in this section uses the entire frequency range of the signal of interest (and corresponding range of values vs. frequency of coaxial cable attenuation and return loss).

C.2.1 Transmission transfer function

Consider a signal transmitted downstream from a tap output to the adjacent tap input as shown in Figure 81 with cable impulse response $a(t)$ and amplitude response $A(f)$ with linear phase response. The transmitter at the signal source has (nearly) matched impedance to the cable but with a return loss RL_o (dB). In general, the return loss is also a function of frequency $RL(f)$. The signal traverses the cable to the sink tap with propagation delay T which has a (nearly) matched impedance to the cable with return loss RL_i (dB). A portion of the signal equal to the reflection coefficient $\rho_i = 10^{-RLi/20}$ is reflected back to the source tap, which in turn a portion of the reflected signal equal to the reflection coefficient $\rho_o = 10^{-RLo/20}$ is re-reflected back toward the sink tap, and so on ad infinitum. This can be represented as a sum of the signal x(t) incident at the sink tap and the infinite series of reflections each delayed by the round-trip time (i.e., twice the propagation delay or 2T) of the cable as shown in Figure 81.

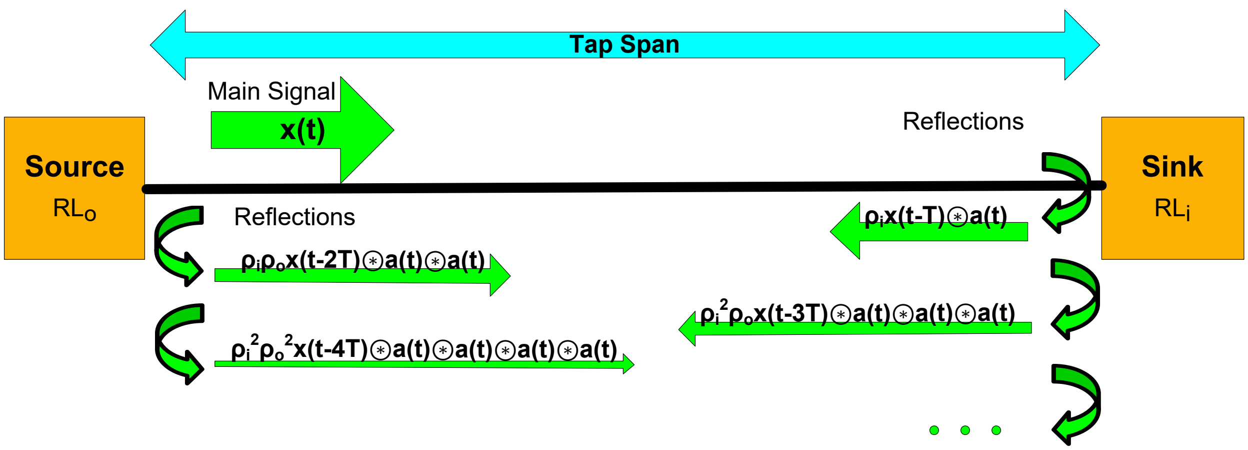


Figure 81. Signal reflections in a cable between adjacent taps.

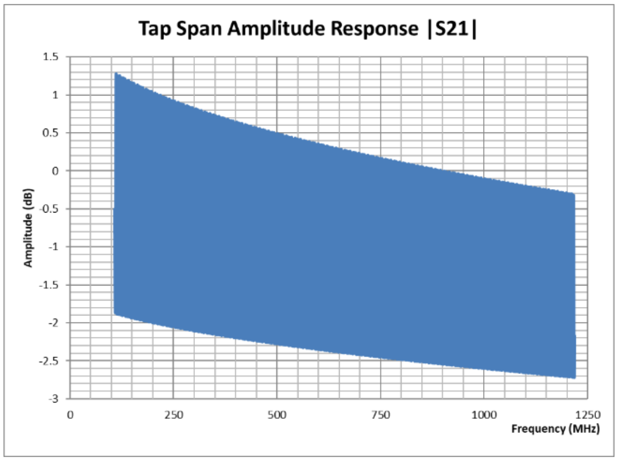
This represents a transmission line with frequency response $H(f)$ from source to sink tap transmitting the input signal $X(f)$ plus the discrete delays of each round-trip reflected signal yielding $Y(f)$. The transmitted signal transfer function $H(f)$ (equal to the scattering parameter S_{21}) is given by:

$$H(f) = Y(f)/X(f)$$

It can be shown [28] that the (complex) transmitted signal transfer function $H(f)$ from source to sink is given by:

$$H(f) = \frac{A(f)\, e^{-j2\pi fT}}{1 - A^2(f)\, 10^{-\frac{(\mathrm{RLi+RLo})}{20}} e^{-j4\pi fT}}$$

An example of the log magnitude transmission frequency response = $10log_{10}|H(f)|$ of a 100 foot hardline cable with 0.87 velocity factor and 1 dB/100 ft attenuation at 500 MHz connecting two taps each with a (constant) return loss of 7 dB is shown in Figure 82:



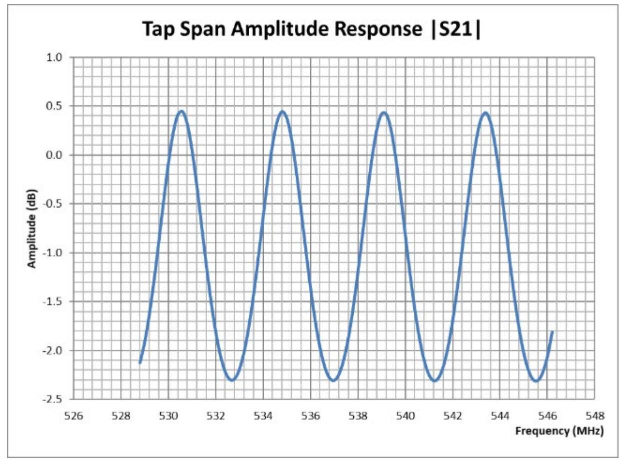


Figure 82. Log magnitude transmission response ($20\log_{10}|S_{21}|$ of Figure 81.

C.3 Echo (reflection) transfer function

Consider a signal transmitted downstream from a tap output to the adjacent tap input as shown in Figure 81 with cable impulse response $a(t)$ and amplitude response $A(f)$ with linear phase response. The transmitter at the signal source has (nearly) matched impedance to the drop cable but with a return loss RL_o (dB). In general, the return loss is also a function of frequency RL(f). The signal traverses the cable to the sink tap with propagation delay T which has a (nearly) matched impedance to the cable with return loss RL_i (dB). A portion of the signal equal to the reflection coefficient $10^{-RLi/20}$ is reflected back toward the source tap, which in turn a portion of the reflected signal equal to the reflection coefficient $10^{-RLo/20}$ is re-reflected back toward the sink tap, and so on ad infinitum. This reflected signal can be represented as a sum of the infinite series of reflections each delayed by the propagation delay T plus multiples of the round-trip time (i.e., twice the propagation delay or 2T) of the cable as shown in Figure 81.

This represents a transmission line with reflected signal frequency response $E_r(f)$ at the source tap (equal to the scattering parameter S_{11}) consisting of the sum of the discrete delays of each round-trip reflected input signal.

It can be shown [28] that the (complex) reflected echo transfer function $E_r(f)$ at the source is given by:

$$E_r(f) = \frac{A^2(f)\,10^{-\frac{(\mathrm{RLi})}{20}} e^{-j4\pi fT}}{1 - A^2(f)\,10^{-\frac{(\mathrm{RLi+RLo})}{20}} e^{-j4\pi fT}} = H(f)\left[A(f)\,10^{-\frac{(\mathrm{RLi})}{20}} e^{-j2\pi fT}\right]$$

An example of the log magnitude reflected echo frequency response $= 20log_{10}|E_r(f)|$ of a 100 foot hardline cable with 0.87 velocity factor and 1 dB/100 ft attenuation at 500 MHz connecting two taps each with a (constant) return loss of 7 dB is shown in Figure 83:

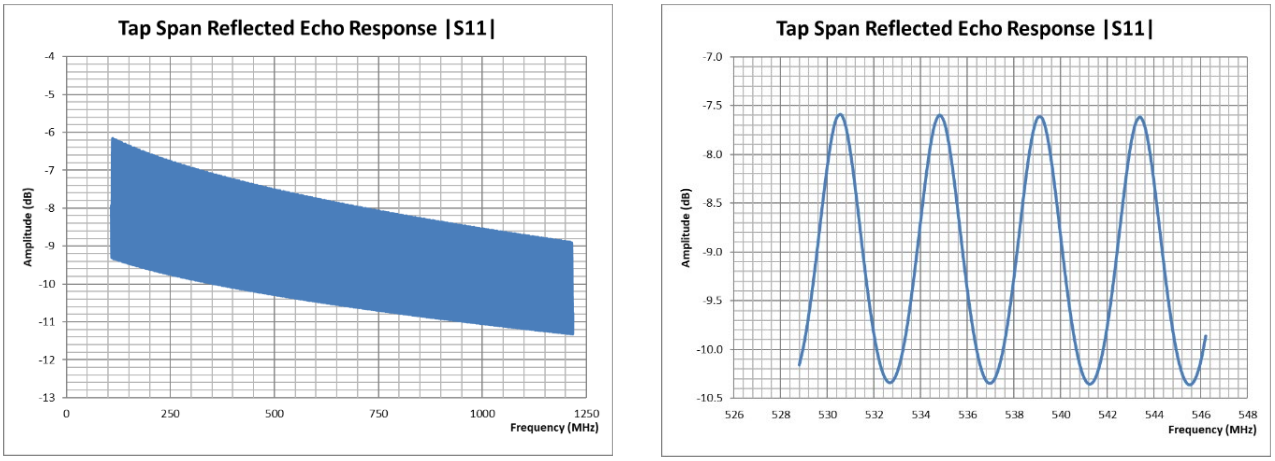


Figure 83. Log magnitude reflected echo response ($20log_{10}|S_{11}|$) of Figure 81.

Similarly, the (complex) incident echo transfer function $E_i(f)$ at the sink is equal to the transmission frequency response $H(f)$ minus the incident source signal portion of the response attenuated by the cable response $A(f)$ and delayed by the propagation delay T which is given by:

$$E_i(f) = H(f) - A(f)\, e^{-j2\pi fT}$$

An example of the log magnitude incident echo frequency response $= 20log_{10}|E_i(f)|$ of a 100 foot hardline cable with 0.87 velocity factor and 1 dB/100 ft attenuation at 500 MHz connecting two taps each with a (constant) return loss of 7 dB is shown in Figure 84:

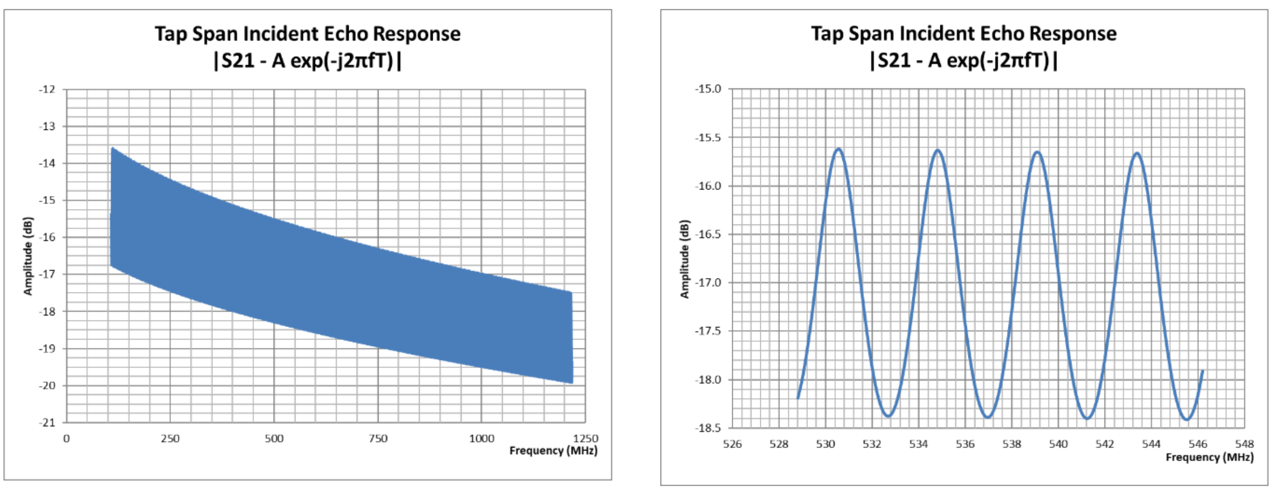


Figure 84: Log magnitude incident echo response ($20log_{10}|S_{21} – A e^{-j\omega T}|$) of Figure 81.

Appendix D Distortion Performance Analysis

This section includes a detailed analysis of distortions in cable networks. The material was excerpted from a document written by Lamar West as the qualifying exam for his Ph.D. at the Georgia Institute of Technology. The exam was entitled "Distortion Characteristics of CATV Broadband Networks," and focused on cable networks carrying analog TV channels. The concepts remain applicable to modern all-digital cable networks. Used with permission of the author.

Distortion Performance Analysis

Lamar West, Ph.D.
LEW Consulting, LLC

The standards that have been developed for characterizing distortion performance in broadband coaxial networks have been based on the assumption that only mild nonlinearities will be encountered. Most devices for use in cable applications exhibit mild nonlinearities when properly operated. It is worthwhile to explain what we mean by mild nonlinearities in this context.

CATV repeater amplifiers (e.g., trunk, line-extender) are designed with a single input and output unless intended for bridging applications. Bridging is the term applied to active devices that are used in location where multiple amplified outputs are required. However, for the purposes of this analysis, the single input, single output case is examined. Consequently, the amplifier is modeled as a two-port device. Initially, we consider the case of a memory-less two-port device. The transfer characteristic h(x) of such a device may be modeled using a Taylor series expansion

$$y = h(x) = a_0 + (a_1 \cdot x) + (a_2 \cdot x^2) + (a_3 \cdot x^3) + (a_4 \cdot x^4) + \cdots \tag{2.18}$$

In the case of CATV RF amplifiers, α_0 is generally assumed to be zero due to AC coupling in and out of the device. The α_1 term is associated with the desired, linear system output. Term α_2 is associated with second order distortion, term α_3 is associated with third order distortion, and so on.

In order to be considered a device with mild nonlinearities in the CATV case, terms α_4, α_5, α_6, and higher must be sufficiently small so that their effect on system performance may be ignored (D. McEwen). Typically, in cable systems, only second order and third distortion terms are considered. In this context we define mild nonlinearities to be nonlinearities in which distortion terms of order greater than three are of sufficiently small amplitude as to be insignificant. Therefore, the memory-less transfer characteristic simplifies to

$$y = (a_1 \cdot x) + (a_2 \cdot x^2) + (a_3 \cdot x^3) \tag{2.19}$$

One may also plot the transfer characteristic as shown in Figure 85.

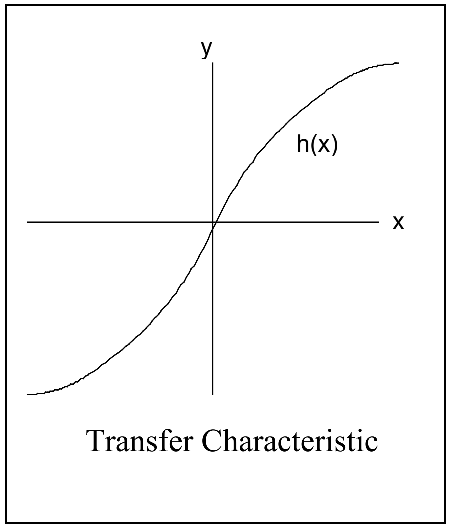


Figure 85 - Typical transfer characteristic.

The nonlinearity exhibited in this transfer characteristic is greatly exaggerated with respect to that shown by typical cable devices.

Let us examine the case of a single RF channel being passed through this device. We model the channel as a sinusoid of amplitude A and frequency ω_1 ($\omega_I = 2\pi f_i$),

$$x = A \cos \omega_1 t \tag{2.20}$$

The phase has been set to zero for simplicity of analysis. The output is given by

$$y = a_1 A \cos \omega_1 t + a_2 A^2 \cos^2 \omega_1 t + a_3 A^3 \cos^3 \omega_1 t \tag{2.21}$$

After application of trigonometric identities we get

$$y = \frac{1}{2} a_2 A^2 + [a_1 A + \frac{3}{4} a_3 A^3] \cos t\omega_1 t + \frac{1}{2} a_2 A^2 \cos 2\omega_1 t + \frac{1}{4} a_3 A^3 \cos 3\omega_1 t \tag{2.22}$$

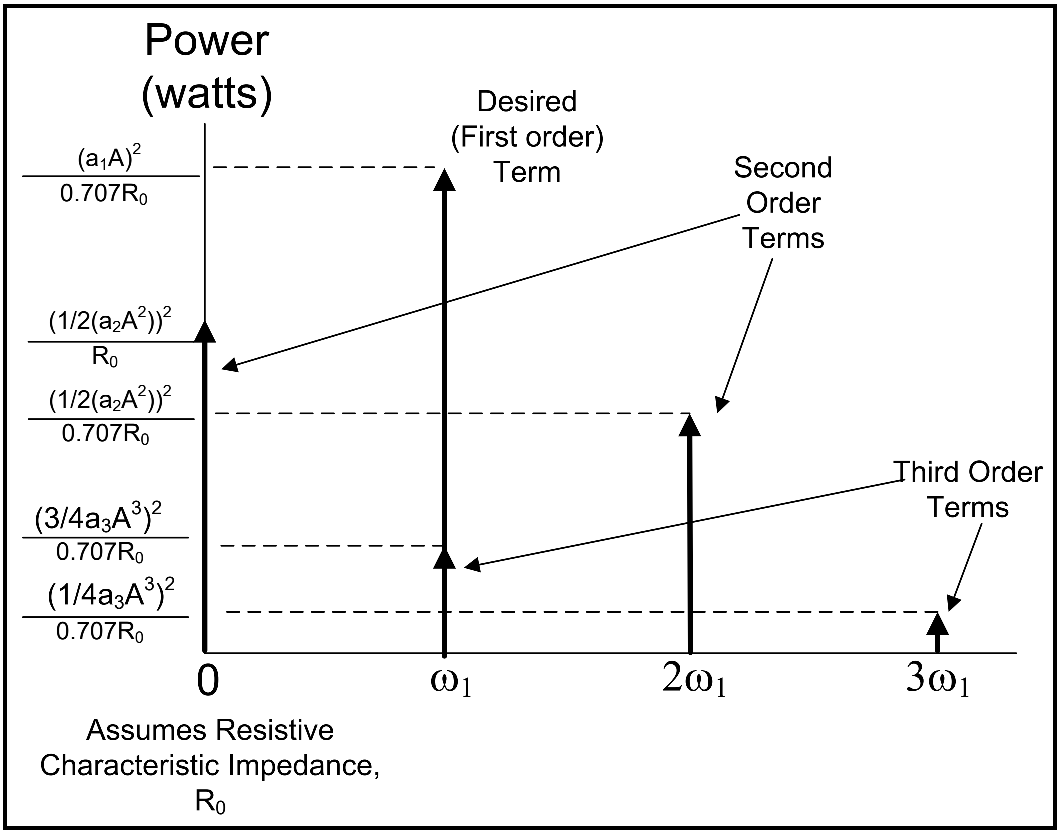


Figure 86. Power spectrum described by equation (2.22)

A power spectrum is shown in Figure 86. A purely real load impedance of value R_0 is assumed in this figure.

Thus, the application of a single frequency sinusoid results in an output containing components at DC, the input frequency and harmonics of the input frequency. The components associated with the second order term of the transfer characteristic are referred to as second order distortion or second order beats and fall at DC and at twice the fundamental frequency. Typically, the term at DC is ignored as long as it is sufficiently small so as not to impact the bias of the devices in question. The resulting components associated with the third order term are referred to as third order distortion or third order beats. Third order beats in this example fall at the fundamental frequency and at the third harmonic of that frequency.

A similar analysis can be performed if the input consists of two sinusoids at different frequencies, ω_1 and ω_2 respectively. Let

$$x = A\cos\omega_1 t + B\cos\omega_2 t \qquad (2.23)$$

If we apply this to the transfer characteristic and then simplify, the result becomes

$$y = a_1\left[A\cos\omega_1 t + B\cos\omega_2 t\right] + a_2\begin{bmatrix} \frac{1}{2}A^2 + \frac{1}{2}A^2\cos 2\omega_1 t + AB\cos(\omega_1 - \omega_2)t + AB\cos_1(\omega_1 + \omega_2)t + \\ \frac{1}{2}B^2 + \frac{1}{2}B^2\cos 2\omega_2 t \end{bmatrix} + a_3\begin{bmatrix} \frac{3}{4}A^3\cos\omega_1 t + \frac{1}{4}A^3\cos 3\omega t_1 + \frac{3}{4}B^3\cos\omega_2 t + \\ \frac{1}{4}B^3\cos 3\omega_2 t + \frac{3}{2}A^2B\cos\omega_2 t + \frac{3}{2}AB^2\cos\omega_1 t + \\ \frac{3}{4}A^2B\cos(2\omega_1 - \omega_2)t + \frac{3}{4}A^2B\cos\left(2\omega_1 + \omega_2\right) + \\ \frac{3}{4}AB^2\cos\left(\omega_1 - 2\omega_2\right)t + \frac{3}{4}AB^2\cos\left(\omega_1 + 2\omega_2\right)t \end{bmatrix} \quad (2.24)$$

Note that there are terms up to and including all third order sums and differences of the frequencies ω_1 and ω_2. A similar analysis is given in (Laboratories, 1971) for the case of an input consisting of three sinusoids. As one might expect, that result becomes substantially more complex.

An important conclusion may be reached by examination of these results. A key figure of merit for the performance of a device is the ratio of the power in a desired carrier (the first order term in the above expressions) to the sum of the power in the distortion terms (higher order terms). In particular, the ratio of the carrier power to the sum of the power associated with the second order terms (carrier-to-second order) and the ratio of the carrier power to the sum of the power associated with the third order terms (carrier-to-third order) is a significant figure of merit. It is useful to examine the behavior these ratios as a function of the level of the desired carriers.

First, note that the amplitudes of the desired outputs are linearly proportional to the amplitudes of the inputs. However, the amplitudes of the second order terms are proportional to the square, or a second order product, of the amplitudes of the inputs. In other words, an increase in the absolute amplitude of the input signals of 1 dB will result in increase in absolute amplitude of the desired output signals of 1 dB but an increase in amplitude of the second order distortion products of 2 dB. Such a 1 dB increase in input level will result in a 1 dB *decrease* in carrier-to-second order distortion ratio. Similarly, a 1 dB increase in input level will result in a 3 dB increase in the level of the third order products and consequently a 2 dB decrease in carrier-to-third order distortion ratio. These results are shown graphically in Figure 87.

This important relationship makes it possible to predict the carrier-to-second order distortion ratio and carrier-to-third order distortion ratio at any level based on the known performance at a given level, so long as the levels involved are well below the compression point of the amplifier. The compression point is defined as the output level for a device above which these relationships no longer hold. (Jacobi, November, 1986)

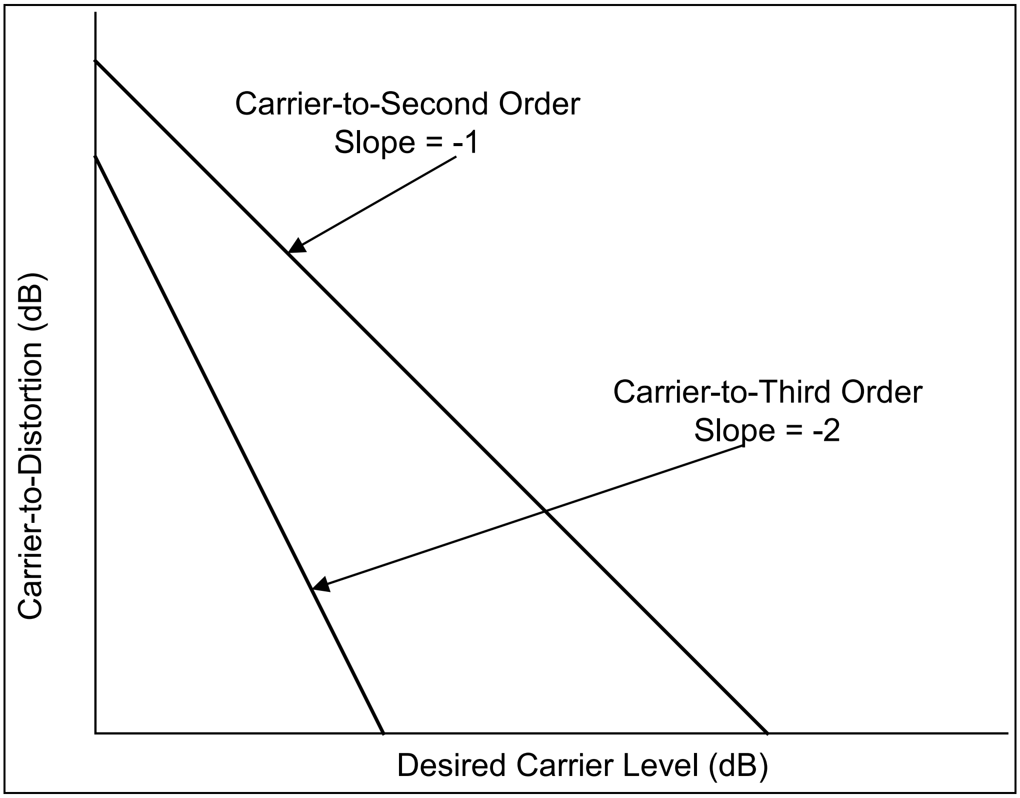


Figure 87 - Relationship between carrier level and carrier-to-distortion.

Carrier-to-second order distortion ratio decreases at a one-to-one rate with desired carrier level. Carrier-to-third order distortion ratio decreases at a two-to-one rate with desired carrier level.

D.1 Results with a large number of carriers

The analysis above indicates the complexity of intermodulation distortion when only a small number of carriers are present. In the case of a conventional CATV system, the number of carriers may be very large. In such a situation, the characterization of the network by purely analytical means would be extremely difficult. The difficulty is complicated by the fact that one of the assumptions in our simplified distortion model is violated.

In the case of a conventional CATV system, the memory-less assumption is not valid. (Chang, 1975) This results in a dependence of the distortion product amplitude on not only the amplitudes of the input carriers but also on the frequency of the input carriers. An analysis of this problem for a small number of carriers using a Volterra series expansion has been described by K.Y. Chang (Chang, 1975). This method begins by generalizing the Taylor series expansion described previously into a generalized series

$$y(t) = y_1(t) + y_2(t) + y_3(t) + \cdots \qquad (2.26)$$

where the individual terms are defined by a series of convolution integrals given by

$$y_i(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_i(\alpha_1, \alpha_2, \cdots, \alpha_i)\, x(t-\alpha_1)\, x(t-\alpha_2)\cdots x(t-\alpha_i)\, d\alpha_1\, d\alpha_2 \cdots d\alpha_i \qquad (2.27)$$

and h_i is a real-valued symmetric function of i real variables. This representation demonstrates that a nonlinear system may be regarded as the combination of a linear and a number of higher order nonlinear subsystems. Each of these subsystems is characterized by a multidimensional impulse response given by $h_i(t_1, t_2, \ldots, t_i)$ as opposed to the single dimensional impulse response, h(t), in the linear case.

It is possible to take the *i*-dimensional Fourier transform of the *i*-dimensional impulse response in order to obtain a transfer function for each subsystem.

$$H_i(f_1,\cdots,f_i)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}h_i(t_1,\cdots,t_i)\cdot\exp\left[-j2\pi\left(f_1t_1+\cdots+f_i\,t_i\right)\right]dt_1\cdots dt_i \qquad (2.28)$$

The result is a family of transfer functions, $H_1(f_1)$, $H_2(f_1, f_2)$, $H_3(f_1, f_2, f_3)$ …

In the case of a conventional CATV system, the input to the system is a spectrum of K conventional analog television channels. Based on the previous models,

$$x(t)=\sum_{i=1}^{K}A_i\cos\left(2\pi f_i+\phi_i\right) \qquad (2.29)$$

The response of the device may then be determined by substitution of this expression into the Volterra series given previously. If we define $\theta_1(f_1)$, $\theta_2(f_1, f_2)$, and $\theta_3(f_1, f_2, f_3)$ as the phase angles of $H_1(f_1)$, $H_2(f_1, f_2)$ and $H_3(f_1, f_2, f_3)$, and we consider only the first, second and third order terms (assuming mild nonlinearities, as before), substitution into the Volterra series results in the following first, second and third order terms

$$y_1(t)=\sum_{i=1}^{K}A_i\left|H_1\left(f_i\right)\right|\cos\left(\omega_i t+\phi_i+\theta_1\right) \qquad (2.30)$$

$$\begin{aligned}y_2(t)=&\frac{1}{2}\sum_{i=1}^{K}A_i^2\left\{\left|H_2\left(f_i,-f_i\right)\right|\cos\left(\theta_2\right)+\left|H_2\left(f_i,f_i\right)\right|\cos\left(2\omega_i t+2\phi_i+\theta_2\right)\right\}\\&+\sum_{i=2}^{K}\sum_{j=1}^{i-1}A_iA_j\begin{bmatrix}\left|H_2\left(f_i,f_j\right)\right|\cos\left\{\left(\omega_i+\omega_j\right)t+\phi_i+\phi_j+\theta_2\right\}+\\\left|H_2\left(f_i,-f_j\right)\right|\cos\left\{\left(\omega_i-\omega_j\right)t+\phi_i-\phi_j+\theta_2\right\}\end{bmatrix}\end{aligned} \qquad (2.31)$$

$$
\begin{aligned}
y_3(t) = {} & \frac{1}{4}\sum_{i=1}^{K} A_i^3 \begin{bmatrix} \left|H_3\left(f_i, f_i, f_i\right)\right| \cos\left(3\omega_i t + 3\phi_i + \theta_3\right) + \\ 3\left|H_3\left(f_i, f_i, -f_i\right)\right| \cos\left(\omega_i t + \phi_i + \theta_3\right) \end{bmatrix} + \\
& \frac{3}{4}\sum_{i=1}^{K}\sum_{\substack{j=1 \\ i \neq j}}^{K} A_i A_j^2 \begin{bmatrix} \left|H_3\left(f_i, f_j, f_j\right)\right| \cos\left\{\left(\omega_i + 2\omega_j\right)t + \phi_i + 2\phi_j + \theta_3\right\} + \\ \left|H_3\left(-f_i, f_j, f_j\right)\right| \cos\left\{\left(2\omega_j - \omega_i\right)t - \phi_i + 2\phi_j + \theta_3\right\} + \\ 2\left|H_3\left(f_i, f_j, -f_j\right)\right| \cos\left\{\omega_i t + \phi_i + \theta_3\right\} \end{bmatrix} + \\
& \frac{3}{2}\sum_{1=3}^{K}\sum_{j=2}^{i-1}\sum_{k=1}^{j-1} A_k A_j A_i \begin{bmatrix} \left|H_3\left(f_i, f_j, f_k\right)\right| \cos\left\{\left(\omega_i + \omega_j + \omega_k\right)t + \phi_i + \phi_j + \phi_k + \theta_3\right\} + \\ \left|H_3\left(f_i, f_j, -f_k\right)\right| \cos\left\{\left(\omega_i + \omega_j - \omega_k\right)t + \phi_i + \phi_j - \phi_k + \theta_3\right\} + \\ \left|H_3\left(f_i, -f_j, f_k\right)\right| \cos\left\{\left(\omega_i - \omega_j + \omega_k\right)t + \phi_i - \phi_j + \phi_k + \theta_3\right\} + \\ \left|H_3\left(-f_i, f_j, f_k\right)\right| \cos\left\{\left(-\omega_i + \omega_j + \omega_k\right)t - \phi_i + \phi_j + \phi_k + \theta_3\right\} \end{bmatrix}
\end{aligned} \tag{2.32}
$$

The term y_1 is the desired linear term. Chang associates the term y_2 with composite second order (CSO) and associates the term y_3 with composite third order, or as it is known in the CATV literature, composite triple beat (CTB). In the special case where K=2, the term y_2 is referred to as discrete second order (DSO) and contains terms at DC, the harmonics of the input carrier frequencies and at the sum and differences of the frequencies of the input carriers, as in the Taylor series model.

Unfortunately, the application of the Volterra series model to a real world CATV device is a very difficult task. In the case of a system with 110 channels (K=110), the determination of composite distortion requires the determination of over 900,000 complex quantities that must be summed in order to determine the composite distortion. An empirically based technique is generally used to characterize distortion performance.

D.2 Empirical methods

D.2.1 NCTA standard measurements

It is useful at this time to examine the frequencies used for carriers in a CATV system. The exact frequency assignments have evolved from those initially chosen for broadcast television. The individual broadcast channels are 6 MHz wide with the main energy in the channel centered around the picture carrier at 1.25 MHz above the lower edge of the channel. CATV channels follow this convention. For the purpose of intermodulation distortion analysis, CATV channels are modeled as CW carriers located at the visual carrier frequency of the channel.[87]

[87] Editor's Note: The discussion here is based on information in "NCTA Recommended Practices for Measurements on Cable Television Systems," which has since been replaced by "SCTE Measurement Recommended Practices for Cable Systems, Fourth Edition."

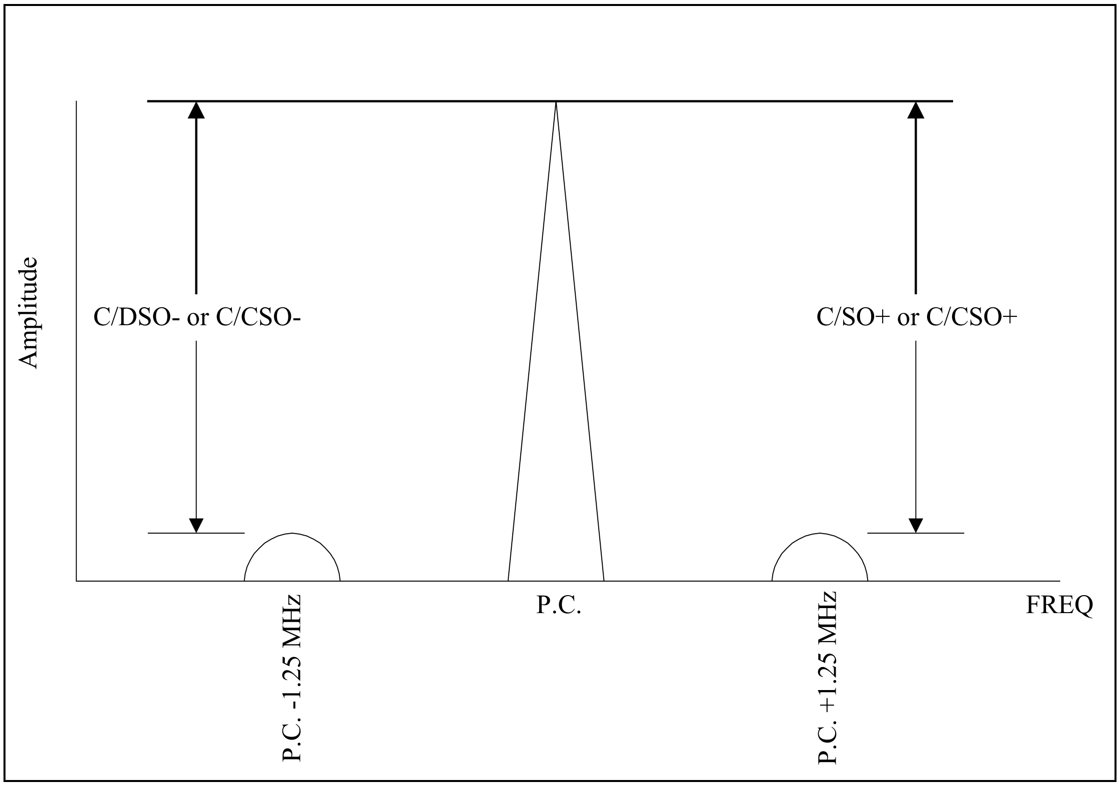


Figure 88 - Second order distortion.

As shown previously, the second order distortion products formed by a pair of carriers at frequency F_A and F_B respectively will fall at DC, $2F_A$, $2F_B$, $F_A + F_B$ and at $F_A – F_B$. An examination of the frequency assignments of the CATV channels indicates that the vast majority of visual carriers are located at 0.25 MHz above an odd frequency (e.g., 55.25 MHz, 199.25 MHz, etc.). It can easily be shown that when two carriers produce second order distortion that falls on another channel, the distortion will typically fall at 1.25 MHz above or below the picture carrier of that channel (see Figure 88). The resulting distortion product has the effect of producing a beat (diagonal cross-hatching) in the picture of the [analog] channel on which the distortion falls, as shown in Figure 89. When these distortion products fall within the CATV band, a figure of merit is the ratio of the amplitude of this distortion to the amplitude of the carrier of the channel on which the distortion falls. This ratio, called carrier-to-discrete second order (C/DSO– and C/CSO+) is defined in two parts as (Staff, 1989)

$$C/DSO- = 10\log_{10}\left(\frac{\text{Carrier Power}}{\text{DSO Power at Picture Carrier } -1.25\text{ MHz}}\right) \qquad (2.33)$$

$$C/DSO+ = 10\log_{10}\left(\frac{\text{Carrier Power}}{\text{DSO Power at Picture Carrier } +1.25\text{ MHz}}\right) \qquad (2.34)$$

and is measured in decibels (dB).

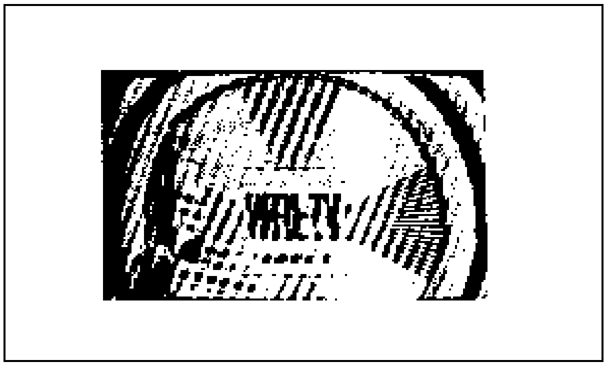


Figure 89 - Subjective effect of CSO on received picture quality; note the diagonal cross-hatching in the image.

Second order is a useful number for making comparisons between the performance of two or more devices. However, a more useful measurement for characterizing the subjective effects of distortion is carrier-to-composite second order (C/CSO– and C/CSO+) which results from fully loading the device with carriers. (Staff, 1989)

$$C / CSO- = 10\log_{10}\left(\frac{\text{Carrier Power}}{\text{Average CSO Power in a 30 kHz BW at P. C. } -1.25 \text{ MHz}}\right) \quad (2.35)$$

$$C / CSO+ = 10\log_{10}\left(\frac{\text{Carrier Power}}{\text{Average CSO Power in a 30 kHz BW at P. C. } +1.25 \text{ MHz}}\right) \quad (2.36)$$

Unfortunately, in the CATV literature, the terms CSO and CTB are used to refer to both the absolute distortion amplitude and the carrier-to-distortion ratio. In this discussion, separate terms will be used as defined above.

In a standard frequency CATV system, the individual carriers are generated by a series of individual crystal-controlled oscillators. These oscillators are therefore not phase locked, and vary slightly from the ideal frequencies of the picture carriers. As a consequence, the second order beats that ideally fall at 1.25 MHz above the picture carrier for channel 2, for example, in practice do not all fall at exactly 56.500000 MHz. The composite distortion manifests itself as a cluster of individual beats clustered around the ideal frequencies of 56.5 MHz and 54.0 MHz. Similarly, the composite second order on all other channels consists of a cluster of beats grouped around the ideal second order frequencies. Hence the requirement for an average power measurement in a 30 kHz bandwidth in the definition. This characteristic of CSO greatly affects its perceptibility and hence the acceptable limits for this type of distortion.

A similar situation exists for carrier-to-third order distortion (C/CTB) as defined by (Staff, 1989)

$$C / CTB = 10\log_{10}\left(\frac{\text{Carrier Power}}{\text{Average CTB Power in a 30 kHz BW at P. C}}\right) \quad (2.37)$$

A simple examination of the frequencies of the standard frequency CATV carriers will show that third order distortion products that fall within the CATV band will fall at the picture carrier frequency of a channel as shown in Figure 90.

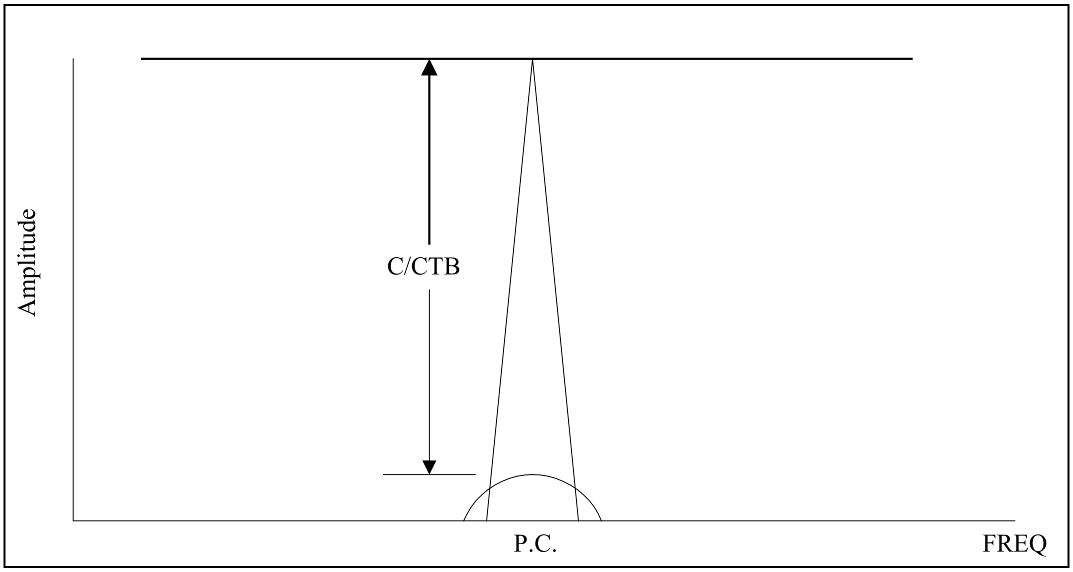


Figure 90 - Carrier-to-composite triple beat (C/CTB).

Consequently, the CTB spectrum looks very much like the CSO spectrum except for its location. Because of this location, the subjective effect of CTB is low frequency noise (or streaking) in the video as shown in Figure 91. As with CSO, the CATV literature uses the term CTB interchangeably to refer to the absolute level of the distortion as well as the carrier-to-distortion ratio.

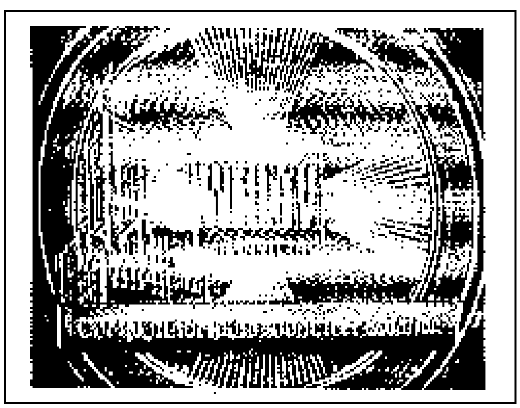


Figure 91 - Subjective effect of CTB on received picture quality.

An actual CTB measurement is shown in Figure 92. The value of C/CTB in this example is approximately 55 dB.

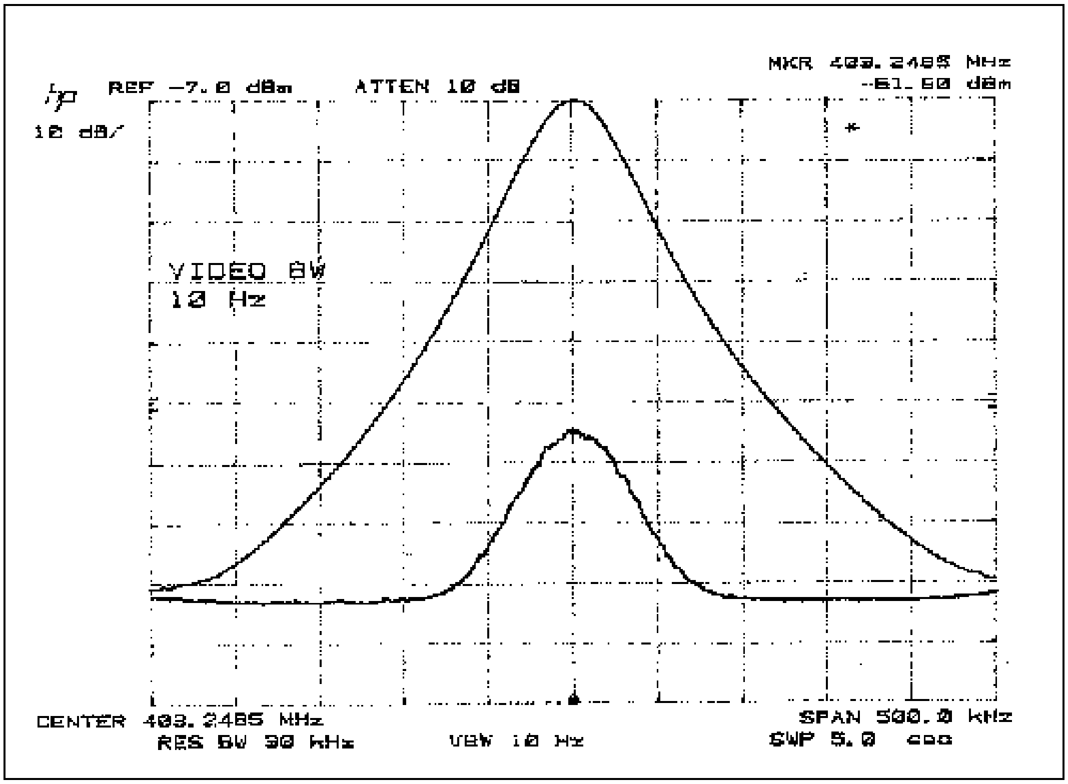


Figure 92 - Analyzer display of CTB measurement.

D.3 Cascade effects

The discussion so far has assumed that devices in CATV systems have mild nonlinearities. As a consequence, the distortion generated by any one device is small when compared with the overall system distortion goals. What is of concern, however, is how these small distortions add up throughout the system cascade to produce the end of line performance.

For the purposes of analysis, it is assumed that any given device (amplifier) has a linear phase-versus-frequency characteristic. Phase and delay are related by (Laboratories, 1971)

$$D = \frac{\partial \phi}{\partial \omega} \tag{2.38}$$

where D = delay, ϕ = phase and ω = frequency. Hence, linear phase means constant delay.

In this context, phase is defined as the equivalent electrical phase length of the network, not the output phase minus the input phase. For example, consider a piece of coaxial cable that has a length, l, of 2.0 meters. Assume the cable has a velocity factor of 0.6667. Then at an operating frequency of 100 MHz, the cable has a phase length given by

$$\phi = \frac{l \cdot f}{c\,(0.6667)}(360°) = 360° \tag{2.39}$$

As television signals are extremely sensitive to delay distortion, the CATV system delay must be constant with frequency, and hence the assumption of linear phase-versus-frequency is valid (Grob, 1984).

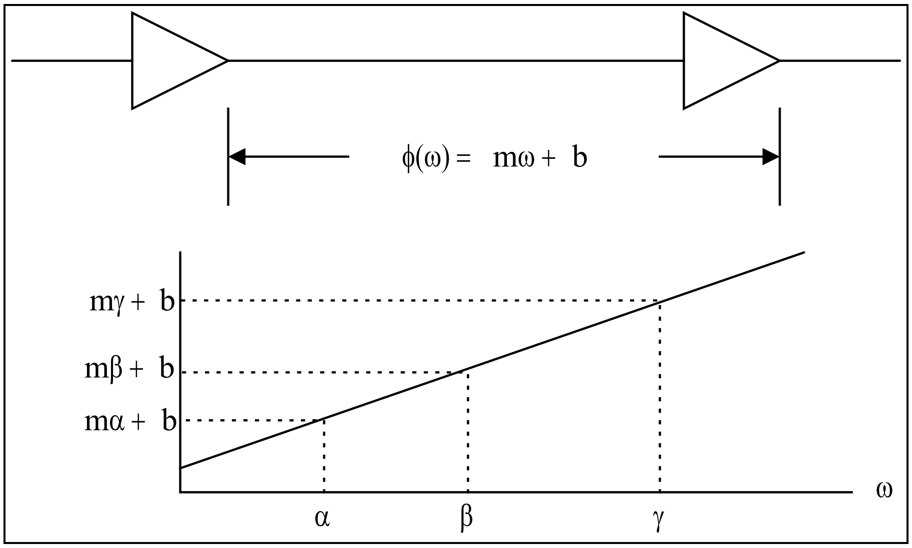


Figure 93 - Linear phase-versus-frequency.

Consider the case of two identical amplifiers separated by a length of cable with a constant delay as shown in Figure 93. The phase characteristic of the cable plus second amplifier will be given by

$$\phi(\omega) = m\omega + b \tag{2.40}$$

where m is the slope of the phase-versus-frequency characteristic and b is a constant phase offset. For example, when a carrier at frequency α has zero phase at the output of the first amplifier, the phase of that carrier at the output of the second amplifier will be

$$\phi_\alpha = m\alpha + b \tag{2.41}$$

When two signals of the form

$$\cos(\alpha\, t), \cos(\beta\, t) \tag{2.42}$$

are applied to the input of the first amplifier, a component proportional to

$$\cos((\alpha + \beta)t) \tag{2.43}$$

will appear at the output of the first amplifier resulting from the second order distortion. Note here that the phases of these signals have been chosen to be zero at the output of the first amplifier for simplicity's sake. At the output of the second amplifier, the three signals will be proportional to

$$\cos(\alpha\, t + \phi_\alpha), \cos(\beta\, t + \phi_\beta), A\cos((\alpha + \beta)t + \phi_{\alpha+\beta}) \tag{2.44}$$

where

$$\phi_\alpha = m\alpha + b,\ \ \phi_\beta = m\beta + b,\ \ \phi_{\alpha+\beta} = m(\alpha+\beta) + b \tag{2.45}$$

In the second amplifier, a second order distortion component is generated that is proportional to

$$\cos\left((\alpha+\beta)t + \phi_\alpha + \phi_\beta\right) \tag{2.46}$$

with

$$\phi_\alpha + \phi_\beta = m(\alpha+\beta) + 2b \tag{2.47}$$

At the output of the second amplifier, the second order distortion generated in the first amplifier has a phase of m(α+β) + b. At the output of the second amplifier, the second order distortion generated in the second amplifier has a phase of m(α+β) + 2b. The difference in the phase of the second order products generated in the two amplifiers is given by b. As the value of b is not known *a-priori* and will vary from span to span within the system, second order distortion products will combine on a random phase (power) basis throughout the distribution plant. For a cascade of N identical amplifiers where N is large (typically N ≥ 10), all operating at the same output power levels and each having composite second order distortion of value CSO dB, the total composite second order distortion of the cascade will be given by

$$CSO_{total}(dB) = 10\log_{10}\sum_{i=1}^{N}\left(10^{\frac{CSO}{10}}\right) = CSO + 10\log_{10}(N) \tag{2.48}$$

$$C/CSO+_{total} = (C/CSO+) - 10\log_{10}(N) \tag{2.49}$$

$$C/CSO-_{total} = (C/CSO-) - 10\log_{10}(N) \tag{2.50}$$

Note that this relationship only holds when N is large, and the devices are identical. If only a small number of devices are in cascade the dependence on the phase length of the transmission media separating the devices becomes significant. In such a case the assumption of power addition of the distortion no longer holds and the $10\log_{10}$ relationship falls apart.[88]

A similar analysis can be done for third order distortion. Assume three carriers are applied to the first amplifier resulting in signals at the output of that amplifier that are proportional to

$$\cos(\alpha t), \cos(\beta t), \cos(\gamma t), B\cos\left((\alpha+\beta-\gamma)t\right) \tag{2.51}$$

where the fourth signal comprises a third order distortion product generated in that amplifier. At the output of the second amplifier they will have undergone phase shifts of ϕ_α, ϕ_β, ϕ_γ and $\phi_{\alpha+\beta-\gamma}$ respectively, where

[88] Editor's Note: In the 1980s and 1990s some calculations of carrier-to-composite second order distortion ratio used $15\log_{10}$ to accommodate the CSO in short cascades "breathing" or varying in amplitude depending on when it was measured. The variation was a result of the phasors rotating around creating constructive and destructive interference of the individual second order products. Once there are enough distortion products things start to average out to power addition.

$$\phi_{\alpha+\beta-\gamma} = m(\alpha+\beta-\gamma)+b \tag{2.52}$$

There will also be a third order distortion product generated in the second amplifier that will be proportional to

$$\cos((\alpha+\beta-\gamma)t + \phi_\alpha + \phi_\beta - \phi_\gamma) \tag{2.53}$$

and

$$\phi_\alpha + \phi_\beta - \phi_\gamma = m(\alpha+\beta-\gamma) + b \tag{2.54}$$

Hence at the output of the second amplifier, the phase of the third order distortion product generated in the first amplifier and the phase of the third order distortion product generated in the second amplifier are the same. These two products will combine on an in-phase (voltage) basis. For a cascade of N identical amplifiers, all operating at the same output power levels and each having composite triple beat distortion of value CTB dB, the total composite triple beat distortion of the cascade will be given by

$$CTB_{total}(dB) = 20\log_{10}\sum_{i=1}^{N}\left(10^{\frac{CTB}{20}}\right) = CTB + 20\log_{10}(N) \tag{2.55}$$

and

$$C / CTB_{total}(dB) = CTB - 20\log_{10}(N) \tag{2.56}$$

Note that the results for cascaded third order distortion does not rely on the phase length of the transmission media between the devices. Therefore, this relationship holds regardless of the number of devices in cascade.

By making use of either power addition or voltage addition and the rules presented earlier for the calculation of distortion as a function of operating level, it is possible to calculate the overall cascade distortion of a series of amplifiers even if these amplifiers are not operated at equivalent power levels.

Bibliography

Chang, K. Y. (1975). Intermodulation Noise and Products due to Frequency-Dependent Nonlinearities in CATV Systems. IEEE Transactions on Communication, COM-23.

D. McEwen, N. S. (n.d.). Distortion Accumulation in Trunks. 1990 Proceedings from the Technical Sessions, 44th Annual Convention and Exposition of the National Cable Television Association.

Grob, B. (1984). Basic Television and Video Systems. McGraw Hill, Inc.

Jacobi, J. H. (November, 1986). IMD, Still Unclear After 20 Years. Microwaves and RF.

Laboratories, B. T. (1971). Transmission Systems for Communications, (Revised Fourth ed.). Western Electric Company, Inc. Technical Publications.

Staff, N. T. (1989). NCTA Recommended Practices for Measurements on Cable Television Systems. NCTA Technical Publications.

Appendix E Error Performance Testing Confidence Level

As mentioned in Section 29.7.1, among the factors that can affect BER measurements are the number of bits transmitted during the measurement, the duration of the measurement, and whether the bit errors are independent and identically distributed (IID). This section includes an overview and analysis of the aforementioned factors, written by Tom Kolze. Used with permission.

Error Performance Testing Confidence Level

Tom Kolze, Ph.D.
Broadcom, Inc.

E.1 Introduction

One of the primary figures of merit for digital communication systems is the error performance, and for many decades the predominant figure of merit for the error performance was the bit error ratio (BER). Historically a lot of emphasis has been placed on making reliable measurements of a communication system's error performance, and this section presents a few of the practical "take-aways" while referring readers to references for a more rigorous treatment.

Generally, a measurement of error performance, a "test," is characterized by counting a number of "trials" and "errors," for example, counting the total number of transmitted bits and the number of bit errors that occurred during the test. The number of errors is generally a random variable, but the ratio of bit errors to total bits typically converges for an increasingly larger number of transmitted bits.

The greater the number of bits, the better the quality of the BER estimation. Ideally, an infinite number of bits will give a perfect estimate of error probability, but that's simply not practical! So, how many bits are enough?

The two primary considerations in a BER measurement, or test, are a) how accurately does the ratio of the number of bit errors to total bits compare with the "true" (long term) average for the system; and b) how much time does it take to make the measurement, which is also "how many bits have to be transmitted to get the reliable measurement"? The "confidence level" describes a measure of accuracy for the testing, and is given a brief overview in the following section.

E.2 Confidence level of a test

In qualification testing a buyer is wanting to be assured that the system or product meets its requirements, so the buyer's goal is to identify, and not accept, a product which fails to meet the requirements. Both the buyer and seller want to avoid unnecessarily long testing. Methodology has been developed to quantitatively describe the buyer's goal, and define satisfactory testing which meets the goal.

A buyer determines their acceptable amount of risk of "passing" a product which actually fails to meet the requirements; for example, a buyer may decide it is okay to accept a non-compliant product 5% of the time. Continuing with this example of allowing a PASS on 5% of non-compliant product, the methodology is to design a test to meet the buyer's goal and acceptable risk: Any product that doesn't meet the requirements will be detected with a FAIL in the testing 95% of the time, or more. Equivalently, the non-compliant product may slide by, undetected as non-compliant, up to 5% of the time. Such a test is said to have a "95% confidence level." *With a qualifying test of 95% confidence level, the buyer is confident that a non-compliant product will be detected 95% of the time (or more).*

As an example to illustrate the concept, consider a modem which has a BER requirement of 10^{-8} or lower in a certain test environment (SNR, received power level, etc.). It can be shown that a qualifying test that has $3 * 10^{8}$ (that is, 300 million) transmitted bits, with "independent and identically distributed" (IID) bit errors, and operating at the spec BER amount of 10^{-8}, yields a probability of no errors at all in the 300 million transmitted bits, at 4.98%. This is just under 5%. Thus, this qualifying test for the modem which involves transmitting 300 million bits and requires no bit errors to obtain a PASS has a 95% confidence level. To explain, if the modem had a BER of just a little worse (higher) than 10^{-8}, there would be AT LEAST a 95% chance of the modem having at least one bit error out of the 300 million, and thus FAILING. *There is 95% confidence for the buyer to detect and weed out a non-compliant modem.*

Summarizing, the confidence level of a test is related to the buyer's confidence of detecting a FAILING product. The risk of a non-compliant system actually PASSING the test, quantitatively, is 1 minus the stated confidence level (5% risk equals 1 minus 95%, where 95% is the confidence level in the example).

It is worth mentioning that there are many qualifying tests, even in this specific example of a modem with a 10^{-8} BER requirement (in specified conditions), which have a 95% confidence level. For example, another such 95% confidence level test would be to transmit 1.06 billion bits ($1.06 * 10^{9}$), and a PASS requires five or fewer bit errors. This test requires more than three times the number of transmitted bits as the test described previously (1.06 billion versus 300 million), but has the same 95% confidence level. A key difference is that five bit errors are allowed while still achieving a PASS in the longer test, whereas in the test with 300 million transmitted bits, the occurrence of any bit error will result in a FAIL.

Finally, it is worth pointing out that a system that is precisely compliant with the specified BER, with no margin, will FAIL the test 95% of the time! *This is not a great deal for the seller!*

In practice, the seller would either go to lengths to ensure that their products (modems in this example) have lots of margin compared to the 10^{-8} BER, or, if they could not ensure such margin (could not provide very much lower BER than the requirement), they would recognize that they needed a much longer qualifying test. With the much longer qualifying test, the 95% confidence level corresponds to a higher limit on the number of bit errors which would PASS. It is also the case that the qualifying limit for the number of bit errors that still passes ALSO corresponds to a higher BER average, as the test length gets longer. In the example, if the modem has an average BER of $3 * 10^{-9}$, which is much better than the spec of 10^{-8}, when the qualifying test is $1.06 * 10^{9}$ transmitted bits, the cable modem will incur five or fewer bit errors, and thus PASS, 90% of the time; *however, this means that this satisfactory cable modem still FAILs 10% of the time*. If the cable modem has an even lower average BER, 10^{-9}, it will PASS the test with five or fewer bit errors 99.9% of the time.

Once the buyer has determined the confidence level they desire for the qualifying test, the seller determines the test duration (or jointly the buyer and seller agree), with a large consideration being the expected margin for the product. A product performing vastly better than the requirement will allow shorter duration qualifying tests and still achieve a high PASS rate for the compliant product.

In practice, a buyer and seller can agree to allow retesting in the event of FAILs, but the cases are very carefully monitored, recorded, and fully reported. Often the specific conditions allowing and executing retesting are spelled out in the qualifying test procedure. Sometimes in the event of a FAIL, a review by buyer and seller is held to determine the next steps.

E.3 Rules of thumb and examples

One rule of thumb used in an example in the previous section is that transmitting 3 times the reciprocal of the specified BER *without an error* gives 95% confidence level that the device or network meets the desired BER specification, when the bit errors are IID. If one wants 99% confidence level, the multiplier is 4.61 rather than 3. Note: These multipliers are derived from statistics involving the binomial distribution function and Poisson theorem, as demonstrated below.

A critically important assumption in the rule of thumb, and the application of the binomial statistics, is that all the bit errors are “identically distributed and statistically independent”: the IID assumption. This means (for practical purposes) that if a bit error occurs in one of the transmitted bits, other bits (in particular bits “adjacent” in the bit stream) have no greater (or lesser) likelihood of being in error even with the presence of the observed bit error. This is a key assumption and we explore the impact when errors are not IID in Sections E.4 and E.5, but for now, note that the “independent bit errors” and “identically distributed bit errors” were assumed in the previous section pertaining to confidence level of “qualifying” tests.

With the assumption of IID bit errors, if there is a specified requirement of no more than one bit error in X transmitted bits, on average, then a rule of thumb is that a test with $3 * X$ transmitted bits, and requires NO bit errors in order to PASS, has a “95% confidence level that the system is at or better than the error performance requirement, which is BER less than or equal to 1/X.” Similarly, another rule of thumb (also mentioned above) is that a measurement with $4.61 * X$ transmitted bits, and NO bit errors to PASS, has a “99% confidence level that the system is at or better than the error performance requirement.”

In general, any confidence level can be selected for a qualifying test (strictly upper bounded by 100%). With a BER requirement for no more than one bit error in X transmitted bits, on average, and a qualifying test requiring no bit errors to achieve a PASS, any confidence level less than 100% can be provided. To achieve a confidence level, Y, the number of transmitted bits, N, should be:

> $N = [\,-\{\ln(1 - Y)\} * X]$ number of transmitted bits; BER requirement is $\leq 1/X$;
> confidence level Y; test limit to PASS is zero bit errors.

Such a value for N provides a lower bound for designing the test and is tight for BER requirements of 10^{-2} and smaller, and confidence levels of 95% and larger.

Let’s say we want to ensure the BER at a modem which is specified at $1 * 10^{-8}$ (1.0E–08), at a 95% confidence level. As mentioned previously, one such qualifying test with 95% confidence level requires transmitting 3 times the reciprocal of the specified BER without an error. Note that $\ln(1 - 0.95)$ is $\ln(0.05)$ which is –2.9957; thus $N = 2.996 * X$, or we can conveniently round up (increasing the test duration without changing the PASS limit will increase the confidence level, so rounding up is “safe” for maintaining the 95% confidence level). Thus, for this example $N = 3 * X = 3 * 10^{8}$; this is the first rule of thumb qualifying test mentioned above.

To satisfy the IID assumption on the bit errors, assume a modulation such as BPSK or QPSK, and no forward error correction (FEC) coding. These assumptions will be discussed in more detail in Section E.5; under AWGN and other ideal conditions, the bit errors with BPSK or QPSK are IID.

Calculate the reciprocal of the BER in question and multiply by a multiplier of 3:

$$= \frac{1}{1 * 10^{-8}} * 3$$

$$= \frac{1}{0.00000001} * 3$$

$$= 300{,}000{,}000$$

For a 95% confidence level, the required number of bits to be transmitted error free is 300,000,000, which is the same as $3 * 10^8$ bits.

The next question is how long it takes to transmit the desired number of bits.

Assume QPSK at 10.72 Mbps:

$$Time\ (seconds) = \frac{3 * 10^8 bits}{10.72 * 10^6 bits\ per\ second}$$

$$Time = 27.984$$

Answer: The time to transmit 300,000,000 bits at 10.72 Mbps is 27.984 seconds.

If the BER target is something more aggressive like $1*10^{-10}$ (1.0E–10) at 99% confidence level (4.61 multiplier), the required number of bits that must be transmitted error free is

$$= \frac{1}{1 * 10^{-10}} * 4.61$$

$$= 46{,}100{,}000{,}000$$

The minimum test time for QPSK at 10.72 Mbps is

$$Time\ (seconds) = \frac{4.61 * 10^{10} bits}{10.72 * 10^6 bits\ per\ second}$$

$$Time = 4{,}300$$

Answer: The time to transmit 46,100,000,000 bits at 10.72 Mbps is 4,300 seconds or 71 minutes 40 seconds.

Now consider a system using 256-QAM at the same symbol rate. With 256-QAM there are eight bits per QAM symbol. At this point the necessary IID assumptions for application of the confidence level for qualifying tests become problematic. Even if the QAM symbols are Gray coded, and there is no FEC, and even with the very low error rates (such as 10^{-8} BER), it can be shown the bits are not IID. The bit errors are not identically distributed and the bit errors are not independent, even with the Gray coding.

Still, with the Gray coding, and no FEC, and a fairly low BER requirement (even 10^{-2} would suffice), the confidence level analysis that holds with IID can be applied here for practical purposes, but even this has a subtlety. With the Gray coding, each of the four bit positions of a 256-QAM symbol rail has half the probability of error of the preceding bit position, with the LSB having the largest probability of error. Thus, the bit errors are not identically distributed. Another violation of the IID assumption is the independence of the bit errors; even with the Gray coding, and low overall error ratios, it is extremely unlikely to have more than one bit error per “rail” in a 256-QAM symbol which is Gray coded, compared to the probability of one error. In fact, that is the entire point of Gray coding. Thus, of the four bits per rail per 256-QAM symbol, the probability of two or more of the four bits being in error is negligible. So, this is the subtlety: Given a bit error in one of the four bits of a QAM symbol rail, the other three bits are extremely unlikely to be in error (much less than the average BER). This is a strong violation of the independence of bit errors requirement! However, it can be shown that the effect on the confidence level “cancels” in a fashion, and the standard analysis with IID bit errors can still be applied even with this strong bit error dependence (in this case, no errors “near” a given bit error). While these factors are violations of the conditions for applicability of the confidence level analysis, it can be shown (beyond the scope here) that the confidence level results will still apply for the Gray coded 256-QAM close enough for practice.

Assume 256-QAM at 42.88 Mbps:

$$Time\ (seconds) = \frac{3 * 10^{8} bits}{42.88 * 10^{6} bits\ per\ second}$$

$$Time = 6.996$$

Answer: The time to transmit 300,000,000 bits at 42.88 Mbps is 6.996 seconds.

If the BER target is something more aggressive like $1*10^{-10}$ (1.0E–10) at 99% confidence level (4.61 multiplier), the required number of bits that must be transmitted error free is

$$= \frac{1}{1 * 10^{-10}} * 4.61$$

$$= 46{,}100{,}000{,}000$$

The minimum test time for 256-QAM at 42.88 Mbps is

$$Time\ (seconds) = \frac{4.61 * 10^{10} bits}{42.88 * 10^{6} bits\ per\ second}$$

$$Time = 1{,}075$$

Answer: The time to transmit 46,100,000,000 bits at 42.88 Mbps is 1,075 seconds or 17.9 minutes.

E.4 FEC and M-ary modulation

BER testing, and application of the confidence level to determine qualification testing, was straightforward for decades. However, with the advent of M-ary constellations, and in particular the application of FEC, the accurate determination of a BER test’s confidence level can be unwieldy. [See

(Atul Shah and Thomas Kolze)]. And without an accurate determination of the confidence level, for example, ignoring the dependence of the bit error occurrences (incorrectly assuming bit error independence) can lead to inadequate qualification testing.

In one example, with a fairly modest FEC by today's standards, neglecting the bit error dependence would lead to the qualification testing providing a confidence level of 79% when the intention was 95%. See (Atul Shah and Thomas Kolze). This is a stark difference; neglecting the bit error dependence of the system would cause the confidence level of the qualification testing (21% of non-compliant product could PASS erroneously) to fall far short of the buyer's goal (the 95% identification of a non-compliant system). Including the effect of the bit error dependence showed that to achieve the 95% confidence level, the test time (the number of transmitted bits) for this system had to be increased by 68% compared to the number of transmitted bits if the independence had been incorrectly assumed.

The modulation and FEC in the system in Reference (Atul Shah and Thomas Kolze) have much less bit error dependence than the more recently developed communication systems, such as DOCSIS 3.1. In the DOCSIS 3.1 downstream, for example, codewords can be as large as 16,200 bits, including the parity bits, or as large as 14,232 bits after decoding.

The effect of the dependence of bit error occurrences upon confidence level of qualification testing is problematic with even modest FEC. See (Jeruchim, Michel C.). For more recent communication systems, such as DOCSIS 3.1, with powerful FEC, and especially with the extremely long codewords compared to a generation ago (e.g., DOCSIS 1.0, which itself was quite an achievement at its onset), even a tight approximation of confidence level in BER testing would likely be unattainable. And, more to the point, hardly worth the heroic effort necessary.

Due to the extreme bit error dependence in modern communication systems, owing to the powerful FEC techniques employed, many recently developed communications systems have moved away from specifying BER. There is impact of that dependence on the supported applications of the links, in addition to impacting the error performance testing (confidence level). For example, what good does it do to know a BER when the bit errors, which may be rare, are bursted out in spans of a few thousand bits (highly correlated bit error occurrences), when the ultimate goal is to determine an Ethernet packet error ratio (PER)? To calculate the Ethernet packet error ratio from a BER measurement, or specification, requires knowing fairly precisely the burstiness or distribution of bit errors, i.e., the correlation or dependence of bit error events; thus, a daunting further analysis would be needed to determine the Ethernet performance even knowing the BER performance! For these reasons, in addition to the entire issue of uncertainty regarding the confidence level of the testing*, DOCSIS moved away from exclusively specifying BER over a decade ago, for example, in DOCSIS 3.0.* [Reference (PHYv3.0); I01 was in 2006.]

DOCSIS downstream error performance was originally specified as a BER (DOCSIS 1.0 and forward), but within a few years DOCSIS developed a codeword error ratio (CER) performance requirement, and vendors were allowed to use either. The determination of an "equivalent" codeword error performance with the BER requirement was based on measurements taken by cable modem vendors at the time, not by analysis or simulation. In DOCSIS 3.1 the downstream link performance is specified exclusively via Ethernet packet error ratio. [Reference (PHYv3.1); I01 was in 2013.]

For the DOCSIS 3.0 specification PHYv3.0, the downstream SC-QAM ITU-T J.83B performance specifies the original DOCSIS BER of 10^{-8} and is accompanied with the CER requirement of $9 * 10^{-7}$. Note that BER = CER $*$ (bit errors per errored codeword/total bits per codeword), and that there are $122 * 7 = 854$ bits per codeword after decoding the Reed-Solomon outer codewords of the concatenated coding scheme. The testing program for setting a CER equivalent to the BER requirement found there was an average of about $9 * 10^{-7}$ errored Reed-Solomon codewords when the BER was close to threshold, just meeting the requirement at around 10^{-8}. The CER value corresponding to the BER threshold performance was obtained by averaging the CER results from many vendors, each cable modem operating where it was just achieving 10^{-8} BER. The average bit errors per errored codeword can be obtained from this testing program, and the relationship between CER and BER, such that the average bit errors per errored codeword = (BER/CER) ∗ (total bits per codeword) = $((10^{-8})/(9 * 10^{-7})) * (854) = (85.4/9) = 9.5$ bits.

The FEC for ITU-T-J.83B has an inner convolutional code and an outer Reed-Solomon code, with an interleaver in-between. Even in AWGN with no burst noise or other impairments, there are bursted error events at the output of the convolutional decoder; the de-interleaver will spread the errors of a single burst error event across multiple Reed-Solomon codewords. In AWGN, it will generally take multiple convolutional decoding burst error events to cause even a single Reed-Solomon decoding failure, but this also means that the Reed-Solomon codeword errors are not IID. When there are sufficient inner code errors to cause the Reed-Solomon decoder to fail, there are errors from those same convolutional decoding error events spread into adjacent Reed-Solomon codewords, too. The impact of the correlation of Reed-Solomon codeword failures on confidence interval will be deferred for another day. However, the DOCSIS 3.1 OFDM FEC is examined below. Note that even if the DOCSIS requirements remained solely in terms of the BER, the correlation of the bit errors at the output of the Reed-Solomon decoder (due to multiple bit errors per codeword error), and the correlated decoder error events, both serve to increase the required number of bits to achieve a given confidence level compared to IID bit errors.

As an illustration of the impact of M-ary modulation, and powerful FEC, on the confidence level, the DOCSIS 3.1 OFDM PHY layer is examined.

While the performance requirements of DOCSIS 3.1 are in terms of Ethernet packet errors, we consider the codeword errors, make a few assumptions, and see what impact correlated bit errors may have on the confidence level of the testing.

E.4.1 Exploring confidence level for DOCSIS 3.1 OFDM FEC testing

The DOCSIS 3.1 OFDM codewords can have as many as 14,232 bits, and let’s assume there are 70 bits in error when a codeword is decoded with an error (i.e., fails to decode). In actuality, the number of bit errors that occur when a codeword fails to decode is variable and can cover a large range, but to illustrate the impact of correlated bit errors, simplifying assumptions are used here. In the errored codewords we see that the bit errors per errored codeword divided by the total bits per codeword is 70/14,232 which is (roughly) 1/200.

When there is an average BER of 10^{-8}, and the codewords contain the bit errors as described above (70 bit errors in each errored codeword, out of the 14,232 bits), this dictates that the CER is (roughly) $2 * 10^{-6}$. This is because BER = CER ∗ (bit errors per errored codeword/total bits per codeword), where the ratio in parentheses is 1/200 (by our assumption for this example).

Let’s assume that in AWGN that codeword errors are IID. Then a confidence level of 95% with ZERO errors for a PASS can be constructed based on the $2 * 10^{-6}$ CER which would correspond to the system BER when operating at the BER specification of 10^{-8} (in this example we are “allowing” or presuming a BER specification, though in DOCSIS 3.1 OFDM there isn’t such a specification). The number of codewords that must be tested to achieve the desired confidence level is

$$N\_codewords = 3 * (1/CER) = 3/(2 * 10^{-6}) = 1.5 * 10^{6}$$

Let's compare the number of bits in this test, compared to the number of bits if we (incorrectly) assumed the bit errors were IID and designed a qualifying BER test. Recall that in this case we determined that $3 * 10^8$ transmitted bits would be needed. But with the correlated error events and the large codewords, we actually need $1.5 * 10^6$ codewords to achieve the confidence level desired, and this requires $1.5 * 10^6 *$ 14,232 bits, which is more than $2 * 10^{10}$ transmitted bits; this is more than 60 times the number of bits determined with the IID assumption, neglecting the correlated bit errors.

E.4.2 Exploring confidence level for DOCSIS 3.1 downstream Ethernet PER performance specification

It should be noted that the DOCSIS 3.1 modem error performance for OFDM is specified for 1500-byte Ethernet packets, rather than codewords, and the requirement is 10^{-6}. The bits in full length OFDM codewords are distributed into two or even three Ethernet 1500-byte packets, since they are longer than the specified Ethernet packets (14,232 bits compared to 12,000 bits). A 1500-byte Ethernet packet comprises about 84.3% of a full length codeword, so (assuming randomized alignments occur) two Ethernet packets contain bits from the same codeword in 81.4% ([84.3 – 15.7]/84.3 = 81.4) of the codewords, and in the remaining 18.6% (15.7/84.3 = 18.6) of the codewords the bits are distributed among three of the Ethernet packets. When a codeword error occurs, there is always at least one Ethernet packet in error that occurs, but there could be two or even three Ethernet packet errors arising from a single codeword error. With full length OFDM codewords and 1500-byte Ethernet packets, the Ethernet packet errors are not IID. Even with the codeword errors being IID, the Ethernet packet errors often occur in pairs or even triplets. It is possible that all the codeword errored bits concentrate in one Ethernet packet, but with an average of 70 bit errors per errored codeword (assumed for the example), the likelihood is very small (less than $2 * 10^{-5}$) for none of those bit errors falling in the 2,232 bits (or more) outside the Ethernet packet containing the most bits in the codeword. Thus, when there is an Ethernet packet error, there is greater than 0.99998 probability that one or both of the two adjacent Ethernet packets will be in error, too (a pair or a triplet of errors out of three consecutive packets).

After a fair amount of calculation, it can be shown that under the assumptions above there is an average of 2.15 Ethernet packet errors when a full length codeword error occurs. For the case where the number of codeword errors is very small (such as our area of interest with the CER less than 10^{-5}), we can neglect the occurrences of consecutive codeword errors (for the accuracy we are using), so the ratio of the number of Ethernet packet errors to codeword errors remains at the value 2.15.

The ratio of the number of Ethernet packets to codewords is 14,232/12,000 in a long-term interval of time of continuous transmission. Using the ratios of Ethernet packets to codewords, and the ratio of the number of errors of the two, we see the relationship

$$PER/CER = 2.15 * (12{,}000/14{,}232) = 1.8$$

Let's examine the number of 1500-byte Ethernet packets that need to be transmitted for a confidence level of 95% with ZERO errors for a PASS. If we were to assume that the Ethernet packet errors are IID, since the requirement is 10^{-6} PER, then the rule of thumb dictates that the number of packets that must be transmitted to achieve the desired confidence level is

$$N\_packets = 3 * (1/PER) = 3/(10^{-6}) = 3 * 10^6$$

However, we have seen that the Ethernet packet errors usually occur in pairs or even triplets, owing to a single FEC codeword error. The Ethernet packet errors are not IID, and in fact more closely fit a model of occurring in pairs (the average number of Ethernet packet errors arising from a single codeword error was 2.15). We will see in Section E.5.4 that such error behavior, due to the correlated error events (errors are not IID), with errors occurring in pairs, results in a doubling of the number of trials that have to be tested to achieve the same confidence level as with IID errors. Thus, in DOCSIS 3.1 OFDM, when testing Ethernet packets with the requirement of 10^{-6} PER, about $2 * (3 * 10^6)$ or $6 * 10^6$ transmitted packets are needed for a confidence level of 95% with ZERO errors for a PASS. Since these packets are 1500-byte Ethernet packets, this means the testing involves $(1500 * 8) * 6 * 10^6$ bits, which is $7.2 * 10^{10}$ bits.

This result ($7.2 * 10^{10}$ bits) is more than 3.5 times the number of transmitted bits determined in Section E.4.1 for the corresponding 95% confidence level test based on CER. The difference is attributable to the fact that the CER performance target in Section E.4.1 is about 3.5 times larger than the CER associated with the Ethernet PER requirement. In Section E.4.1 the CER target is $2 * 10^{-6}$, while in this section the Ethernet PER performance target equates to a CER performance of $0.6 * 10^{-6}$. That ratio of roughly 3.5 more bits *for the same confidence level*, derived by applying analysis to adjust (increase) the testing due to the dependent errors of the Ethernet packets, is *entirely* attributable to the performance targets of Ethernet PER (10^{-6}) and BER (10^{-8}) corresponding to different codeword error ratios. Properly accounting for error dependence in the Ethernet packets (shown in this section) would provide the same result for the number of transmitted bit errors as derived in testing the codeword errors, which are independent (previous section), if the CER target is the same in both cases. The reason the CER targets of the two sections are different is because Section E.4.1 *assumes* a BER requirement of 10^{-8} as a starting point in that section's analysis, and that does not correspond to the DOCSIS 3.1 OFDM Ethernet PER requirement, which is roughly 3.5 times more demanding, as we have just seen.

E.4.3 Relating BER, Ethernet PER, and FEC codeword CER in DOCSIS 3.1 OFDM

It is worth mention that FEC statistics are available via DOCSIS 3.1 proactive network maintenance (PNM) tools and are often used by cable technicians and network maintenance operations personnel, rather than the Ethernet PER. A CER of 10^{-6} is often used as the ad hoc performance requirement or goal, although the DOCSIS 3.1 downstream performance requirement is officially 10^{-6} for the Ethernet PER and is not in terms of the FEC CER. The BER is not available to technicians or network operations personnel in DOCSIS 3.1 OFDM at all.

If we had chosen a value of 140 bit errors per errored codeword in the example in Section E.4.1, then the CER corresponding to a BER of 10^{-8} would be 10^{-6}, instead of $2 * 10^{-6}$ which arose from 70 bit errors per errored codeword.

We saw in the Section E.4.2 that with full length codewords the CER is about 1/1.8 of the Ethernet PER. Though not discussed in this Appendix, for shorter FEC codewords the number of Ethernet packet errors to codeword errors reduces to 1, while the ratio of total codewords to total Ethernet packets increases from less than 1 to greater than 2 for increasingly shorter codewords. The result is that the Ethernet PER remains generally in the neighborhood of $2 *$ CER.

Thus, we have seen that whether the DOCSIS 3.1 OFDM downstream performance is targeted at 10^{-8} BER or 10^{-6} Ethernet PER (the actual DOCSIS requirement), the corresponding FEC performance is in the range of $2 * 10^{-6}$ to $(1/2) * 10^{-6}$, respectively. Also, in practice the CER versus SNR performance difference between CER of $(1/2) * 10^{-6}$ and $2 * 10^{-6}$ is practically negligible (in the neighborhood of 0.1 dB). This is why all three descriptions of the DOCSIS 3.1 target performance can be found in literature (10^{-8} BER, 10^{-6} Ethernet PER, and 10^{-6} FEC CER).

E.5 Analysis

This section includes calculation details of some of the computations in the previous section.

E.5.1 Exploring Confidence Level for tests with test limit of ZERO errors required to PASS

Recall:

In general, any confidence level can be selected for a qualifying test (upper bounded by 100%). With a BER requirement for no more than one bit error in X transmitted bits, on average, and a qualifying test requiring no bit errors to achieve a PASS, any confidence level less than 100% can be provided. To achieve a confidence level, Y, the number of transmitted bits, N, should be:

> $N = [-(\ln(1 - Y)) * X]$ number of transmitted bits; BER requirement is ≤ 1/X; confidence level Y; test limit to PASS is zero bit errors.

IID assumption, of course.

Let's derive the formula.

The binomial distribution provides

p = probability of single trial failure, e.g., probability of bit error
q = 1 – p = probability of trial success, e.g., probability of correct bit
Y = confidence level desired

[Note that in the formula in the previous pages, and copied above, X = 1/p since it was assumed there is 1 error in X trials on average.]

The binomial distribution has the probability of each number of bit errors, with N trials, given by expanding:

$$(q + px)^N = q^N + N * \left(q^{(N-1)}\right) * p * x + (N(N-1)/2) * \left(q^{(N-2)}\right) * (p^2) * x^2 + \cdots$$

The coefficient of x^n is the probability of n errors in the N trials.

Note that the "little" x in the binomial expansion giving the binomial PDF is NOT at all related to the uppercase X which was used as the reciprocal of the single-trial error probability.

The probability of ZERO errors is given by the first term:

Probability of zero errors $= q^N = (1 - p)^N$

For a confidence level of Y = 95%, we want 1 – Y = probability of ZERO errors.

This provides $1 - Y = (1 - p)^N$, and we have Y and p and want to solve for N.

Taking the natural log of both sides and multiplying through by –1: $-\ln(1 - Y) = N * ln(1/(1 - p))$

Rearranging to solve for N:

$$N = -\ln(1-Y) \ * \left[\frac{1}{\ln(1/(1-p))}\right]$$

Then factoring in a p/p term:

$$N = -\ln(1-Y) * (p/p) * \left[\frac{1}{\ln(1/(1-p))}\right] = -\ln(1-Y) * (1/p) * \left[\frac{p}{\ln(1/(1-p))}\right]$$

It will be shown that the term $\left[\frac{p}{\ln(1/(1-p))}\right]$ is less than 1, but approaches 1 as p gets small (as p approaches 0).

Thus, letting $N = -\ln(1-Y) * (1/p)$ provides that $N > -\ln(1-Y) * (1/p) * \left[\frac{p}{ln(1/(1-p))}\right]$, so setting N equal to the less complicated formula will provide more trials than necessary, but *sufficient* to yield the desired confidence level. The fact that $\left[\frac{p}{\ln(1/(1-p))}\right]$ approaches 1 for small p provides that setting N equal to the less complicated formula will cause little additional testing.

It can be shown that the Taylor series expansion of the reciprocal of $\left[\frac{p}{\ln(1/(1-p))}\right]$, is

$$(1/p) * ln(1/(1-p)) = 1 + p/2 + (p^2)/3 + (p^3)/4 + (p^4)/5 + \cdots > 1$$

For p = 0.01 or less, the reduced term is within 1 percent of unity (well approximated by 1 + p/2, so of course less than 1 + p), and calculating N using the simplified formula yields less than 1 percent additional testing compared to an exact computation of the number of trials for the Y confidence level.

So, for confidence level Y, and probability of single-trial error of p, the number of trials, N, when PASS requires ZERO errors, is bounded by:

$$N = -\ln(1-Y) * (1/p)$$

E.5.2 Binomial Distribution basics

With IID single-trial errors, the mean number of errors in N trials is Np.
The standard deviation of the number of errors is

$$\sqrt{(Npq)} = \sqrt{\left(Np(1-p)\right)}$$

For small values of p, such as 10^{-4} and lower, $\sqrt{\left(Np(1-p)\right)} \cong \sqrt{(Np)}$, is within 1 percent.

E.5.3 Gray coded SC-QAM

With Gray coding and square SC-QAM constellations, each rail contains half the bits of the SC-QAM symbol, and in AWGN each rail (I and Q) has independent errors.

With Gray coding, when there is a rail error, and the symbol error probability is small (less than 1 percent for example), the probability of more than one bit error when there is a rail error is much smaller than if the bit errors were independent. The Gray coding provides that crossing a single decision boundary will always result in only one bit error, and crossing two symbol boundaries is much less than crossing a single boundary (three times the distance for the noise to cover, or about 9.5 dB higher "effective" SNR).

With Gray coded 256-QAM, there are four bits per rail. If the probability of a rail error is p_rail, the probability of symbol error is $1 - (1 - p\_rail)^2 = 2 * p\_rail - p\_rail^2$.

As long as the p_rail is less than 1 percent, the following **approximations** hold well:

The probability of MSB error is $p\_msb = (1/15) * p\_rail$.
The probability of the second MSB error is $p\_2sb = 2 * p\_msb = (2/15) * p\_rail$.
The probability of the third MSB error is $p\_3sb = 2 * p\_2sb = (4/15) * p\_rail$.
The probability of the LSB error is $p\_lsb = 2 * p\_3sb = (8/15) * p\_rail$.

The above are the probabilities of error when only one decision boundary is "crossed" by the noise.

For two bit errors to occur in a rail error with Gray coding requires the noise crossing two decision boundaries, as mentioned. For example, if the probability of a rail error is about 1 percent, 0.01, the probability of crossing two decision boundaries is less than 10^{-14}.

When the probability of a rail error is about 10 percent, the probability of crossing three decision boundaries is about 10^{-6}.

The probability of noise crossing more than one decision boundary is far less than the square of the probability of crossing one boundary. This is important because of the independence property.

For example, for independence of bit errors in the LSB and 3sb requires $p\_lsb * p\_3sb$ = probability of both bits in error. The left-hand side is $(32/225) * p\_rail^2$, and the right-hand side is $\ll p\_rail^2$. Thus, we know that the two bit positions do not have independent error events. In this case of dependence, or correlated bit errors, we have the situation where having one bit in error means the proximity bits are NOT in error. We will look at other cases where a bit error occurrence means other bit errors are more likely, not less likely as in this Gray coded situation.

Accepting these probabilities above, let's examine the confidence level for BER testing with Gray coded 256-QAM.

First, let's examine the probability of a bit error, not just the bit error of a specific bit in the constellation rail. Each rail trial contains four bits, and we know that with low error probabilities (less than 1 percent), there is at most one bit error of the four due to the Gray coding (for all practical purposes). For each rail error there is one bit error, and four bits total; thus $p\_bit = (1/4) * p\_rail$. This is a key result in examining the confidence level of the testing.

We see that the If we want to have a confidence level of 95% and a test limit of ZERO errors required for PASS, the rail errors are independent and we construct the test to see how many “rail trials” need to be transmitted.

If the spec is for p_bit = 10^{-8}, we have seen that N = $3 * 10^8$ bits with IID bits.

With 256-QAM Gray coding and p_bit = 10^{-8} as the spec, that means the spec corresponds to p_rail = $4 * 10^{-8}$. The 95% confidence level with ZERO error test limit to PASS means we need N = $3/(4 * 10^{-8})$ transmitted rails which means N = $0.75 * 10^8$ rails transmitted. But this equates to the N = $3 * 10^8$ transmitted bits which is needed for the 95% confidence level with IID bit errors.

So, in THIS case, with Gray coding, the confidence level for the BER testing, with ZERO errors required to PASS, results in the same number of transmitted bits as with IID bit errors, even though the bit errors are not independent, and very decidedly so.

The same thing happens for BER testing with Gray coded SC-QAM for PASS test limits which are non-zero. This isn't a proof, but that this occurs with Gray coding can be justified by examining the mean and standard deviation of the testing in terms of bits and also for number of rails. With N rails transmitted, the average number of rails received in error will be N ∗ p_rail, and the standard deviation is

$$\sqrt{[N * p\_rail * (1 - p\_rail)]}$$

Looking at the testing in terms of the bits, there are 4N bits transmitted, and the probability of bit error is (1/4)p_rail. Thus, the mean for the number of bit errors is

$$(4N) * (p\_bit) = (4N) * (p\_rail/4) = N * p\_rail$$

The mean number of errors is the same for the test with Gray coded SC-QAM as with a bit stream which was IID with the same bit error probability, which of course we knew it had to be.

Examining the standard deviation for the number of errors is slightly different, though!

The standard deviation of the number of rails in error in the Gray coded SC-QAM case is given as $\sqrt{[N * p\_rail * (1 - p\_rail)]}$ (as above). If we look at a bit stream with IID with the same mean error with the Gray coding, we see that the standard deviation is

$$\sqrt{\left[4N * \left(\frac{p\_rail}{4}\right) * \left(1 - \frac{p\_rail}{4}\right)\right]}$$

This simplifies to

$$\sqrt{\left[N * p\_rail * \left(1 - \frac{p\_rail}{4}\right)\right]}$$

This is NOT the same as with the SC-QAM Gray coded standard deviation!

However, for even modestly low probability of bit error or rail error, the terms are only negligibly different, since the difference involves factors which are so close to 1.

The cases of a) repeated errors, as opposed to b) the above case where an error means adjacent bits are not in error, behave very differently in regard to the standard deviation of the number of bit errors in N transmitted bits:

a) with one error indicating the adjacent bits are not in error (as in Gray coded SC-QAM), the standard deviation of the number of bit errors in N transmitted bits is similar to when the bit errors are IID, although they are not IID;

b) when errors occur in pairs (or larger multiples), we will see in the next section the standard deviation of the number of bit errors in N transmitted bits is much larger than when the N bits are IID. This is explored in the following sections.

E.5.4 Correlated error events

We have examined testing and confidence level with the IID assumption, and we have examined Gray coded constellations and seen bit errors that are not IID and examined the impact on confidence testing. It is instructive to examine a case which is superficially constructed but provides insight into relevant examples.

A useful construction is to consider trials which always consist of a pair of transmitted bits, and both bits are received correctly or both bits are received in error. Let D = number of trials, and they are IID. Let p_t be the probability of a trial error, and $q_t = 1 – p_t$ is the probability of a trial being correct. We know that in D trials the average number of errors will be $D * p_t$, and the standard deviation of the number of errors in D trials will be the square root of $D * p_t * q_t$. We have seen for a 95% confidence level test, with the PASS criterion being zero errors, we solve $0.05 = (1 – p_t)^D$, for D, and that we find $D = 3 * (p_t)$.

Now, let's examine this test, with significant and straightforward correlated bit errors, and see the impact of the correlation on the confidence level in terms of testing the bit errors.

First, we can that in the case of the "zero errors for a PASS" test design, the same testing when looking at the independent trials D applies when counting the number of transmitted bits and number of bit errors, because "0 errors" is the same whether looking at the trials or counting bits. So, to obtain the same 95% confidence level with zero bit errors as the PASS, the number of transmitted bits has to be $2 * (3 * p)$. This is twice the number of bits that have to be transmitted if the bit errors themselves are IID; the impact of the correlated bit errors on the required number of bits in the testing is not surprising, but it is good to justify it rigorously.

The impact of the correlation of bit errors in this example can be formalized rigorously for tests other than those with "0 to PASS" criterion. With N being the number of transmitted bits, we see $N = 2 * D$. It is straightforward to show that the probability of bit error, p, is given by $p = p_t$. If d is the number of trials of D that are errored, this means that the number of bit errors, b, is given by $b = 2 * d$. The average value of b is 2 times the average value of d which is $2 * D * p_t$ but this is also $N * p$. This is the same formula for the average of b as if it were binomially distributed and IID, but it is not.

The probability of the values of d, from 0 to D, is given by the binomial distribution, and these binomial probabilities align to the values of b, where $b = 2 * d$, for b values of 0, 2, 4, … , $2 * D$. From this it is straightforward that the expected value of b^2 is 4 times the expected value of d^2. The standard deviation of b is expected_value{ b^2 } minus (expected_value{ b })^2, which we see is 4 times expected_value{ d^2}

minus 4 times (expected_value{ d })^2. Therefore, the variance of b is 4 times the variance of d. Finally, we see that the standard deviation of b is 2 times the standard deviation of d. None of this is a surprise.

Collecting the results above:

The average value of $b = 2 * D * p_t = N * p$

The standard deviation of b

$$= 2 * standard\ deviation\ of\ d$$

$$= 2 * \sqrt{(D * p_t * q_t)}$$

$$= 2 * \sqrt{\left((N/2) * p * q\right)}$$

$$= \left(2/\sqrt{(2)}\right) * \sqrt{(N * p * q)}$$

$$= \sqrt{(2)} * \left(\sqrt{(N * p * q)}\right)$$

Perhaps this is not a startling result, either. It is good if this is intuitively pleasing or even obvious, but it is worth explicitly pointing out that the average of the number of bit errors in N transmitted bits is the same as given by the formula N ∗ p, which is the same as the formula that would apply if b were an IID binomial distributed random variable, but it is not IID. So, this is interesting, as noted in the previous paragraph. But even more importantly, we see that the standard deviation of b is $\sqrt{2}$ times the standard deviation which would occur if b were binomially distributed with the same number of transmitted bits, N, and same bit error probability, p. Thus, while not surprising, we definitively see that the standard deviation of b is larger in this construction than if it were IID and binomially distributed with the same parameters N and p. The correlation of the bit errors has led to the standard deviation of the number of bit errors in N transmitted bits to be larger than when the bit errors are IID, although the expected number of bit errors is the same.

Heuristically, the number of bits is N in both cases, but in the case with the pairs of errors (the case with the correlated errors) there is only half as many independent trials which have occurred. Thus, *the case with IID has had more independent trials, and the resulting number of bit errors tends more strongly to the mean (i.e., smaller standard deviation) than the case with correlated errors.*

Because the standard deviation of the number of bit errors is larger with the correlation than it is with IID, the number of bits that must be transmitted to achieve a given confidence level is larger. With the same average value of bit errors in a trial of N bits, but larger standard deviation, the number of bit errors falling below the test threshold and into the PASS region (low number of errors) is a higher probability with these correlated bit errors than it is with IID bit errors, with the same parameters N and p. *That is the take-away: Correlated bit errors imply larger number of transmitted bits to achieve the same confidence level as with IID bit errors*.

In the example above, with two bits per trial, both correct or both in error, we saw that the standard deviation of the number of bit errors in N transmitted bits is the square root of 2 times the standard deviation for the IID case. It can be readily shown that if each trial involves the repetition of R bits per trial, all in error or all correct, then the standard deviation of the number of bit errors in N transmitted bits is the square root of R times the standard deviation with IID errors. Noting that the standard deviation in the IID case is proportional to the square root of N, it is not surprising *it can be shown* that *with the R repetitions of bit errors or correct bits, it is required to increase the number of transmitted bits by a factor of R to achieve a comparable confidence level test as with IID bit errors*.

A real example from communications practice is instructive, which is similar to the previous (and somewhat artificial) example of multiple bits per trial; in that previous example, all of the bits in a single trial are correct or all bits are in error in each trial result. This example occurs with the real-world case of differentially demodulated data, where the carrier and phase reference for a BPSK bit decision is the previously received bit. For example, a phase shift of 0 degrees from the previous bit could be decided as “0” and a phase shift of 180 degrees could be decided as “1”. It can be shown that demodulated bit errors occur in pairs in this scheme. Consider each bit interval as a “symbol,” and two such “symbols” in succession are processed to make a bit decision of one bit; but each subsequent new “symbol” received is processed with the preceding “symbol” and produces another bit decision.

In this demodulation, an error in a single symbol will result in two demodulated bit errors: 1) the bit determined when the symbol is received, and 2) the bit determined when the next symbol is received, and the errored symbol is used as the reference. It is a fair assumption to say that the “symbols” are each an independent trial. Unlike the previous example with a pair of bits for each trial, there is a one-to-one alignment of the “trials” or “symbols” and the number of bits, but a single trial error will result in a pair of bit errors.

Complicating matters, two consecutive symbol errors will result in an error pattern of {Bit Error, Correct Bit, Bit Error}. The pattern continues, with consecutive symbol errors yielding only a pair of bit errors, corresponding to the first symbol error of the string, and the bit following the last symbol error.

Working through some algebra for the expression for the variance of the number bit errors with N bits transmitted provides the following:

p_s = probability of symbol error
q_s = probability symbol is correct = 1 – p_s
p = probability of bit error = 2 ∗ p_s ∗ q_s
b = number of bit errors in N transmitted bits
expected value of b = E{ b } = Np

The variance of b yields a complicated expression:

$$Var(b) = 2Np[1 - 1/(2N)]\left[1 - (3/2)p\,\frac{[1 - ((2)/(3N))]}{[1 - 1/(2N)]}\right]$$

A little bit of simplification can be provided by collecting the term:

$$Factor = \frac{[1 - (2/(3N))]}{[1 - (1/(2N))]}$$

Note that Factor approaches unity as N increases.

So that

$$Var(b) = 2Np[1 - 1/(2N)][1 - (3/2)p * Factor]$$

For small values of the differentially demodulated bit error, such as less than 10^{-2}, and number of trials large, such as greater than 100, we see

$$Var(b) \cong 2Np$$

Recall that for IID bits with probability of correct = 1 – p = q, we had the variance of the number of bit errors in N bits transmitted given by $N * p * q$, so that we can write the variance for the number of bit errors in the differentially demodulated stream in terms of the expression for the variance under IID conditions of the bit errors:

$$Var(b) \cong 2 * Variance\ of\ bit\ errors\ with\ N\ bits\ transmitted * \left(\frac{1}{q}\right)$$

Since q is close to 1 under the assumption of p being less than 0.01, we see that the standard deviation of the number of bit errors in the differentially demodulated stream tends to the square root of 2 times the variance which would occur for the N trials with probability of bit error, p, if those bit errors were IID. This is the same result for the standard deviation of the number of errors, in the limit as N increases, and the error probability of a single trial is small, as we saw in the previous example of two bits per trial, both in error or both correct, with the square root of 2 multiplying the binomial IID variance. The square root of 2 increase in the standard deviation of the number of bit errors in N transmitted bits in this example translates into requiring a doubling of the number of transmitted bits to achieve a comparable confidence level test as with IID bit errors.

References

Atul Shah and Thomas Kolze, “Evaluating Bit Error Rate Test Confidence Level for Systems with Bit Error Dependence,” AIAA 14th International Communication Satellite Systems Conference and Exhibit, 22 March 1992 - 26 March 1992, Washington, DC, U.S.A.

Jeruchim, Michel C., “Techniques for Estimating the Bit Error Rate in the Simulation of Digital Communications Systems,” IEEE Journal on Selected Areas in Communications, Vol. SAC-2, No. 1, January 1984.

PHYv3.0 – Data-Over-Cable Service Interface Specifications, DOCSIS® 3.0 Physical Layer Specification

PHYv3.1 – Data-Over-Cable Service Interface Specifications, DOCSIS® 3.1 Physical Layer Specification

Appendix F Analog Optical Links

The majority of optical links deployed in HFC networks use linear fiber optic signal transmission, based upon analog intensity modulation. Many in the field use the term "AM fiber link" or similar. This section includes a brief overview of analog intensity modulation, written by Lamar West. For additional information about linear fiber optical signal transmission, see Chapter 12 of [7] or Chapter 4 of [4].

F.1 Analog intensity modulation

Analog Intensity Modulation

Lamar West, Ph.D.
LEW Consulting, LLC

Analog intensity modulation is accomplished by varying the intensity of the light in proportion to an analog electrical signal. Consider an electrical signal consisting of an RF carrier amplitude modulated by a sine wave. This is shown in the time domain in Figure 94.

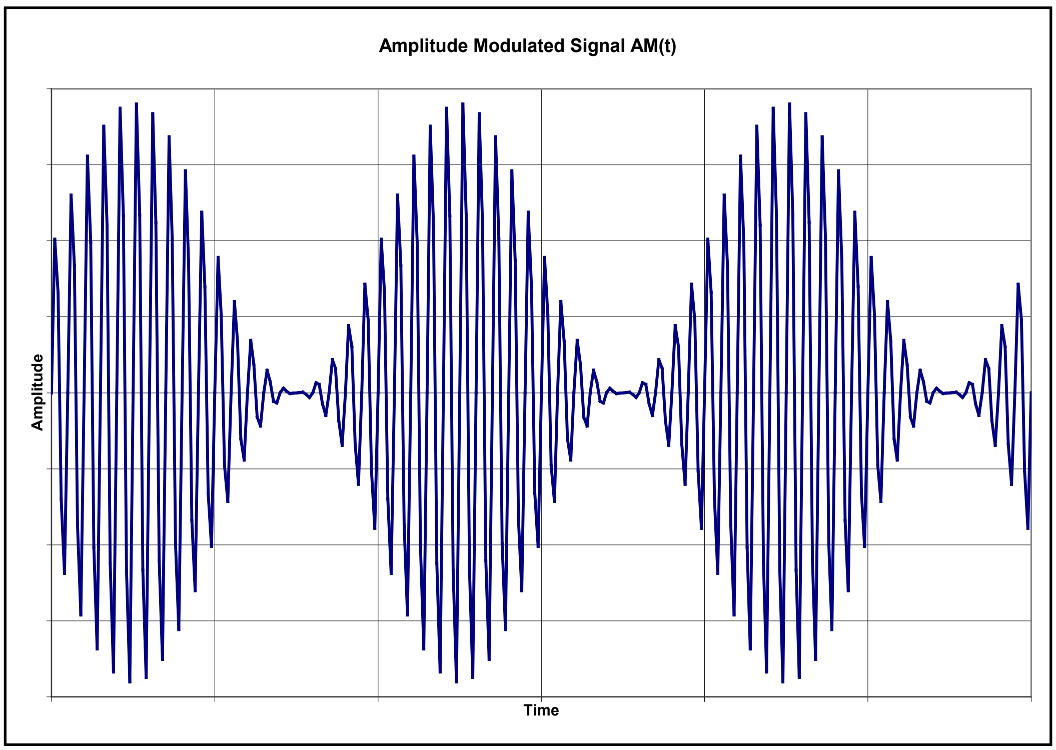


Figure 94. Electrical signal in the time domain comprising an RF carrier amplitude modulated by a sine wave.

If we then use this electrical signal to analog intensity modulate an optical source the result would be as shown in Figure 95.

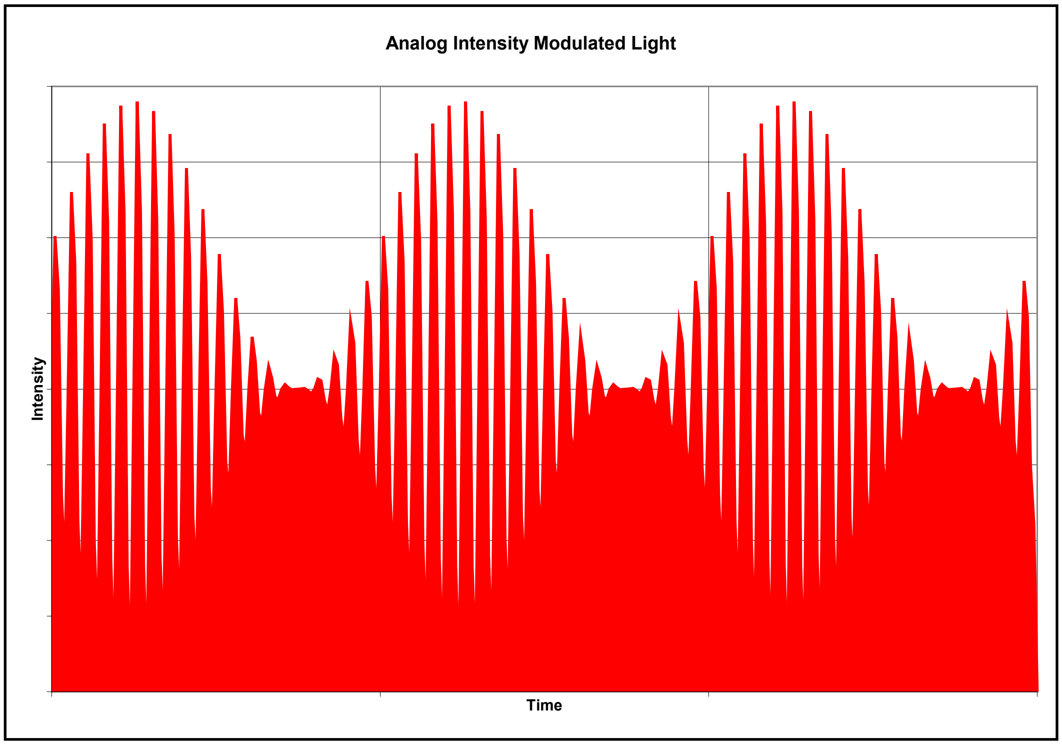


Figure 95. Analog intensity modulated optical source.

An optical detector such as a photodiode can be used to recover the original RF signal at the output of the optical link. The recovered RF signal is directly compatible with conventional RF coaxial networks. However, highly linear optical sources and detectors are required for high quality analog intensity modulated optical links.

The signal amplitude of the electrical signal at the output of an analog intensity modulated optical link depends on the amplitude of the optical signal at the input to the optical receiver. The level of the optical signal at the input to the optical receiver in turn depends on the loss encountered in the optical link. As the length of the fiber increases, the amplitude of the electrical output of the receiver decreases. A 1 dB change in optical loss will result in a 2 dB change in electrical output.

The laser drive current used for analog intensity modulation consists of the sum of an AC signal current and a DC bias current, I_B. The bias current is greater than the threshold current, I_{TH}, and is chosen to produce the desired average optical power level, APL. Analog intensity modulation is illustrated in Figure 96.

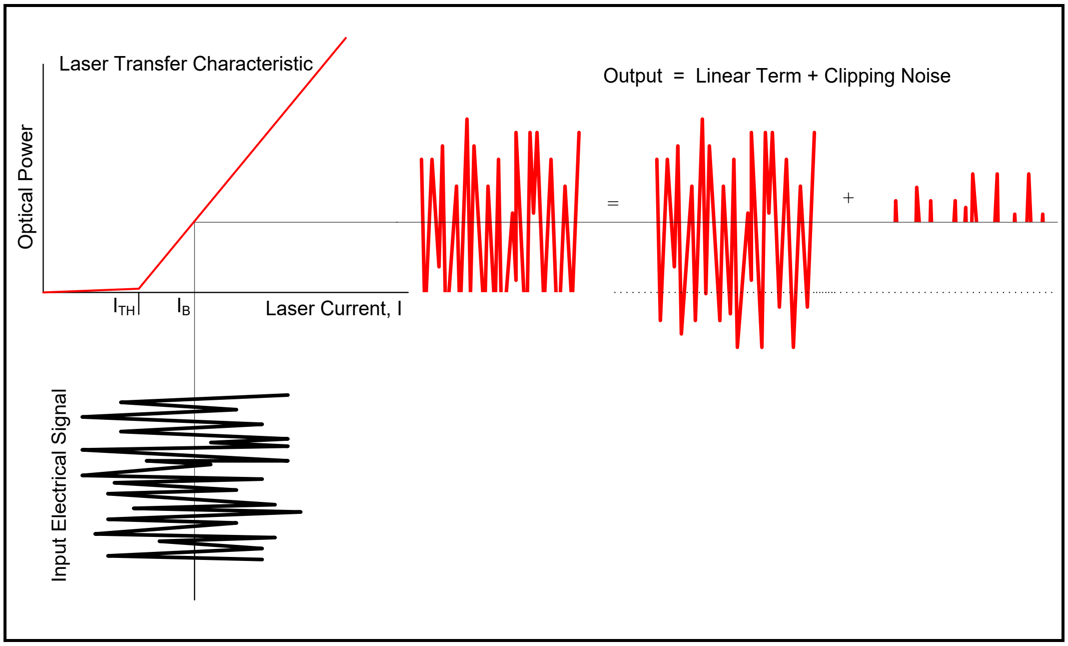


Figure 96. Analog intensity modulation.

There is a limit to the amplitude of the input signal current. If the peak value of the input current is greater than the difference between the bias current and the threshold current the optical output power will drop to zero. The negative peaks of the input signal will be clipped off in this case. This type of distortion is known as *laser clipping*. When laser clipping occurs it is often useful to think of the output as consisting of the desired, undistorted signal plus a clipping noise term that just cancels out the peaks of the undistorted signal.

It is desirable to operate the input electrical signal at high amplitude in order to maximize carrier-to-noise ratios at the output of the fiber-optic link. However, the amplitude must be limited so that any clipping noise produced is of sufficiently small amplitude that it will not significantly deteriorate the quality of the output signals. Laser drive level is of critical importance in obtaining maximum link performance.

We define the maximum allowable peak current that will not result in clipping, I_0,

$$I_0 = \left(I_B - I_{TH} \right)$$

If the input signal consists of the sum of a group of N equal-amplitude RF carriers, such as one would encounter in the downstream of a cable network, each with a peak current of A, then we define the optical modulation index of each carrier, m, as

$$m = \frac{A}{I_0}$$

We also define an RMS modulation index, μ, for the composite signal as

$$\mu = \sqrt{\sum_{i=1}^{N} \frac{m^2}{2}} = m\sqrt{\frac{N}{2}}$$

There are minute, random fluctuations in the laser output optical power that are not directly related to the drive current. These result in fluctuations of the electrical power at the output of the optical link. These fluctuations are referred to as relative intensity noise or RIN. Typical vales for RIN in DFB laser links are –150 dB/Hz to –170 dB/Hz. This RIN is generally made worse by light that is reflected back into the laser output. RIN can be minimized by the inclusion of an optical isolator at the laser output.

Appendix G Derivation of Return Loss of Cascaded Systems

Return loss measurements are commonly performed on individual components or devices, such as nodes, amplifiers, splitters, directional couplers, taps, connectors, set-top boxes, and so on. But what is the return loss when components or devices are cascaded? Examples of the latter include back-to-back taps, or a line passive and tap, interconnected with a housing-to-housing adapter. The following analysis, authored by Richard S. Prodan, Ph.D., shows how to derive a formula that can be used to calculate the return loss of two cascaded components or devices. In the analysis, each component or device is referred to as a system.

Return Loss of Cascaded Systems

Richard S. Prodan

If SYSTEM 1 has RL1 dB return loss and SYSTEM 2 has RL2 dB return loss, then what is the overall return loss of the cascade of SYSTEM 1 and SYSTEM 2?

Assume the systems have no internal loss (which contributes to insertion loss).

The reflection coefficient $\Gamma = E_r/E_i = 10^{-RL/20}$, where E_r is reflected voltage and E_i is incident voltage.

Reflected power ratio for SYSTEM 1 is $\Gamma_1^2=10^{-RL1/10}$, and for SYSTEM 2 is $\Gamma_2^2=10^{-RL2/10}$.

Power transmitted ratio for SYSTEM 1 is $1 - \Gamma_1^2$, and for SYSTEM 2 is $1 - \Gamma_2^2$.

The first reflection back from each system will be the largest and subsequent reflections will be heavily attenuated, so we approximate by only considering the first reflections.

If we take A to be the power incident on SYSTEM 1:

first reflection from SYSTEM 1 is $\Gamma_1^2 * A$

incident power on SYSTEM 2 is $(1- \Gamma_1^2) * A$

reflected power from SYSTEM 2 is $\Gamma_2^2 * (1- \Gamma_1^2) * A$

reverse-transmission of SYSTEM 2 reflected power through SYSTEM 1 is $(1 - \Gamma_1^2) * \Gamma_2^2 * (1- \Gamma_1^2) * A$

total power reflected from both systems (first reflection only) is $\Gamma_1^2 * A + (1 - \Gamma_1^2) * \Gamma_2^2 * (1- \Gamma_1^2) * A$

return loss of total system:
$RL = -10 * \log_{10}[\Gamma_1^2 + (1 - \Gamma_1^2) * \Gamma_2^2 * (1- \Gamma_1^2)] = 10 * \log_{10}[\Gamma_1^2 + \Gamma_2^2 * (1 - \Gamma_1^2)^2]$

Assume the systems have insertion loss (which includes reflection and internal power losses). Then the transmitted power loss $1 - \Gamma_1^2$ due to refection only is replaced with the insertion loss $L_1 = 10^{-IL1/10}$, where IL1 is the insertion loss of SYSTEM 1 in dB. This yields the following result:

return loss of total cascaded system $RL = -10 * \log_{10}[\Gamma_1^2 + L_1 * \Gamma_2^2 * L_1] = -10 * \log_{10}[\Gamma_1^2 + \Gamma_2^2 * L_1^2]$

For example, a cascade of two taps back-to-back each with a return loss of 18 dB and insertion loss of 0.5 dB would have a cascaded return loss of: $RL = -10 * \log [10^{-18/10} + 10^{-18/10} * (10^{-0.5/10})^2] = 15.46$ dB

Appendix H Mathematics of Field Strength

The material in this section is adapted from an article by Ron Hranac about the mathematics behind signal leakage field strength measurements, in particular the origin of the “0.021” factor used when converting between microvolts and microvolts per meter. That article originally appeared in the July 1991 issue of *Communications Technology*, with an update published in June of 2008. Used with permission of the author.

My original 1991 article was inspired by a call from long-time industry friend and colleague Ralph Haimowitz, who commented that he had seen a recent paper discussing, among other things, the conversion between microvolts (µV) and microvolts per meter (µV/m). The paper used a somewhat different multiplier than the familiar “0.021” that’s in the formulas we still use today! Ralph’s concern was the effect the multiplier used in the paper would have on leakage measurement calculations performed by cable operators. So I sat down with a variety of references and did some number crunching related to the math of field strength measurements. Grab a cup of hot coffee and join me as we revisit where “0.021” comes from and why it is indeed “0.021.”

When we measure signal leakage, we’re dealing with a source of RF power (the leak), its propagation from the source to a location where the measurement is made, and determination of the radio frequency (RF) signal’s field strength at the point of measurement. The receive antenna is normally a half-wave dipole that is resonant at the frequency of interest. In fact, Federal Communications Commission (FCC) rules require that signal leakage measurements be made with a dipole, or at least correlated to a dipole measurement.

Most discussions of RF propagation and field strength measurements reference an isotropic radiator or antenna. This is a theoretical and ideal point source that radiates equally in all directions and has unity gain. Even though an isotropic antenna does not actually exist, it’s useful as a mathematical basis against which real-world comparisons can be made.

Imagine an isotropic antenna in free space, radiating a certain amount of RF power (P_t) uniformly in all directions. Now imagine a sphere whose surface is some distance from the point of radiation, with that point (the isotropic antenna) at the center of the sphere. A good analogy here is a very tiny but bright light bulb – an “isotropic radiator” – in the center of a large balloon, with the bulb illuminating the balloon from the inside. You could define the amount of power in a given area at the surface of the sphere (or balloon) – say, in a square meter – as so many watts per unit of area. Indeed, that’s exactly how RF power density is characterized.

In this example, the power density P_d (in watts per square meter) at the surface of the sphere is simply the total transmitted power P_t divided by the surface area of the sphere ($4\pi R^2$):

$$P_d = \frac{P_t}{4\pi R^2} \qquad \text{(Eq 1)}$$

The level of the radiated RF power also can be expressed in volts per meter (V/m). If the field strength on our imaginary sphere has an intensity of E in V/m, then the power density is:

$$P_d = \frac{E^2}{120\pi} \qquad \text{(Eq 2)}$$

You'll notice that Equation 2 is an expression of the basic power equation with Ohm's Law equivalents substituted for E and I, or $P = E^2/R$. In this case, 120π is the "resistance," or impedance of free space (approximately 377 ohms). In order for power density to be useful, though, it has to be converted to received power. To do that, multiply power density in watts per square meter by the area in square meters of the receiving antenna. Keep in mind there is little relationship between the actual size of a dipole antenna and its *effective* area. If you know the linear ("numerical") gain (G) of an antenna you can calculate its effective area (A_e) with the formula:

$$A_e = \frac{G\lambda^2}{4\pi} \tag{Eq 3}$$

For a half-wave dipole, the effective area is:

$$A_e = \frac{1.64\lambda^2}{4\pi} \tag{Eq 4}$$

Note that in Equation 4, 1.64 is the linear gain of a half-wave dipole in free space. Its gain in decibels relative to an isotropic antenna is $10\log_{10}(1.64) = 2.15$ dBi.

As mentioned previously, the power intercepted by a receiving antenna is found by multiplying the antenna's effective area by the power density at the point of measurement:

$$P_r = A_e P_d \tag{Eq 5}$$

Substituting Equations 2 and 4 for P_d and A_e in Equation 5:

$$P_r = \frac{1.64\lambda^2 E^2}{480\pi^2} \tag{Eq 6}$$

We can now calculate the received voltage at the terminals of a half-wave dipole antenna using the power equation variation of Ohm's Law:

$$E_r = \sqrt{(P_r Z)} \tag{Eq 7}$$

where P_r is the received power (from Equation 6) and Z is the impedance of a half-wave dipole in free space. The latter is $Z = 73.13 + j42.5$ ohms for an infinitely thin dipole in free space, but we'll assume no reactance and call the dipole's impedance approximately 73.2 ohms. Substituting Equation 6 for P_r and 73.2 for Z, Equation 7 becomes:

$$E_r = \sqrt{\left(\frac{1.64\lambda^2 E^2}{480\pi^2}\right)73.2} \tag{Eq 8}$$

The free space wavelength of an RF signal can be found with the formula:

$$\lambda = \frac{c}{f_{Hz}} \tag{Eq 9}$$

where c is the speed of light (299,792,458 meters per second) and f_{Hz} is the frequency in hertz. When dealing with frequencies in megahertz the formula becomes:

$$\lambda = \frac{299.792458}{f_{MHz}} \quad \text{(Eq 10)}$$

Substituting Equation 10 for λ in Equation 8, a little algebra will reduce Equation 8 to a simpler form:

$$E_r = \frac{47.72299333E}{f_{MHz}} \quad \text{(Eq 11)}$$

With Equation 11 we can calculate the received voltage (in volts) from a half-wave dipole in free space when we know the field strength in V/m (*E*) and the frequency in MHz (f_{MHz}). If you want to change units so that the received voltage is in µV and the field strength is in µV/m, the same equation is used. Just make sure you use the same units; don't mix units – for example volts for E_r and µV/m for *E*.

To convert a received voltage in µV to dBmV, use:

$$dBmV = 20log_{10}\left(\frac{E_r}{1000}\right) \quad \text{(Eq 12)}$$

This can be used to modify Equation 11:

$$E_r = 20log_{10}\left(\frac{0.04772299333E}{f_{MHz}}\right) \quad \text{(Eq 13)}$$

E_r is the received voltage in µV from the dipole, and *E* is the field strength in µV/m. Going the other way, you can determine the field strength in µV/m if you know the frequency in MHz and the dipole's received signal level in dBmV:

$$\mu V/m = 20.9543 * f_{MHz} * 10^{(dBmV/20)} \quad \text{(Eq 14)}$$

Here 20.9543 is the reciprocal of 0.04772299333 from Equation 13. If you divide by 1,000, you'll have the familiar 0.021 (actually 0.0209543) multiplication factor used in the more common conversion from µV to µV/m:

$$\mu V/m = \mu V * 0.021 * f_{MHz} \quad \text{(Eq 15)}$$

dBmV can be converted to microvolts with the formula:

$$\mu V = 10^{(dBmV/20)} * 1000 \quad \text{(Eq 16)}$$

Thus, when you know the received signal level in dBmV, the conversion to µV/m becomes a slight variation of Equation 14:

$$\mu V/m = 10^{(dBmV/20)} * 1000 * 0.021 * f_{MHz} \quad \text{(Eq 17)}$$

Now let's put this to work. Assume that you've measured a leaking continuous wave (CW) carrier on CTA Ch. 14's visual carrier frequency (121.2625 MHz) with a half-wave dipole held 3 meters (approx. 10 feet) from the plant, and the leak's level at the dipole terminals is –44.6 dBmV. What's the field strength in µV/m? Use Equation 17:

$$\mu V/m = 10^{(-44.6/20)} * 1000 * 0.021 * 121.2625$$

$$\mu V/m = 10^{(-2.23)} * 1000 * 0.021 * 121.2625$$

$$\mu V/m = 0.01 * 1000 * 0.021 * 121.2625$$

$$\mu V/m = 14.99$$

The leak's field strength is about 15 µV/m, which is below the FCC's 20 µV/m limit. Technically it's legal, but good engineering practice says if you find a leak, fix it. After all, where there's leakage, there's most likely ingress, too!

Appendix I Total Power and Power Spectral Density

The material in this section is adapted from an article by Ron Hranac, that originally appeared in the Summer 2019 issue of *Broadband Library*. Used with permission of the author and the publisher.

Two RF power-related parameters that can cause confusion are *total power* (also called total composite power) and *power spectral density* (PSD). Grab a cup of coffee and a scientific calculator. We're going to look at these two parameters a little more closely.

Quick side note: When we measure RF signal level, we are measuring RF power. You might wonder why decibel millivolt (dBmV) is used instead of watt (W) for RF power in cable networks. The first reason is the typical power levels we deal with are very small. For example, 0 dBmV is only 13.33 nanowatts (about 13 billionths of a watt!). Working in the world of the decibel (dB) makes dealing with very small and very large numbers much easier. The second reason we use dBmV is because that metric expresses power in terms of voltage. For more on the latter, see my Summer 2017 *Broadband Library* article "The Wise and Mighty Decibel," available on-line at https://broadbandlibrary.com/wise-and-mighty-decibel/

I.1 Total power

As I noted in the Summer 2017 article, "Total power is the combined power of all signals in a given frequency range – for instance, the downstream. It's of concern because excessive total power is what overdrives lasers, set-tops, modems, and other devices."

Calculating total power does require some number crunching, since you can't simply add the individual signal levels in dBmV to get total power. Consider the example in Figure 97, which shows a single RF signal whose power is +20 dBmV. For this and all subsequent examples, assume the impedance is 75 ohms.

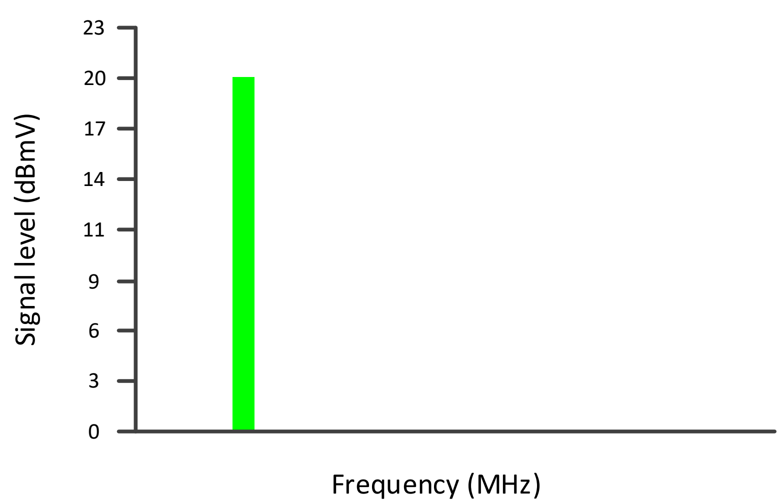


Figure 97. One RF signal whose power is +20 dBmV.

Converting +20 dBmV to power in watts is done as follows. First, convert dBmV to voltage (millivolts in this case):

mV = $10^{(dBmV/20)}$
mV = $10^{(20/20)}$
mV = 10^1
mV = 10 (which equals 0.010 volt)

Next, convert voltage to watts:
P = E^2/R
P = $(0.010 \text{ volt})^2/75$ ohms
P = 0.0001/75
P = 0.00000133 watt, or 1.33 microwatt (µW)

Since there is only one signal, the total power is +20 dBmV or 1.33 µW. What happens to the total power if the number of RF signals is increased to four, each at +20 dBmV? Refer to Figure 98.

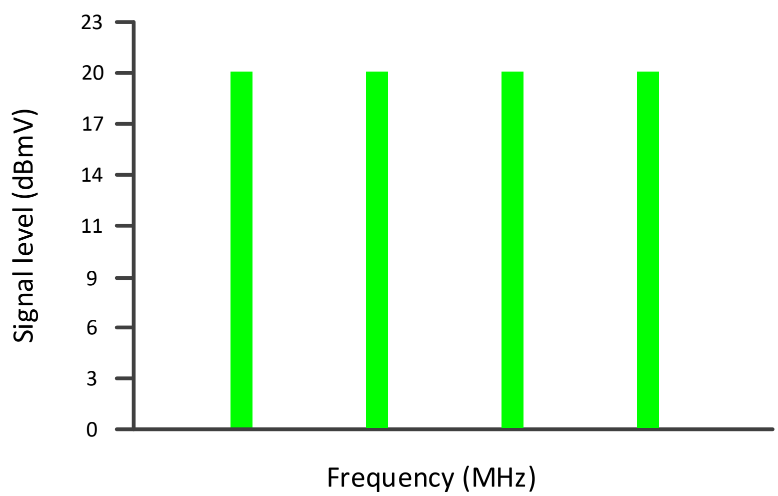


Figure 98. Four RF signals, each whose power is +20 dBmV. What is the total power?

There are a few ways to solve this. The first is to convert the signal level in dBmV to watts, add the watt values, then convert back to dBmV. Since +20 dBmV = 1.33 µW, then 1.33 µW + 1.33 µW + 1.33 µW + 1.33 µW = 5.33 µW (0.00000533 W) total power.

Next, convert watts to voltage:
$E^2 = PR$
$E^2 = 0.00000533 * 75$
$E^2 = 0.0004$
E = 0.02 volt, or 20 mV

Finally convert mV to dBmV:
dBmV = $20\log_{10}(\text{mV}/1 \text{ mV})$
dBmV = $20\log_{10}(20 \text{ mV}/1 \text{ mV})$
dBmV = $20 * [\log_{10}(20)]$
dBmV = 20 * [1.301]
dBmV = 26.02 dBmV

When all signals have identical power, the following formula can be used to calculate total power: $P_{total} = P_{one} + 10\log_{10}(N)$, where P_{total} is total power, P_{one} is the power of one signal, and N is the number of signals. For the previous example, P_{total} = 20 dBmV + $10\log_{10}(4)$ = 26.02 dBmV.

Had the four +20 dBmV values simply been added together as is, the resulting +80 dBmV would have been wrong. That's equal to 1.33 watts!

If the power of each channel is different, a more typical situation, total power is calculated with the formula $P_{total} = 10\log_{10}[10^{(P1/10)} + 10^{(P2/10)} + 10^{(P3/10)} + \ldots + 10^{(PN/10)}]$, where P_{total} is total power in dBmV, and P1, P2, P3…PN are the levels of each channel or signal in dBmV. If you use this formula for the previous example, you'll get +26.02 dBmV total power.

Ok, go refill that coffee cup. More number crunching is on the way.

I.2 Power spectral density

This parameter is a bit trickier to grasp. If you really like gnarly math, see the in-depth article on Wikipedia at https://en.wikipedia.org/wiki/Spectral_density. For the purposes of the discussion here, PSD "describes how power of a signal…is distributed over frequency." PSD is commonly expressed in power per hertz (Hz).

For the next example (see Figure 99), assume a 1.6 MHz-wide upstream single carrier quadrature amplitude modulation (SC-QAM) signal whose digital channel power is +65 dBmV.

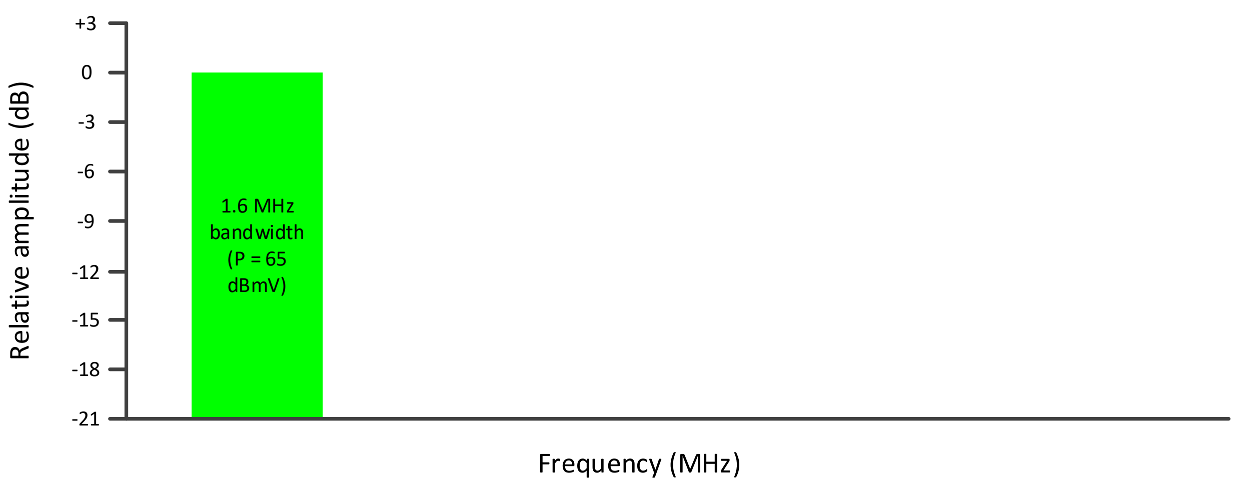


Figure 99. Upstream 1.6 MHz wide SC-QAM signal; P = +65 dBmV.

The PSD is:

PSD = 65 dBmV – $10\log_{10}$(1,600,000 Hz)

PSD = 65 – [10 ∗ $\log_{10}$(1,600,000)]

PSD = 65 – [10 ∗ (6.20)]

PSD = 65 – [62.04]

PSD = 2.96 dBmV/Hz

Now let's replace the 1.6 MHz wide signal with one that's 3.2 MHz wide, but with the same PSD as before (Figure 100). As viewed on a spectrum analyzer, the "haystack" also would be the same height as before. Because the width of the SC-QAM signal is doubled, so is its power. That means the digital channel power is now +68.01 dBmV, a 3.01 dB increase. What about the PSD?

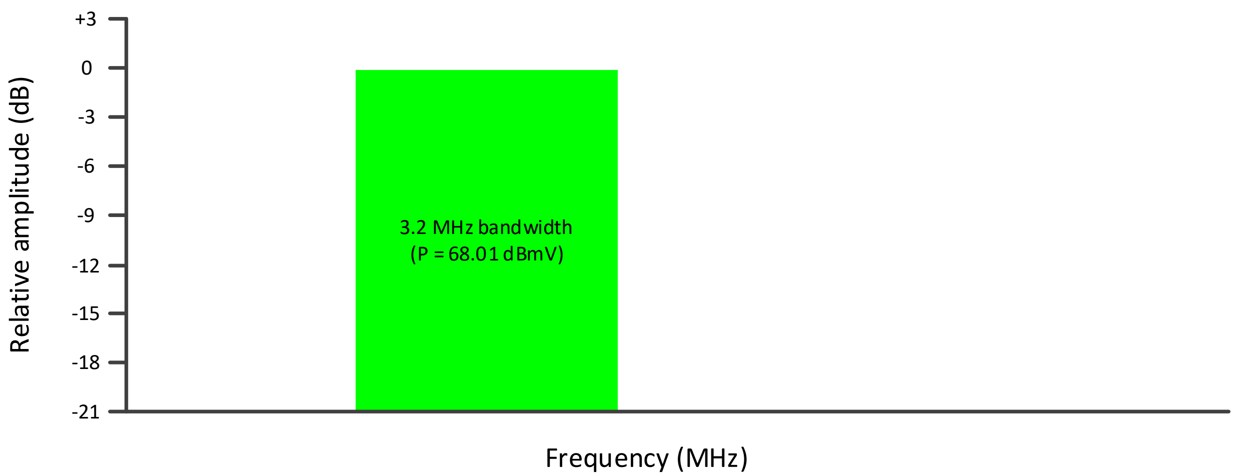


Figure 100. Upstream 3.2 MHz wide SC-QAM signal; P = +68.01 dBmV.

PSD = 68.01 dBmV – $10\log_{10}$(3,200,000 Hz)
PSD = 68.01 – [10 ∗ $\log_{10}$(3,200,000)]
PSD = 68.01 – [10 ∗ (6.51)]
PSD = 68.01 – [65.05]
PSD = 2.96 dBmV/Hz

Next, replace the 3.2 MHz wide SC-QAM signal with one whose bandwidth is 6.4 MHz. As before, we're going to maintain the same PSD (same "haystack" height as viewed on a spectrum analyzer), illustrated in Figure 101. Here, too, since the bandwidth of the SC-QAM signal doubled, so did the digital channel power, which is now +71.02 dBmV.

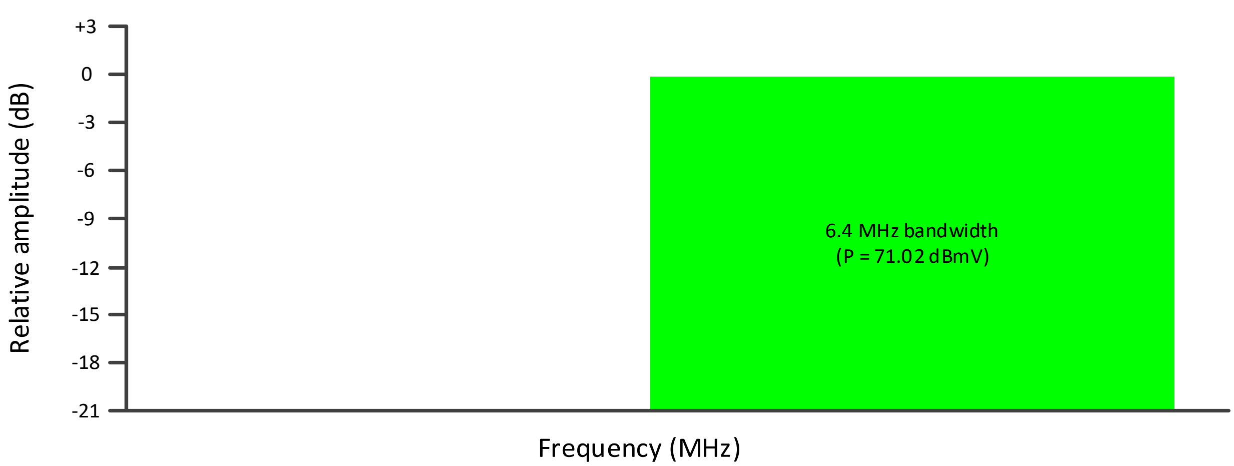


Figure 101 - Upstream 6.4 MHz wide SC-QAM signal; P = +71.02 dBmV.

PSD = 71.02 dBmV – $10\log_{10}$(6,400,000 Hz)

PSD = 71.02 – [10 ∗ $\log_{10}$(6,400,000)]
PSD = 71.02 – [10 ∗ (6.81)]
PSD = 71.02 – [68.06]
PSD = 2.96 dBmV/Hz

Showing all three SC-QAM signals together (Figure 102), we see that they have the same PSD of 2.96 dBmV/Hz and equal haystack heights. The total power is $P_{total} = 10\log_{10}[10^{(65/10)} + 10^{(68.01/10)} + 10^{(71.02/10)}]$ = 73.45 dBmV.

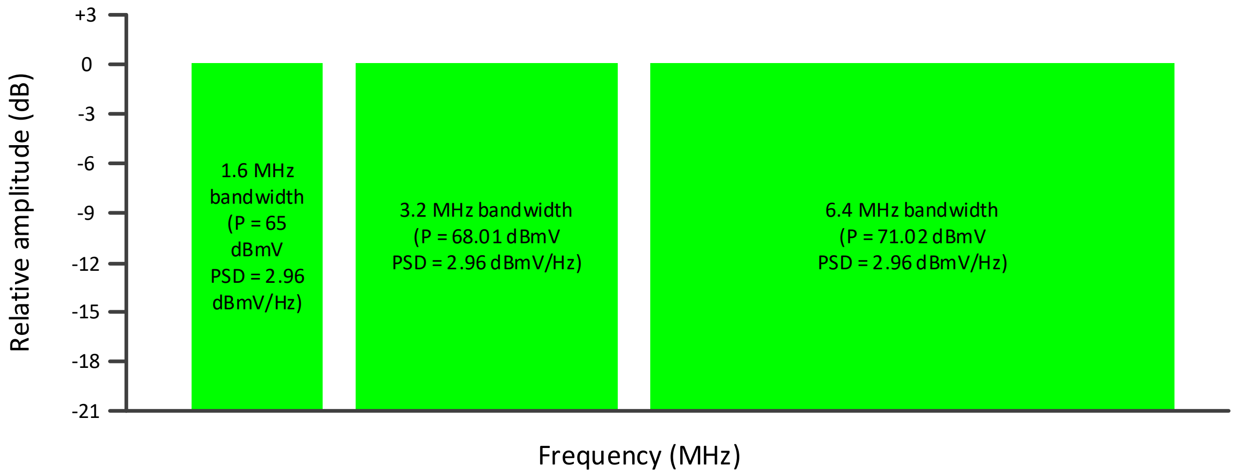


Figure 102 - All three upstream SC-QAM signals. PSD = 2.96 dBmV/Hz, and Ptotal = 73.45 dBmV.

I.3 Constant power per carrier versus constant PSD per carrier

Most cable modem terminations systems (CMTSs) and cable modems are configured for constant power per carrier in the upstream. In the next two examples, the total power of the four signals is the same (+57 dBmV). Figure 103 shows an example of constant power per carrier, with each SC-QAM signal's digital channel power equal to +50.98 dBmV. Note that the haystack heights are different for different bandwidth signals, even though each has the same digital channel power. The PSD, however, is different for each bandwidth channel: 6.4 MHz channel: –17.08 dBmV/Hz; 3.2 MHz channel: –14.07 dBmV/Hz; and the 1.6 MHz channel: –11.06 dBmV/Hz.

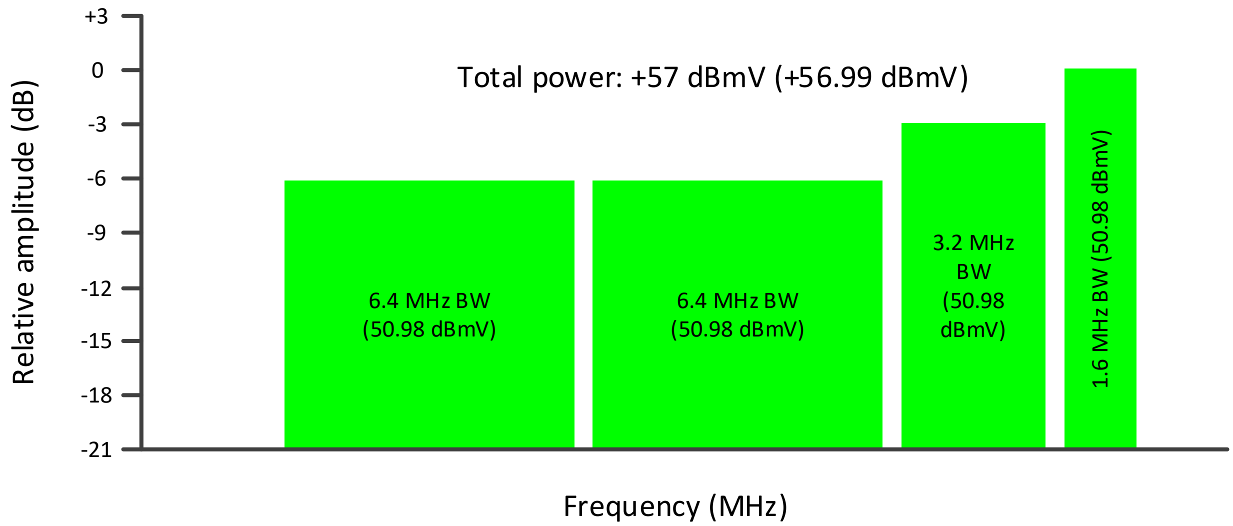


Figure 103 - Constant power per carrier. Ptotal = +57 dBmV.

If the four upstream SC-QAM signals were instead set to constant PSD per carrier (–15.46 dBmV/Hz in this example), the spectrum display would look like Figure 104. The haystack heights are the same, but the per-signal digital channel power is different!

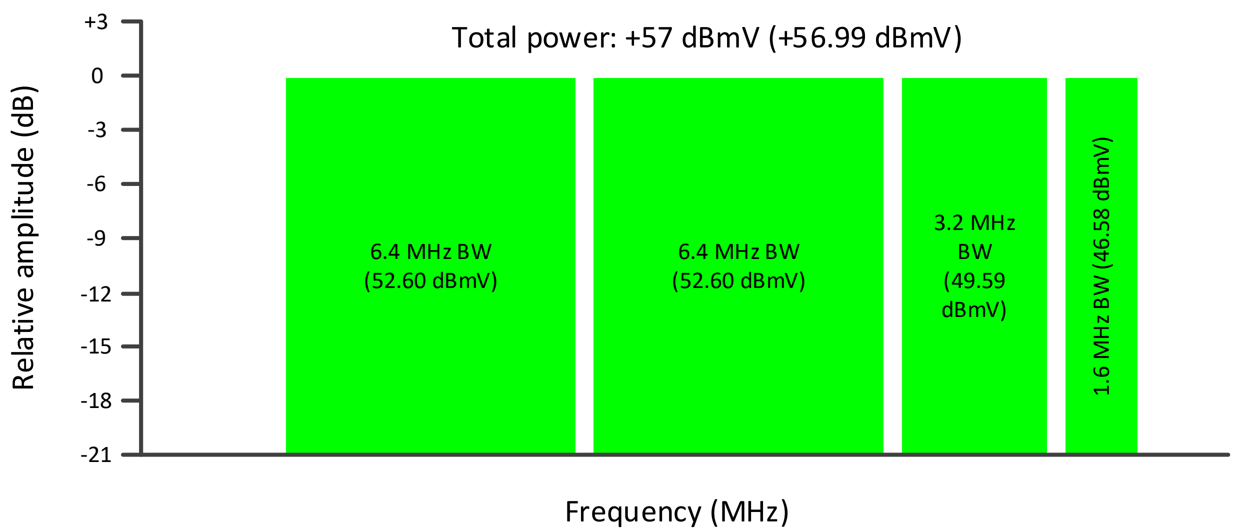


Figure 104 - Constant PSD per carrier; Ptotal = +57 dBmV.

I don't know about you, but my coffee just ran out. That's enough math for now. Class dismissed!

Appendix J Derivation of Node and Amplifier Equalizer Total Composite Power Formulas

The following is a mathematical derivation of the node and amplifier equalizer total composite power (TCP) formulas used in Section 30.14, courtesy of Richard S. Prodan, Ph.D. Used with permission.

Linear Equalizer:

$P_{dB}(f) = mf + b$ (dBmV/MHz) ; $m \equiv$ slope, $b \equiv$ intercept

The total composite power TCP in decibel millivolt (dBmV) from the equalizer start frequency f_{start} to the equalizer stop frequency f_{stop} is given by the logarithm of the integral of $P_{dB}(f)$ as:

$$TCP = 10\ log_{10}\left(\int_{f_{start}}^{f_{stop}} 10^{(mf+b)/10}\, df\right)$$

Integration by variable substitution:

$\int e^u\, du = e^u$ and $e^{\ln(10)} = 10$, so

$$\int 10^{(mf+b)/10}\, df = \int e^{\ln(10)(mf+b)/10}\, df$$

Let $u = \ln(10)(mf+b)/10$

$du = \frac{m}{10} ln(10)\, df$

$$\int e^{\ln(10)(mf+b)/10}\, df = \int e^u\, du \Big/ \left(\frac{m}{10} ln(10)\right) = \frac{e^u}{\left(\frac{m}{10} ln(10)\right)} = \frac{e^{\ln(10)(mf+b)/10}}{\left(\frac{m}{10} ln(10)\right)} = \frac{10^{(mf+b)/10}}{\left(\frac{m}{10} ln(10)\right)}$$

Therefore

$$\int 10^{(mf+b)/10}\, df = \frac{10^{(mf+b)/10}}{\left(\frac{m}{10} ln(10)\right)}$$

Cable Equalizer:

$$P_{dB}(f) = Equalizer\ Tilt\left(1 - \frac{\sqrt{f_{stop}} - \sqrt{f}}{\sqrt{f_{stop}} - \sqrt{f_{start}}}\right) + Transmit\ Level\ @\ Equalizer\ Start\ Frequency - 10\, log_{10}(6)\ \left(\frac{dBmV}{MHz}\right)$$

$$= Equalizer\ Tilt + Transmit\ Level\ @\ Equalizer\ Start\ Frequency - \frac{Equalizer\ Tilt\ \sqrt{f_{stop}}}{\sqrt{f_{stop}} - \sqrt{f_{start}}} + \frac{Equalizer\ Tilt}{\sqrt{f_{stop}} - \sqrt{f_{start}}}\sqrt{f}$$

where

$$m = \frac{Equalizer\ Tilt}{\sqrt{f_{stop}} - \sqrt{f_{start}}} \equiv\ slope$$

$$b = Equalizer\ Tilt + Transmit\ Level\ @\ Equalizer\ Start\ Frequency - \frac{Equalizer\ Tilt\ \sqrt{f_{stop}}}{\sqrt{f_{stop}} - \sqrt{f_{start}}}$$

$$= Equalizer\ Tilt + Transmit\ Level\ @\ Equalizer\ Start\ Frequency - m\sqrt{f_{stop}} \equiv\ intercept$$

Substituting the linear function slope and intercept into $P_{dB}\ (f)$ yields:

$P_{dB}\ (f) = m\sqrt{f} + b\ \ (dBmV/MHz)$

The total composite power TCP in decibel millivolt (dBmV) from the equalizer start frequency f_{start} to the equalizer stop frequency f_{stop} is given by the logarithm of the integral of $P_{dB}(f)$ as:

$$TCP = 10\ log_{10}\left(\int_{f_{start}}^{f_{stop}} 10^{(\ m\sqrt{f}+b\ )/10}\ df\right)$$

Using $e^{\ln(10)} = 10$

$$\int 10^{(m\sqrt{f}+b)/10}\ df = \int e^{\ln(10)(m\sqrt{f}+b)/10}\ df$$

Let $u = \sqrt{f} = f^{\frac{1}{2}}$

Using the Chain Rule $\frac{d}{df}f^n = nf^{n-1}$

$du = \frac{1}{2\sqrt{f}}df$ hence $df = 2u\ du$

Substituting variables:

$$\int e^{\ln(10)(m\sqrt{f}+b)/10}\ df = 2\int u\ e^{\ln(10)(mu+b)/10}\ du$$

Let $v = \frac{m}{10}ln(10)\ u$ and $dv = \frac{m}{10}ln(10)\ du$

$$2\int u\, e^{\ln(10)(mu+b)/10}\, du = 2\, e^{\ln(10)\, b/10} \int \frac{v}{\frac{m}{10} ln(10)}\; e^{\ln(10) m \frac{v}{\frac{m}{10} ln(10)}/10}\, \frac{dv}{\frac{m}{10} ln(10)}$$

$$= \frac{2\left(10^{b/10}\right)}{\left(\frac{m}{10} ln(10)\right)^2} \int v\, e^v\, dv$$

Integrating $\int v\, e^v\, dv$ by parts results in

$$\int v\, e^v\, dv = v\, e^v - \int\; e^v\, dv = e^v(v-1)$$

Therefore

$$\int 10^{\frac{(m\sqrt{f}+b)}{10}} df\, = \frac{2\left(10^{\frac{b}{10}}\right)}{\left(\frac{m}{10} ln(10)\right)^2} e^{\frac{m}{10} ln(10)\sqrt{f}} \left(\frac{m}{10} ln(10)\sqrt{f}\; - 1\right) = \frac{2\left(10^{(m\sqrt{f}+b)/10}\right)}{\left(\frac{m}{10} ln(10)\right)^2} \left(\frac{m}{10} ln(10)\sqrt{f}\; - 1\right)$$

Appendix K A Closer Look at the Basics of Phase Noise

Phase noise is introduced and briefly discussed in Section 30.13 and illustrated in Figure 66. The material in this appendix provides an overview of a few basics and “breaking down” or simplifying some nuances of traditional phase noise literature. The goal is to provide engineers and technologists a “working familiarity” with the most fundamental phase noise considerations and ease the reading of literature and understanding of commonly occurring phase-noise-related requirements. This section was written by Tom Kolze. Used with permission.

A Closer Look at the Basics of Phase Noise

Tom Kolze, Ph.D.
Broadcom, Inc.

K.1 Introduction

Phase noise is an important element of communications systems and can impact system performance negatively if not managed suitably. Oscillators are at the heart of a system’s phase noise performance, but clocking subsystems, synthesis of all the frequencies needed in an implementation, and modulation and demodulation functions all involve analyzing and managing phase noise performance.

This appendix is not addressing how aforementioned functions and subsystems are impacted by phase noise nor how to flow-down or allocate requirements regarding phase noise. Rather the objective of this appendix is to provide a framework for engineers and technologists who work with communications systems, and especially cable communications systems, to enhance a heuristic understanding of manifestation, specification, and measurement of phase noise, and do so through a treatment that is mathematically rigorous, yet involving little math beyond trigonometry and calculus. A goal is to especially avoid the introduction and use of “special” functions and unnecessary terminology common to some of the more rigorous treatments of phase noise.

Regarding prerequisites for this appendix: the reader should have a familiarity with additive white Gaussian noise (AWGN) processes, Fourier transform theory, power spectral density (PSD) of signals, systems and signal processing involving lowpass and bandpass filtering, and some basic modulation theory. A quick review of these concepts is provided at the beginning, primarily to establish notation conventions.

K.2 One-sided, two-sided, single-sideband, double-sideband

Modulation is fundamental in treatments of phase noise, and so is analysis in the frequency domain, such as measurements made using spectrum analyzers. Negative frequencies appear in Fourier transform theory and analysis of modulation. Terminology such as “one-sided” PSD, and “two-sided” PSD, are common and probably familiar. These terms are encountered and need to be understood to benefit from this appendix. Two related, but completely different terms, are “single-sideband” and “double-sideband,” which are introduced and clarified, and will be seen to relate to measurements of modulated signals on a spectrum analyzer.

Power spectral density, S(f). When a signal is measured with a spectrum analyzer, and averaged significantly, the spectrum analyzer display shows the PSD of the signal. If a signal has a random nature, but is stable, averaging is necessary to obtain repeatable measurements. Stability means that averages will converge when taken over a long enough observation time. The PSD has units of power per Hz, such as watts/Hz or perhaps volts squared/Hz. With a PSD, the amount of power in a particular finite band, such as f_L Hz to f_H Hz, is the integral of the PSD function S(f) over the frequency band f_L to f_H. If there is a sinusoid at frequency fc, with power P, then the PSD does not have a "power density" in a literal sense, defined at fc, but instead has a delta function at fc, with weight P. The delta function of weight P indicates the PSD, as a true density does not exist (it is infinite and undefined at fc), but it is such that any integral of S(f) which includes fc within its limits will incorporate the power, P, of that delta function at fc.

It is important to note that the PSD, S(f), is not the Fourier transform of the signal itself.

A spectrum analyzer shows the PSD for positive frequencies, $f \geq 0$, up to the limit for which the equipment is designed to measure. Integrating the function S(f) from 0 Hz to beyond the highest frequency component of the signal will produce the total power of the signal. This is called a **one-sided PSD**.

When dealing with Fourier transforms, mathematically a time domain-signal can be evaluated for the frequency (power) content in negative values of frequency, f. A **two-sided PSD** can be defined such that $S_{two\text{-}sided}(f) = S(f)/2$ for $f \geq 0$, and $S_{two\text{-}sided}(f) = S(-f)/2$ for $f \leq 0$.

Additive white Gaussian noise, n(t). A noise process in the time domain, n(t), which is generated in electronic equipment and adds to other signals. When measured with a spectrum analyzer and averaged significantly, the spectrum is flat versus frequency, or "white," with an understanding that the domain of frequencies containing energy is finite. The display on the spectrum analyzer, versus frequency, f, whatever it may be, is a function S(f). The one-sided PSD of an AWGN process is generally denoted as N_0, which is "N zero" or "N sub zero." The two-sided PSD of an AWGN process is generally denoted by $N_0/2$. The units on N_0 are watts per Hz, or volts squared per Hz, or similarly.

A histogram of the voltage of the AWGN process n(t) will converge to the Gaussian probability distribution. The mean of n(t) is "0" and letting the power of n(t) be P, the standard deviation of the Gaussian distribution is σ_P = square root of P.

Lowpass AWGN. A lowpass AWGN process will be defined by its one-sided PSD value, N_0, and its bandwidth, B Hz. Consider that the units on the PSD are in watts/Hz, and assume the power of n(t) is P watts. The one-sided PSD is such that $S(f) = N_0$ for f between 0 Hz and B Hz, and is zero for $f > B$ Hz. Integrating the one-sided PSD from f = 0 Hz to a value larger than B Hz, produces the power of the AWGN signal, n(t), and is $P = N_0B$ watts.

The two-sided PSD in this example is $S_{two\text{-}sided}(f) = N_0/2$, and integrating $S_{two\text{-}sided}(f)$ from very negative f to very large positive f, we obtain the integral $(N_0/2)(2B)$ because the non-zero range of the two-sided PSD covers from –B Hz to +B Hz. The integral $(N_0/2)(2B) = N_0B = P$, the total power of the lowpass AWGN signal, n(t).

Bandpass AWGN. A bandpass AWGN process will be defined by its one-sided PSD value, N_0, and its lowest frequency B_L and highest frequency, B_H, for which it contains energy. The bandwidth of the bandpass AWGN signal, B Hz, is given by the frequency difference $B_H - B_L$. The value of S(f) is N_0, for f between B_L and B_H, and is 0 for all other frequencies, f. Consider that the units on the PSD are in watts/Hz, and assume the power of n(t) is P watts. Integrating the one-sided PSD from f = 0 Hz to a value larger than B_H Hz, produces the power of the AWGN signal, n(t), and is $P = N_0(B_H - B_L) = N_0B$ watts.

In-phase (I) and quadrature (Q) representation of bandpass AWGN. For a bandpass AWGN process, n(t), with center frequency fc = $(B_H + B_L)/2$ which is larger than B, and typically much larger than B, there is a common representation of the bandpass process n(t) as a composition of two lowpass AWGN processes which are independent (statistically).

The (sufficiently narrowband) bandpass AWGN process n(t), of power P, bandwidth B, center frequency fc, and one-sided PSD N_0, can be written as:

$$n(t) = n_I(t)*2^{1/2}*\sin(2\pi \text{ fc } t) + n_Q(t)*2^{1/2}*\cos(2\pi \text{ fc } t),$$

where $n_I(t)$ and $n_Q(t)$ are statistically independent lowpass AWGN processes, each of power P/2, bandwidth B/2, and one-sided PSD N_0. The bandpass AWGN process, n(t), still has power P, of course.

The noise processes $n_I(t)$ and $n_Q(t)$ are termed the in-phase and quadrature noise components, respectively. They are also called "quadrature components" or "baseband quadrature components" of the bandpass noise. The $2^{1/2}$*sin() and $2^{1/2}$*cos() signals are the quadrature carriers, and each has unity power. An alternative decomposition where the quadrature carriers have unity amplitude, and thus each has power of "1/2," results in the baseband quadrature noise components which each have power P:

$$n(t) = n_I(t)*\sin(2\pi \text{ fc } t) + n_Q(t)*\cos(2\pi \text{ fc } t),$$

where $n_I(t)$ and $n_Q(t)$ are statistically independent lowpass AWGN processes, each of power P, bandwidth B/2, and one-sided PSD $2*N_0$.

The decomposition with unity amplitude quadrature carriers (the latter shown) helps simplify and unify some aspects of the treatment of phase noise; it is beneficial to be familiar with both decompositions, which differ only in a scaling factor. There is no distinction between the two decompositions other than the strength of the baseband quadrature noise components. The relationship and strength (power) of a phase noise process, φ(t), relative to, and associated with, the quadrature AWGN baseband process, $n_Q(t)$, *does* depend on which decomposition is used. [The author prefers the latter decomposition, with unity-amplitude quadrature carriers, for dealing with bandpass AWGN together with phase noise; there is no deep "gotcha" involved. The consideration is only to be mindful of the decomposition (scaling) which is used in any given treatment, analysis, or requirements allocations, and be mindful of the various factors of "2," which is always a handful in dealing with phase noise, anyway.]

The decompositions of bandpass AWGN into expressions of two lowpass (also called "baseband") AWGN processes, each multiplying one of a pair of quadrature carriers, can take on other forms than what is shown above. One variation is exchanging the sin() and cos() quadrature signals, pairing the "in-phase" noise with the cos(), and another variation is that the sign of the quadrature noise can be negative. We will not be using these variations.

Comparing and contrasting phase noise and bandpass AWGN. We will see that phase noise on a carrier has similarities to the quadrature noise of bandpass AWGN. Also, two key differences exist between bandpass AWGN and a carrier's phase noise. One difference is that with bandpass AWGN there is an in-phase noise term of equal power (and independent) of the quadrature noise term, while with phase noise, there is only the quadrature noise present on the carrier. Another difference is that with an otherwise ideal carrier that has phase noise, we will see that with a small phase noise process (magnitude less than 0.1 radian), the phase noise, which is in *quadrature* with the carrier, is *proportional* to the carrier amplitude, and has a multiplicative aspect with respect to the carrier (explained further in Section K.13). With bandpass AWGN, the quadrature noise term is *additive* to the carrier, but with bandpass

AWGN it is not proportional to the carrier. As such, changing the carrier strength does not change the AWGN quadrature noise term, but with the phase noise being proportional to the carrier, increasing the carrier amplitude will increase the apparent quadrature noise which is due to the phase noise on the carrier. This is revisited in Section K.13.

Power and bandwidth. We have seen for the AWGN lowpass and bandpass noise, and for signals in general, that integrating the one-sided PSD over the positive frequencies, or the two-sided PSD over both positive and negative frequencies, will provide the power of the noise.

Modulated signals and usefulness of two-sided PSD. The utility of negative frequencies in signal spectra is illustrated when the signal is used to modulate a carrier. Consider a lowpass signal m(t) with absolute value < 1 and energy all contained below B Hz in the one-sided PSD, M(f). Note that M(f) is the PSD of m(t) and not the Fourier transform of m(t).

The two-sided PSD of m(t) is given in terms of the one-sided PSD, M(f), as:

$M_{two\text{-}sided}(f) = M(-f)/2,\ f < 0,$
$M_{two\text{-}sided}(f) = M(f)/2,\ f \geq 0.$

When the signal is used as the message signal in amplitude modulation of a carrier A*sin(2π fc t) the resulting signal is:

$$x(t) = (1 + m(t))*A*\sin(2\pi\, fc\, t) = A*\sin(2\pi\, fc\, t) + m(t)*A*\sin(2\pi\, fc\, t)$$

A spectrum analyzer will display the PSD of the amplitude modulated carrier, x(t):

$$(M(f_c - f)/2)*(A^2/2) + (A^2/2)*\delta(f - f_c) + (M(f - f_c)/2)*(A^2/2).$$

The one-sided PSD of x(t), displayed on a spectrum analyzer, shows the frequencies below the carrier frequency, f_c, contain the frequency-translated (shifted "right" in the frequency domain by the amount of the carrier frequency, f_c Hz) negative frequencies of the two-sided PSD of m(t), and it is multiplied by the carrier power, $A^2/2$.

The fact that the modulated, or "upconverted" signal, 1 + m(t), has its one-sided PSD revealed as a right-shifted (in the frequency domain) version of the two-sided PSD of 1 + m(t) is sufficient to justify the acceptance of negative frequencies and the utility of two-sided PSDs.

While this example deals with amplitude modulation, when dealing with a carrier with phase modulation or a phase noise process, the utility of the two-sided PSD of the baseband phase noise process is useful in the same way.

Phase noise process φ(t) and one-sided PSD of φ(t), $S_\varphi(f)$. In the subsequent sections we will analyze a carrier which has a baseband phase process φ(t):

$$x(t) = A*\sin(2\pi\, fc\, t + \varphi(t))$$

and the one-sided PSD of φ(t) is denoted $S_\varphi(f)$, and the two-sided PSD of φ(t) is given by:

$S_{\varphi\_two\text{-}sided}(f) = S_\varphi(-f)/2,\ f < 0,$
$S_{\varphi\_two\text{-}sided}(f) = S_\varphi(f)/2,\ f \geq 0.$

Applying a trigonometry relationship to the phase modulated carrier, x(t), we can see:

$$x(t) = \cos(\varphi(t))*A*\sin(2\pi\ fc\ t) + \sin(\varphi(t))*A*\cos(2\pi\ fc\ t).$$

We can see that the phase process φ(t) results in quadrature components of the carrier, each amplitude modulated by nonlinear functions (sine and cosine) operating on φ(t). When a carrier is amplitude modulated by a baseband signal m(t) we saw it was straightforward to find the PSD of the resulting signal, but with a phase modulated signal, *in general*, the resulting PSD is complex due to the fact that the resultant is a quadrature upconversion of highly nonlinear functions of φ(t). The quadrature upconversion is not prohibitive, but the nonlinear functions operating on φ(t) require more sophisticated math than trigonometry and Fourier analysis to develop an informative description, in the general case.

Single-sideband measurement, high-side and low-side. Above, we saw the spectrum analyzer display of an amplitude modulated signal, x(t), with baseband signal, m(t):

The spectrum analyzer will display the one-sided PSD of the amplitude modulated carrier, x(t):

$$X(f) = (M(f_c - f)/2)*(A^2/2) + (A^2/2)*\delta(f - f_c) + (M(f - f_c)/2)*(A^2/2).$$

The residual carrier power (at $f = f_c$) is $A^2/2$ and the PSD at frequencies higher than the carrier frequency is given by:

$$X(f) = (M(f - f_c)/2)*(A^2/2),\ f > f_c.$$

We define the relative value, compared to the residual carrier power, of the spectrum measured above the carrier frequency as the **single-sideband measurement**, and denote it as $\mathscr{L}(f)$, which is spoken as "script L of f," and is also often written as L(f):

$$L(f - f_c) = \{\text{by definition}\}\ X(f)/[\text{carrier power}] = (M(f - f_c)/2)*(A^2/2)/[A^2/2],\ f > f_c.$$

$$L(f - f_c) = (M(f - f_c)/2),\ f > f_c.$$

In general, a single-sideband measurement can be made for either the "high-side," which are the frequencies larger than the carrier frequency, or for the "low-side," which are the frequencies smaller than carrier. In the latter, the argument of L(f) is $(f_c - f)$. *Double-sideband* refers to both the high-side and the low-side spectrum about the carrier.

K.3 Phase modulation of a carrier and the small signal assumption

This section addresses phase modulation of a carrier by a signal which introduces a small amount of phase deviation, such as less than 0.1 radian (which is less than 5.73 degrees). With the "small signal assumption," there is tremendous simplification in that harmonics (higher orders, nonlinearity) of the modulating signal may be neglected. Thus, there is a similarity to amplitude modulation and its simplicity, compared to unrestricted phase modulation.

As an example where the small signal assumption with phase modulation does not apply, consider traditional FM broadcasting (which is a form of phase modulation). By intention, the phase deviation introduced by the frequency modulation in FM broadcasting is large; the "bandwidth expansion" associated with traditional "frequency modulation" is significant and contains much of the energy of the resultant signal. The significant presence of harmonics (nonlinear terms) of the modulating signal is responsible for the bandwidth expansion characteristic of FM broadcasting, and is even critical to the advantages and performance of commercial FM broadcasting.

In the treatment and analysis of phase noise for communications and clock synthesis purposes it is reasonable to have a goal, and expectation, for the applicability of the "small signal assumption," and it is advantageous to make use of this simplification, since it is almost always applicable in practical phase noise analyses, and it is easy to validate the applicability.

Phase modulation of a carrier and the small signal assumption. The following pages show the derivation of a simplified (approximate) expression of the signal resulting when a carrier is phase modulated with a baseband phase process of small maximum amplitude (such as amplitude less than 0.1 radian).

Taylor's Series expansion around $\varphi = 0$ radians:

$\sin(\varphi) = \varphi - \varphi^3/3! + \varphi^5/5! - \dots$

For $|\varphi| < 0.1$ radians, $\sin(\varphi) \approx \varphi$, within 1 percent (the error is less than 1 percent of φ).

For $|\varphi| < 0.01$ radians, $\sin(\varphi) \approx \varphi$, within 0.01 percent ($10^{-4}$);
the magnitude of the error is less than $|\varphi|*10^{-4}$.

Note that for the same small angle, the cos() function has a corresponding Taylor's Series expansion:

$\cos(\varphi) = 1 - \varphi^2/2! + \varphi^4/4! - \dots$

For $|\varphi| < 0.1$ radian, $\cos(\varphi) \approx 1$, within 1 percent.

Note that 0.1 radian is approximately 5.73 degrees.

We have analyzed $\sin(\varphi)$ for small φ, and demonstrated that $\sin(\varphi) \approx \varphi$, now consider that the phase of a carrier is $\varphi(t)$, and φ is small:

$\varphi(t)$ = angle in radians, absolute value less than 0.1 radian;

$$x(t) = A*\sin(2\pi\, fc\, t + \varphi(t)) = A*\sin(2\pi\, fc\, t)\cos(\varphi(t)) + A*\cos(2\pi\, fc\, t)\sin(\varphi(t))$$
$$\approx A*\sin(2\pi\, fc\, t) + A*\varphi(t)*\cos(2\pi\, fc\, t)$$

Thus, neglecting the (insignificant) higher order terms, it can be seen that with the small angle (lower power) phase process, the resultant is the ideal carrier term, plus a modulated signal term. The phase process, $\varphi(t)$, is amplitude-modulating the quadrature of the ideal carrier. Thus, the baseband process, $\varphi(t)$, becomes a modulated process, spectrally "centered" on the ideal carrier and additive to the ideal carrier, and significantly, *proportional* to the carrier, due to the factor "A" which multiplies $\varphi(t)$.

If the power of $\varphi(t)$ is P_φ radians squared, and the power of the ideal carrier is $A^2/2$ watts, and assuming the spectrum of $\varphi(t)$ is insignificant for frequencies approaching fc, the power of the signal (due to the small-amplitude phase noise) added to the ideal carrier is $P_\varphi * (A^2/2)$.

In terms relative to the ideal carrier's power, "1" is the power equal to the ideal carrier, the relative power in the additive process centered on the ideal carrier is P_φ, and is unitless, since it is watts over watts (or volts^2 over volts^2, or similar). Recall that P_φ is also the power in the baseband phase process, φ(t), in radians^2, which is thus also (literally) unitless (because "radians" is literally unitless, as a ratio of lengths).

In terms relative to the ideal carrier's power, using decibel notation, 0 dBc is the ideal carrier power. The entirety of the relative power in the signal or process due to the phase noise, which is centered on the ideal carrier, is $10*\log_{10}(P_\varphi)$, dBc.

PSD of the carrier with phase noise φ(t), where φ(t) is small. We have seen that the carrier with small-signal phase modulation is approximated by:

$$x(t) = A*\sin(2\pi\ fc\ t + \varphi(t)) \approx A*\sin(2\pi\ fc\ t) + A* \varphi(t) * \cos(2\pi\ fc\ t),$$

which is similar to the amplitude modulation we analyzed in Section K.2, except now the modulating function is φ(t) and the carrier component multiplying the modulating function is the quadrature of the unmodulated carrier component.

The PSD of the phase modulated carrier is thus similarly obtained as was the PSD of the amplitude modulated signal of Section K.2, where a spectrum analyzer will display the PSD of the phase modulated carrier, x(t):

$$X(f) = (S_\varphi(f_c - f)/2)*(A^2/2) + (A^2/2)*\delta(f - f_c) + (S_\varphi(f - f_c)/2)*(A^2/2).$$

This is illustrated in Figure 66.

Note that the unmodulated carrier has power $A^2/2$. The PSD in the frequencies higher than f_c are seen to be $(S_\varphi(f - f_c)/2)*(A^2/2)$. As mentioned in Section K.2, the spectrum measured above the carrier frequency, scaled or normalized relative to the residual carrier power, is defined as the **single-sideband measurement**, and denoted as $\mathscr{L}(f)$. This is spoken as "script L of f," and is also often written as L(f).

The single-sideband measurement of the carrier with phase noise is:

$$L(f - f_c) = \{\text{by definition}\}\ X(f)/[\text{carrier power}] = (S_\varphi(f - f_c)/2)*(A^2/2)/[A^2/2],\ f > f_c.$$

$$L(f - f_c) = S_\varphi(f - f_c)/2,\ f > f_c.$$

$$L(f) = S_\varphi(f)/2\ ,\ f > 0.$$

L(f) is highlighted in Figure 66. Since the value L(f) is a relative measurement, it is literally unitless. However, it can be noted that L(f) numerically equals the value of the two-sided PSD of the baseband phase noise process. When expressed in decibels, the units of L(f) are dBc/Hz, where 0 dBc is the carrier power. The values of L(f) may also be interpreted as radians squared per Hz of a two-sided PSD; the practical aspect means that integrating 2 * L(f) over the positive frequencies, a semi-infinite range of frequency, will result in the total phase noise power in units of radians squared. Alternatively, integrating 2 * L(f) from 0 Hz to F Hz results in the phase noise power in units of radians squared for all phase noise below F Hz.

As mentioned in Section K.2, in general, a single-sideband phase noise measurement can be made for either the "high-side," which are the frequencies larger than the carrier frequency, or for the "low-side," which are the frequencies smaller than carrier. In the latter, the argument of L(f) is $(f_c - f)$. *Double-sideband* refers to both the high-side and the low-side spectrum about the carrier.

K.4 Phase modulation of a carrier by a small amplitude sinusoidal phase process

In this section we will use the trigonometric relationships:

$$\sin(a)\cos(b) = (1/2)\sin(a + b) + (1/2)\sin(a - b)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b).$$

Phase modulation of a carrier by a small amplitude sinusoidal phase process. We have examined a carrier with a generalized small phase process, φ(t). Now we consider a specific, sinusoidal phase process, φ(t) = R*sin(2π fm t) radians, with R < 0.1 radian:

$$x(t) = A*\sin(2\pi\ fc\ t + \varphi(t)) = A*\sin(2\pi\ fc\ t + R*\sin(2\pi\ fm\ t))$$

We saw in Section K.3,

$$x(t) = A*\sin(2\pi\ fc\ t + \varphi(t)) = A*\sin(2\pi\ fc\ t)\cos(\varphi(t)) + A*\cos(2\pi fc\ t)\sin(\varphi(t))$$

Substituting the sinusoidal waveform expression for φ(t), and applying the small angle approximations,

$$x(t) \approx A*\sin(2\pi\ fc\ t) + A* R*\sin(2\pi\ fm\ t) * \cos(2\pi\ fc\ t)$$

$$\approx A*\sin(2\pi\ fc\ t) + (A* R/2)\sin(2\pi\ [fc - fm]\ t) + (A* R/2)\sin(2\pi\ [fc + fm]\ t)$$

$(A^2)/2$ is the power of the carrier component, which is also the ideal carrier (we have applied the small angle assumption on the phase process in this example). There is also a lower frequency sideband sinusoid with power $(A^2)(R^2)/8$, and a higher frequency sideband with the same power, $(A^2)(R^2)/8$. In all three cases, the units of power are watts, or volts squared (we have been using watts in examples).

The sinusoidal additive phase process, φ(t) = R*sin(2π fm t) radians, has power $(R^2)/2$ radians squared.

The sinusoidal additive phase process, φ(t), in a one-sided representation of its power spectrum, has all of its power, $(R^2)/2$ radians squared, at fm Hz.

The sinusoidal additive phase process, φ(t), in a two-sided representation of its power spectrum, has half its power, $(R^2)/4$, at –fm Hz and half its power, $(R^2)/4$, at +fm Hz; the positive frequencies contain half the power and the negative frequencies contain half the power.

In relative terms, the power of the ideal carrier, also known as the *residual carrier component*, is “1” or in decibels, 0 dBc. (In actuality, the power of the ideal carrier is reduced by the power which shows up in the sidebands, so that the total (relative) power of the ideal carrier is “1,” or 0 dBc, and the residual carrier power is slightly smaller than “1.”)

In relative terms, the power of the lower frequency sideband is $(R^2)/4$, and the power of the higher frequency sideband is $(R^2)/4$; both are unitless.

It is KEY to note that we have described above the *one-sided* spectral representation of the carrier as impacted by a small-angle sinusoidal phase process (*one-sided* must be the case with a spectrum analyzer display). However, the two offset (from the carrier itself) one-sided delta functions correspond to the *two-sided* spectral representation of the (baseband) sinusoidal phase process; it can be recognized that the pair of baseband sinusoidal spectral delta functions originate in the two-sided representation, and are then *right-shifted in the frequency domain* by multiplication with the carrier, ending up ultimately as the one-sided delta functions as they are shown (negative frequencies are not existing and all the power of the signal is represented in the positive frequency domain). Whereas in the two-sided spectral representation of the (baseband) sinusoidal phase process the power of each of the spectral delta functions is $(R^2)/4$ radians squared, in the one-sided spectral representation of the impacted carrier, each sideband has power $[(A^2)/2]*[(R^2)/4]$, and each of the *relative* sideband's power is $(R^2)/4$ *unitless*:

The power spectral density of the one-sided spectrum of a carrier impacted by a phase process, i.e., the spectrum displayed in a spectrum analyzer, ***in terms relative to the carrier*** *(i.e., "dBc"), has the same numeric value as the two-sided power spectral density $S_\varphi(f)/2$ of the baseband phase process, $\varphi(t)$, in units of radians squared. (This is illustrated in* Figure 66.*)*

In a one-sided representation of its power spectrum, in this example with a small-angle sinusoidal phase process, there are three "delta functions," one each at fc – fm Hz, fc Hz, and fc + fm Hz. The sum of the power of the three delta functions in the one-sided representation of the modulated spectrum totals the complete signal power.

K.5 Recapping the relation of spectrum analyzer measurement of sideband-to-carrier power, L(f), and two-sided power spectral density of the phase noise process, $S_\varphi(f)/2$, and total phase noise power

The relative power of the combined low-sided and high-sided portions of the modulated spectrum is unitless, and integrates to the same numerical value (unitless) as equals the total power of the baseband phase process, in units of radians squared.

Equivalently, the high-side integration of the single-sideband L(f) plus the low-side integration of the single-sideband L(f), is the double-sideband integration of L(f); while the resulting integrals' units are "unitless" (technically), the numerical result is equal to the total integrated phase noise power in units of radians squared. The total phase noise power is the semi-infinite integration (positive frequencies) of $S_\varphi(f)$, the one-sided phase noise PSD.

Symbolically,

$S_\varphi(f)$ is the one-sided power spectral density of the phase noise process, $\varphi(t)$; defined only for $f > 0$.
The two-sided power spectral density of the phase noise process is $S_\varphi(f)/2$, with $S_\varphi(-f)/2 = S_\varphi(f)/2$.

The single-sideband measurement, L(f), at a frequency offset from the carrier, is the measurement of JUST the ***relative*** power in one sideband of the carrier; using the upper frequency for example, in which case for a measurement $f_{measured}$ Hz, the argument of L(f) is $f = f_{measured} - fc$ Hz.

It is apparent from what we have shown above, $L(f) = S_\varphi(f)/2$, when the small angle assumption holds for the phase noise process. The units of L(f) in decibels are "dBc" or unitless for a "tone," or "dBc/Hz" for a power spectral density.

Directly following from above, $S_\varphi(f) = 2 * L(f)$ in units of "$dB_{radians}{}^2$/Hz."

$S_\varphi(f)/2$ radians squared per Hz, two-sided = L(f) relative power, or unitless, per Hz, "single-sideband."

*$S_\varphi(f)$ radians squared per Hz, one-sided = 2 * L(f) relative power, or unitless, per Hz, "single-sideband."*

K.6 Relationship of phase noise process and time domain jitter of zero-crossings

There is a relationship between phase noise and time domain jitter. Phase noise manifests as jitter in the carrier's zero-crossings viewed in the time domain, as explained in this section. Further, there are key insights we explore in this section that derive from "viewing" phase noise as a time domain jitter of a sinusoidal waveform, as well as measuring time domain jitter of a clock signal or carrier.

A static phase value of φ radians in a carrier of frequency fc Hz causes a static time-domain displacement of the carrier's zero-crossings of –φ/fc seconds, found by setting 2*π*fc*t + φ = 2*π radians, and solving for t. In that definition of the jitter process, an advancement in the zero-crossings (earlier occurring) corresponds to a negative jitter; we can just as well define the jitter with the opposite sense of sign, and let an earlier occurrence of the zero-crossings correspond to a positive time-jitter process. We will do that for the remainder of the discussion.

A dynamic phase process of φ(t) radians in a carrier of frequency fc Hz causes a dynamic time-domain displacement of the carrier's zero-crossings of τ(t) = φ(t)/fc seconds.

Sinusoidal phase noise of amplitude R radians in a carrier of frequency fc Hz causes sinusoidal time-domain jitter of amplitude R/fc seconds.

The RMS phase noise, in radians, is the square root of the total integrated phase noise power (integrating both the high-side frequencies and the low-side frequencies, if using L(f)).

The time domain RMS jitter, in seconds, of a carrier of frequency fc Hz, is the RMS phase noise, in radians, divided by fc in Hz.

Equivalently, the RMS jitter in the time domain (in seconds), divided by the period of the carrier (in seconds), is the RMS value of the phase noise.

Converting between the phase noise integrated power (radians squared) and time-domain jitter (seconds squared) is given as: $\sigma_\tau^2 = \sigma_{radians}^2/(f_c)^2$

Example: sinusoidal phase noise, amplitude R = 0.01 radian, on a carrier of 100 MHz:

$S_\varphi(f)$ is a delta function of $R^2/2$ radians squared, which is $5*10^{-5}$ radians squared at f = fm Hz, which is –43 $dB_{radians}{}^2$.

L(f) = $S_\varphi(f)/2$ is a delta function of $2.5*10^{-5}$ relative to carrier power, which is –46 dBc, at f = fc + fm Hz, and another at fc – fm Hz., which is in total, –43 dBc.

It is seen that the total integrated L(f), both the high-side and the low-side, summed (i.e., double-sideband), numerically equals the total integrated phase noise of the fc Hz carrier, in radians squared: The integrated phase noise of the 100 MHz carrier is –43 dBc, or $5*10^{-5}$ radians squared.

Time domain zero-crossing RMS jitter, in units of seconds, is the RMS phase noise (the square root of the integrated phase noise), in radians, multiplied by 1/fc Hz:

The RMS phase noise on the 100 MHz carrier is square root of ($5*10^{-5}$) radians^2, which is approximately $7*10^{-3}$ radians RMS. Converting to units of degrees, the RMS phase noise on the 100 MHz carrier is $7*10^{-3}*57.3 \approx 4$ deg RMS.

The time domain jitter of the carrier, due to the phase noise, is:

$$7*10^{-3} \text{ radians RMS} * 1/(10^8 \text{ Hz}) = 7*10^{-3} * 10^{-8} \text{ seconds} = 7*10^{-2} \text{ nanoseconds RMS} = 0.07 \text{ ns RMS.}$$

K.7 Ideal frequency synthesis, with phase noise on the reference

Building on the insights of the relationship between phase noise of a carrier, and the corresponding time domain jitter of the carrier's zero-crossings, we can develop further insights into the behavior of ideal frequency divider circuits and ideal frequency multipliers. In the analysis, the reference frequency (input to the multiplier or divider circuits), has phase noise, but all other aspects of the operations are ideal.

A reference sinusoid with frequency f_c and given phase noise process, $\varphi(t)$, will have time domain jitter process (zero-crossing jitter process) of $\tau(t) = \varphi(t)/f_c$ seconds.

Analysis of an ideal synthesis of a harmonic frequency, or a subharmonic frequency, to another frequency, f_{c2}, shows that the same time domain jitter process on the reference signal, $\tau(t)$, also occurs on the output signal as well. Filtering functions are typically applied in practice, which will alter the phase noise PSD, but in this analysis such filtering is not incorporated.

Since the time domain jitter process is the same on the output signal as the input signal, we can use the relationships of Section K.6 and determine the phase process on the output signal corresponding to $\tau(t)$. The output signal frequency is f_{c2} Hz so the phase process, $\varphi_2(t)$, is given by $\varphi_2(t) = \tau(t)*f_{c2}$, radians.

The phase noise process of a second frequency, f_{c2}, synthesized from a reference frequency, f_c, via multiplier and divider circuits, where the reference frequency has phase noise $\varphi(t)$ radians, inherits the phase noise process, scaled by ratio of the output-to-input frequency, $\varphi_2(t) = \varphi(t) *f_{c2}/f_c$.

With the frequency synthesis, a display on a spectrum analyzer of the single-sideband power spectral density, L(f), in units dBc, will verify that the $L_2(f)$ of the carrier with the second frequency, f_{c2}, is indeed $(f_{c2}/f_c)^2 * L(f)$ of single-sideband PSD of the reference frequency, f_c. For example, if L(f) of a 10 MHz reference is –110 dBc/Hz at an offset of 10 kHz, then a synthesized frequency of 200 MHz will have $L_2(f)$ at 10 kHz offset of $20*\log_{10}(f_{c2}/f_c) = 26$ dB added to L(f), so $L_2(f)$ = –110 dBc/Hz + 26 dB = –84 dBc/Hz.

Analog to digital converter (ADC) clock jitter and phase noise. The same analysis (sketched in Section K.6) which showed that a sinusoid of fc Hz with a phase noise process, $\varphi(t)$, experiences time domain jitter related to the phase noise as $\tau(t) = \varphi(t)/fc$ seconds, shows that jitter on an ADC clock, and thus jitter in the sampling times, results in the digitized waveform (which will be processed assuming the sampling was uniformly ideal) having the mirror image jitter, $-\tau(t)$. The sign doesn't matter in this case. So, the time domain jitter on the ADC clock becomes the mirror image time domain jitter, $-\tau(t)$, on the input signals to the ADC.

Interestingly, with a single sinusoid, or with a multitude of sinusoids all presented to the ADC, all are sampled with the same jitter process, and thus each digitized sinusoid will individually manifest the phase noise from the introduced jitter. It does not really matter how the time domain jitter was introduced to the sinusoids, the fact is that the jitter exists for each sinusoid, and thus the "time domain jitter versus carrier phase noise" relationship exists, for each. We can use the equation relating the time domain jitter and phase noise, and determine the corresponding phase noise on each sinusoid presented to the ADC: $\varphi(t) = \tau(t)*f_c$ radians, for each sinusoid, where fc Hz is the frequency of a sinusoid presented to the ADC.

Given the ADC sampling clock has frequency, f_{clock} Hz, and phase noise on this ADC sampling clock, $\varphi_{clock}(t)$ radians, then we can determine the sampling clock time domain jitter, and then determine the phase noise on a sinusoid presented to the ADC and digitized: $\varphi(t) = (\varphi_{clock}(t)/f_{clock})*f_c$ radians = $\varphi_{clock}(t)*(f_c/f_{clock})$ radians.

Note the relationship for the phase noise of a sinusoidal signal ADC'd with a sampling clock which has phase noise (just shown), is the same as if the input signal to the ADC were synthesized with frequency dividers and multipliers from the ADC sample clock. This is an intuitively pleasing result.

Digital to analog converter (DAC) clock jitter and phase noise. The same analysis (sketched above) for the ADC can be shown to be valid for a DAC with phase noise on its reconstruction clock.

K.8 Use of masks in oscillator phase noise specifications

Phase noise specifications of oscillators often use "masks" of PSD, characterized by a given carrier frequency, and a table of offset frequencies, and corresponding maximum allowed $S_\varphi(f)$, in units of $dB_{radians}{}^2$/Hz. Alternatively, the specification may be in terms of single-sideband phase noise, L(f). Care must be taken in determining which is being specified, since there is a factor of 2 (or 3 dB) scaling for $S_\varphi(f)$ and L(f).

For example, a specification may be for L(f) for a carrier at 10 MHz, and at the offset frequencies 100 Hz, 1 kHz, 10 kHz, 100 kHz, 300 kHz, 1 MHz, and 3 MHz. (Note: "Decade" values are not always selected.) The L(f) requirements may be, as an example: –115 dBc/Hz, –125 dBc/Hz, –140 dBc/Hz, –155 dBc/Hz, –163 dBc/Hz, –170 dBc/Hz, and –175 dBc/Hz, at the corresponding offset frequencies.

Note that the requirements for L(f) are technically unitless, but also, we know that they specify the two-sided power spectral density limit of the phase noise process, which has units of radians squared per Hz.

To pass the requirement, an oscillator's phase noise PSD (the two-sided PSD, $S_\varphi(f)/2$, if the specification is on L(f)) would have to be lower than the specified mask at every frequency from 100 Hz to 3 MHz, where the specification mask is interpolated between the given points with a line (linear interpolation) on a log-log graph, so both the x-axis (the offset frequency) and the y-axis (the $S_\varphi(f)$) are log scales.

Unless specifically discounted in the official requirements, a continuous wave tone, aka discrete frequency, such as described in Section K.4, will violate the mask requirement because by definition it has an infinite PSD.

Often, some discrete frequency spurs in oscillators are allowed, specified or listed separately from the PSD requirements. Even the selection of the measurement bandwidth for determination of L(f) in dBc/Hz can be crucial, and a sticking point if not carefully specified; a wider measurement bandwidth will itself "smooth out" variations in the L(f), possibly smoothing out a portion of phase noise PSD of the oscillator being tested which failed the mask limit. This can become an arguing point between a buyer and a seller, if the measurement bandwidth is not listed as part of the requirements.

K.9 Specification of integration of the phase noise power over specified frequency ranges

As an alternative to specifying a phase noise mask for an oscillator, or for a modulator (i.e., for a transmitted signal's carrier), **specification of integration of the phase noise power over specified frequency ranges** may be provided.

The specification of integration regions with a cap on the integrated phase noise in each region is beneficial, i.e., preferred over a PSD mask specification, if system performance and considerations can allow the determination or shaping for such requirements.

The use of integration regions, and caps on the phase noise power within each region, is not really a far deviation from a mask PSD requirement, since careful specification of limits on measurement bandwidth should be performed even with a specified mask PSD. By specifying frequency intervals, each with a cap on the phase noise power in the interval, discrete spurs are included in the integration and need not be distinguished from the true PSD. Separate requirements on the number and power of discrete spurs can still be levied, in addition to the integration regions and their caps.

The integration regions, as opposed to a spectral PSD mask, is beneficial because it allows purchase of oscillators, and design of clocking and synthesizer schemes, which may violate a rigorous mask specification by a small amount over an insignificant range of frequencies, while maintaining acceptably low integrated phase noise amounts over each specified region. This superior latitude in purchase and design of parts and critical subsystems allows cost minimization without sacrificing meaningful performance. The cost is simply doing more due-diligence in analyzing the system needs and flow-down of allocation to the phase noise and the clocking and oscillator subsystems.

The DOCSIS PHY layer requirements, dating back to DOCSIS 1.0 and through to the current DOCSIS 3.1, have examples of specification of caps on integrated phase noise power in specified frequency intervals. See **References** at the end of this appendix, immediately after Section K.13.

K.10 Closed form integration formulas for the linear (in log-log graphs) interpolation of the PSD masks

One-sided PSD of phase noise process is $S_\varphi(f)$, with units of radians^2/Hz, and f Hz.

Given two values of $S_\varphi(f)$:

$S_1 = S_\varphi(f_1)$ and $S_2 = S_\varphi(f_2)$,

with coordinate pairs (f_1, $10*\log_{10}(S_1)$) and (f_2, $10*\log_{10}(S_2)$).

Find the integral of $S_\varphi(f)$ from f_1 to f_2, with linear interpolation of log-log graphing of $S_\varphi(f)$ between the two given coordinate points. This involves linearly interpolating $S_\varphi(f)$ using $10*\log_{10}(S)$ and $\log_{10}(f)$.

The slope of the linear interpolation, in log-log graphing, between the two given coordinates is:

$$m = [\ 10*\log_{10}(S_2) - \log_{10}(S_1)\ ] / [\ \log_{10}(f_2/f_1)\ ].$$

The resulting integral is:

$$\text{Integral} = \int_{f1}^{f2} S_\varphi(f)\,df = [\ 10/(10 + m)\ ] * (S_1 * f_1) * [\ (f_2/f_1)^{(1 + (m/10))} - 1\ ] \ \text{radians}^2\text{, when } m \neq -10$$

$$\text{Integral} = \int_{f1}^{f2} S_\varphi(f)\, df = (S_1 * f_1) * [\ \ln(f_2/f_1)\ ] \ \text{radians}^2, \text{ when } m = -10.$$

When $S_\varphi(f)$ is proportional to f^{-1}, the slope, m, is –10, which results in the special case for the integral, shown above.

K.11 Time domain specification of integrated phase noise

Specification of integrated phase noise can be given in the time domain, making the specification independent of the carrier frequency. When specifying phase noise via integration regions of the offset frequency, as described in Section K.9, the caps on the result of the integration (measurement) of phase noise power over the various offset frequency regions may be given as an RMS limit, in units of time.

Section K.6 shows the relation of RMS time domain jitter to RMS phase noise in radians, and that the relationship holds for the underlying processes, $\tau(t)$ and $\varphi(t)$. The relationship shows a procedure of computing the RMS value of time domain jitter by a) integrating L(f) or $S_\varphi(f)$, with proper factor of 2 if needed, b) taking the square root, yielding units of radians RMS, and c) dividing by the carrier frequency. The approach is the conversion of the requirement in terms of phase noise power into RMS time domain jitter. Any specification of integrated phase noise over any offset frequency region, in units of radians^2, for a specific carrier frequency, can be converted to an equivalent time domain specification, in units of seconds^2, and independent of carrier frequency.

When a system or subsystem has many possible carrier frequencies this may be an attractive approach to the specification of the phase noise.

K.12 Bandpass AWGN and phase noise, not entirely separable

The two processes a) bandpass AWGN added to an unmodulated carrier, and b) a carrier with phase noise, are two distinct processes. In a system the two are usually specified (or a requirement allowance "flowed down") separately, but in practice the measurement of either one will be corrupted or influenced by the presence of the other. This coupling is important to understand. An even stronger statement can be made: *Some portion of bandpass AWGN IS phase noise, and also, phase noise contributes, or IS, a portion of bandpass AWGN.*

We have seen bandpass AWGN, n(t), is described in Section K.2 as:

$$n(t) = n_I(t) * \sin(\ 2\pi\ fc\ t\ ) + n_Q(t) * \cos(\ 2\pi\ fc\ t\ ),$$

where $n_I(t)$ and $n_Q(t)$ are statistically independent lowpass AWGN processes, each of power P, bandwidth B/2, and one-sided PSD $2*N_0$.

And we have seen small power phase noise on a carrier is well-described as:

$$x(t) = A*\sin(\ 2\pi\ fc\ t + \varphi(t)\ ) \approx A*\sin(\ 2\pi\ fc\ t\ ) + A*\ \varphi(t) * \cos(\ 2\pi\ fc\ t\ )$$

We noted in Section K.2, and elsewhere, that the phase noise process of an ideal carrier (with small-amplitude phase noise) is *additive*, similar to the bandpass AWGN quadrature noise term, but unlike AWGN, the phase noise is *proportional* to the carrier amplitude, as seen by the "A" multiplying $\varphi(t)$. The fact that the phase noise is proportional to the carrier amplitude is one aspect of the multiplicative nature of phase noise; that multiplicative nature is explored more in the next section. In the remainder of this section, we will continue examining the proportionality of the phase noise to the carrier amplitude, and the relationship of phase noise and bandpass AWGN.

We can assign the power of φ(t) as P_φ radians squared, and the power of the ideal carrier is $A^2/2$ watts, and assuming the spectrum of φ(t) is insignificant for frequencies approaching fc, the power of the signal added to the ideal carrier is $P_\varphi * (A^2/2)$ watts.

Consider a situation of a carrier with (small) phase noise, and also bandpass AWGN (so the two signals are added):

$$x(t) = A*\sin(2\pi \text{ fc } t + \varphi(t)) + n(t) \approx A*\sin(2\pi \text{ fc } t) + A* \varphi(t) * \cos(2\pi \text{ fc } t) + n(t)$$

$$x(t) \approx (A + n_I(t))*\sin(2\pi \text{ fc } t) + (A*\varphi(t) + n_Q(t)) * \cos(2\pi \text{ fc } t)$$

From the above expression it is apparent that the phase noise and the lowpass AWGN quadrature noise add together and modulate the quadrature carrier. An ideal measurement of phase noise will “see” the lowpass AWGN quadrature noise ($A*\varphi(t) + n_Q(t)$). Likewise, a measurement of bandpass AWGN will be corrupted by the presence of the phase noise. The phase noise will contribute to the measurement of the bandpass AWGN power, e.g., ($n_Q(t)^2 + A^2*\varphi(t)^2 + n_Q(t)^2 + 2*A*\varphi(t)*n_Q(t)$). The latter term will generally average to zero, but the phase noise power, weighted proportionally with the carrier power, will corrupt the measurement if it is large enough.

There are three possibilities (neglecting for a moment that the resulting situation can vary depending on the offset frequencies being measured): a) the phase noise is negligible compared to the bandpass AWGN; b) the bandpass AWGN is negligible compared to the phase noise; and c) the relative strengths are such that neither is negligible in the combination. It may be that in a frequency range close to the carrier that the phase noise dominates the bandpass AWGN, and that for far-out offset frequencies from the carrier, it may be that the bandpass AWGN dominates the phase noise. If this is the case, it is likely that at some offset frequencies the two noise sources or noise types will be comparable in their power.

It IS true that with both bandpass AWGN and phase noise present, with parameters such that the one is not negligible compared to the other, it is even arguable that the lowpass AWGN quadrature noise, $n_Q(t)$, IS phase noise. The only thing that prevents the converse statement, that the phase noise is arguably lowpass AWGN quadrature noise, is that the phase noise is not necessarily spectrally flat.

In fact, it may just be semantics, but a strong argument can be made that given an observation of “white” (spectrally flat) quadrature noise, it is not possible to categorize it as just flat phase noise or just quadrature noise; in essence, the noise is simultaneously *both*.

In making specifications for either of these two types of noise, and in designing testing to measure the performance or amount of presence of these two types of noise, the reader should be left with the correct impression that both must be considered, in their anticipated and allowed amounts. It is necessary to understand that the measurement (and thus validation of meeting the requirements) of each will be potentially impacted by the other. The specifications and test limits for each of the types of noise have to be synergistic.

It is worth noting that if there is some control of the signals, a determination of the decomposition between the phase noise and bandpass AWGN is possible, such as changing the carrier amplitude. This will change the phase noise component relative to the bandpass AWGN. Other means of assessing the presence of the two different types of noise can involve examining different offset frequency bands, and comparing the in-phase lowpass AWGN power (if it is possible in the existing test configurations; it is practically and theoretically possible in most cases) with the quadrature noise power, or other imbalance in the two quadrature noise processes, or correlation with aspects of the signal, if the carrier is data modulated. This is a topic that is beyond the scope of this treatment.

In a well-designed system operating as intended, it will not be important to determine which noise types are dominating, but in a broken system it may be good to know. It is the concern of this writing to understand the fundamentals, without going into designing and trouble-shooting systems.

K.13 Phase noise and a phase-modulated carrier

In this section we illustrate that phase noise has a multiplicative nature, which is most evident when there is a message[89] which is phase modulating the carrier. The fact that the phase noise is proportional to the carrier amplitude is one aspect of the multiplicative nature of phase noise, and that was fully explored in the previous section.

When multiplying complex numbers, the magnitudes multiply, but the multiplication also results in the phases adding. Phase noise is precisely the same as multiplying complex numbers, where a) the phase noise is proportional to the carrier amplitude (multiplication of the magnitudes), and b) overall phase of the modulated carrier is the addition of i) the signal phase due to the message and ii) the phase noise.

Equations help to nail down the multiplicative nature of phase noise upon a carrier with intended phase modulation.

We have analyzed $\sin(\varphi)$ for small φ, and demonstrated that $\sin(\varphi) \approx \varphi$, now consider that the phase of a carrier is the sum of $\theta(t)$ and $\varphi(t)$, where $\theta(t)$ is a message and is not small, covering a full range of 2π radians, and φ is the small phase noise as before:

$\varphi(t)$ = angle in radians, absolute value less than 0.1 radian;

$$x(t) = A*\sin(2\pi\, fc\, t + \theta(t) + \varphi(t)) = A*\sin(2\pi\, fc\, t + \theta(t))\cos(\varphi(t)) + A*\cos(2\pi\, fc\, t + \theta(t))\sin(\varphi(t))$$
$$\approx A*\sin(2\pi\, fc\, t + \theta(t)) + A*\varphi(t) * \cos(2\pi\, fc\, t + \theta(t))$$

It is apparent that after applying the small angle approximations for the phase noise process, what remains is the carrier with its (undistorted) ideal phase modulation by the message, $\theta(t)$, and then another term, a term *additive* to the ideal phase modulated carrier, which is the quadrature of the ideal phase modulated carrier, but this term (quadrature of the ideal signal) is *multiplied* by the phase noise process.

Without the message modulating the carrier, this expression was used in the previous section to show that the additive phase noise process was proportional to the carrier. We see this is still the case even when the carrier is phase modulated by a message. We also see that the phase of the carrier is the sum of the

[89] In this context, the term "message" can be thought of as analogous to data. The concept of "message" has been used in communications theory for decades. For instance, the 1948 "A Mathematical Theory of Communication" by Shannon describes a model for communication (*The Bell System Technical Journal*, Vol. 27, pp. 379–423, 623–656, July, October, 1948). The sender (or information source) is the originator of a message; the encoder (e.g., transmitter) converts the message into signals such as electromagnetic waves; the message is transported from encoder to decoder through a channel; the decoder converts the signals into a message; and the receiver is the destination of the message from the sender.

message-induced phase and the phase noise itself, so this is exactly the result with multiplication of complex numbers.

A phase modulated carrier, even with large phase deviation by the message, still is impacted by small angle phase noise in the same fashion as complex numbers multiply: a) amplitude is multiplied, and b) phases add. Even with the *multiplicative* nature of the phase noise, the equation above shows it is still also an *additive* process onto the ideal carrier. A noise term is added which is proportional to the carrier amplitude and proportional to the phase noise process. This noise term is amplitude modulating the quadrature of the *ideal phase modulated* carrier, and thus this term is always in quadrature to the ideal signal, even when the signal has phase modulation. A figure helps to illustrate this. Figure 105 shows graphs of two sets of clusters around the constellation points of the upper right quadrant of a 16-QAM SC-QAM constellation. One set of clusters, represented with the blue points, shows the same 10 bandpass AWGN sample values added to the four ideal constellation points of the upper right quadrant. Inspection should show that the same 10 noise values are added to the four ideal constellation values; this properly illustrates how the bandpass AWGN process will interact with the ideal modulated carrier to yield the resultant signal-plus-noise, for each of the four signal values which are depicted in the figure. The 10 noise vectors literally add to the signal vector.

The other set of clusters, represented with orange points, shows a phase noise process on the otherwise ideal 16-QAM modulated carrier, where the same 10 phase noise sample values are shown as they impact each of the four constellation points. First, it is seen that the phase noise is not impacting the amplitude of the carrier; there is no in-phase (amplitude) variation contributed with phase noise. All the displacement from the ideal constellation point is in quadrature to the ideal modulated carrier. Secondly, the larger amount of absolute displacement or rotation, due to the phase noise, for the constellation points with larger amplitude is clearly distinguishable. The multiplicative nature of the phase noise acting on the carrier is seen as the phase noise adds to the carrier phase regardless of the carrier phase ideal value (and recall phase adds as complex numbers are multiplied). The multiplicative nature of the phase noise is also seen where the displacement from the ideal constellation point is proportional to the magnitude of the ideal modulated carrier: The phase value of the phase noise is the same with a smaller amplitude ideal carrier as with a larger amplitude ideal carrier, but the displacement with respect to the ideal constellation point location, especially as contrasted to the AWGN, is larger.

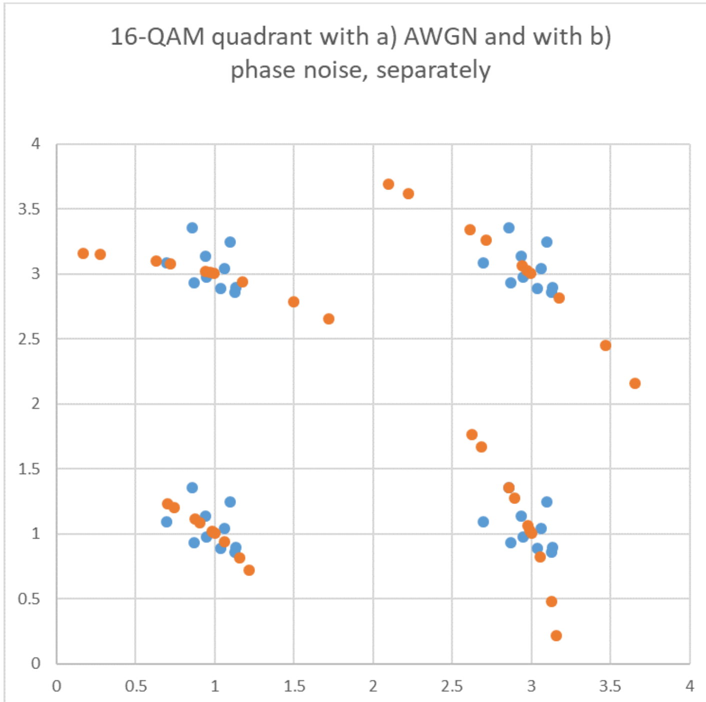


Figure 105 - 16-QAM constellation quadrant with AWGN (blue dots) and phase noise (orange dots), separately.

References

PHYv1.0 – Data-Over-Cable Service Interface Specifications, DOCSIS® 1.0 Radio Frequency Interface Specification

PHYv1.1 – Data-Over-Cable Service Interface Specifications, DOCSIS® 1.1 Radio Frequency Interface Specification

PHYv2.0 – Data-Over-Cable Service Interface Specifications, DOCSIS® 2.0 Radio Frequency Interface Specification

PHYv3.0 – Data-Over-Cable Service Interface Specifications, DOCSIS® 3.0 Physical Layer Specification

PHYv3.1 – Data-Over-Cable Service Interface Specifications, DOCSIS® 3.1 Physical Layer Specification

Appendix L Decomposing Data Communication Efficiency into Four Factors

“Spectral efficiency” is discussed in Section 29.8, and is introduced as being excerpted and adapted from [29]. “QAM-independent system efficiency” is introduced in Section 29.8.2 and is computed for an example in Section 29.8.3. This appendix revisits the QAM-independent system efficiency and generates a definition for system efficiency, and a decomposition which provides insight and utility. New definitions of different but heuristically attractive “efficiencies” are introduced, and their relationships are developed and enlightening. This section was written by Tom Kolze. Used with permission.

Decomposing Data Communication Efficiency into Four Factors

Tom Kolze, Ph.D.
Broadcom, Inc.

L.1 Introduction

This appendix explores “efficiency” of modulation and coding communication systems, building upon the contents of Section 29.8, and the “QAM-independent system efficiency” presented therein. The system efficiency in this appendix is the same as the “QAM-independent system efficiency,” but is generalized to apply to any modulation and coding system. It is demonstrated in this appendix that the efficiency is defined as a measure of the utilization of the time and frequency resource dedicated to the system operation; the efficiency is a measure of the percentage of the time-frequency resource which is allocated or utilized for transmitting “information,” or information bits (also called in this document, net bits). The complement of this efficiency is properly viewed as the “inefficiency” of the system in utilizing resources (in utilizing the frequency allocated, during the time of operation).

This appendix provides a) introduction of clarifying terminology, and b) a decomposition of efficiency (the “QAM-independent system efficiency” of Section 29.8.2) into a product of four intuitively satisfying factors. The approach is generalized, such that it is not restricted or customized for OFDM or SC-QAM, or any particular modulation.

One benefit of this approach is the significant insight provided by the decomposition into the four factors. Another benefit of this efficiency approach is unification of efficiency calculations, unambiguously applicable for any modulation and coding scheme. For example, the same decomposition applies to DOCSIS SC-QAM and DOCSIS OFDM.

With this generalized and unifying efficiency approach, accurate and meaningful direct comparison of SC-QAM and OFDM is finally achievable. These comparisons have proven elusive and/or controversial dating back over a decade, due to differences of opinion regarding “bookkeeping details” – how to account for, whether to include or not, factors which seem to be “apples-and-oranges” between the schemes attempted to be compared. These difficulties are avoided with the derivation and decomposition of efficiency introduced in this appendix.

New definitions of different but heuristically attractive “efficiencies” are introduced, and their relationships are developed and enlightening. Section L.2 presents new terms and modifications to “old” terms (defined in other sections of this document), which are helpful in understanding, defining, and calculating efficiency. Then the four factors are introduced which multiply to generate an overall measure of System Efficiency, which are Information Rate Efficiency, Powered Data Efficiency (the product of these two is Bit Stream Efficiency), Symbol Utilization Efficiency, and Time-Frequency Efficiency (the product of the latter two is Modulation Efficiency).

The Time-Frequency Efficiency is a measure (percentage) of the generation of symbol “opportunities” which are created by the system out of the number possible within a frequency (channel) and time allotted. The Symbol Utilization Efficiency is a measure (percentage) of how many of those symbol “opportunities” are energized and transmitted through the channel, as opposed to left silent. The Powered Data Efficiency (PDE) is the measure (percentage) of how many energized symbols are carrying information, i.e., the percentage of data-carrying symbols to the sum of the total number of energized symbols (data-carrying plus non-data-carrying). The Information Rate Efficiency (IRE) is the measure (percentage) of how much of the data-carrying symbols is dedicated to “information” bits, as opposed to “overhead” bits and/or FEC parity bits, etc.

The Bit Stream Efficiency, the product of the IRE and the PDE, is a measure (percentage) of how much of the energy in the transmitted symbols is dedicated to information, rather than overhead uses such as error correction redundancy, pilots and/or preamble, protocol headers, etc.

Section L.3 shows that Modulation Efficiency is the product of Symbol Utilization Efficiency and Time-Frequency Efficiency; thus, it follows that the System Efficiency is the product of Bit Stream Efficiency and Modulation Efficiency. The Modulation Efficiency is the measure (percentage) of the number of energized symbols compared to the number possible based on the channel bandwidth, in any duration of time. The Modulation Efficiency is also the symbol rate divided by the channel bandwidth, in Hz.

All these terms are defined and explained further in the following pages.

L.2 New terminology and modified terminology

Symbol rate, r_{sym}. Symbol rate, r_{sym}, will be defined in this appendix as the number of symbols containing energy presented to the channel every second (by the modulator or transmitter). Examples are provided in the next section, Section L.3. But as a preview, in OFDM, excluded subcarriers are not counted in the symbol rate.

Bits per symbol, s_{b_net} and s_{b_gross}. This term exists and is in common use, but needs the same modifiers, “net” and “gross,” that we introduced and added in earlier sections to “bits per second” or bps. The term s_b is used for bits per symbol in previous sections, and we introduce s_{b_net} and s_{b_gross} to denote the number of information bits (i.e., net bits) per symbol and the number of gross bits per symbol, respectively. Note that net bits per symbol, s_{b_net}, is given by the equation $s_{b\_net} = bps_{net}/r_{sym}$. It is easy to see by “following the units,” that the net bits per symbol is given by the net bits per second divided by the symbols per second. Similarly, of course, $s_{b\_gross} = bps_{gross}/r_{sym}$.

Note that the bps_{gross} and s_{b_gross} have an ambiguity that currently is treated in this Operational Practice in a particular manner, and this appendix is following that approach. The ambiguity involves transmitted symbols (symbols with energy) which do not contain any data, such as preamble symbols or pilot symbols (e.g., pilot subcarriers in OFDM) or PLC symbols (subcarriers) in OFDM. It is apparent from the definitions in the preceding paragraph that the s_{b_net} is reduced by accounting for non-data-carrying symbols in the total symbols per second, r_{sym}, but this is the convention. When all the data-carrying symbols have the same number of gross bits (channel bits) per symbol, this document assigns that same value of gross bits per symbol to the non-data-carrying symbols. (The ambiguity mentioned above is *how* to handle the accounting of the gross bits or channel bits in regards to the non-data-carrying symbols.) In counting the non-data-carrying symbols for computing the s_{b_gross}, and assigning (i.e., extrapolating) the same gross bits per symbol to the non-data-carrying symbols, the value of s_{b_gross} is the same as the value of gross bits per symbol as occurs in the data-carrying-symbols. The s_{b_net} is not affected by this assumption or assignment of gross bits upon the non-data-carrying symbols.

In the event that the data-carrying symbols have different values of gross bits per symbol among them, the approach recommended here (and applied later in this appendix), is to apply (assign, or extrapolate) the average gross bits per symbol of the data-carrying symbols to the non-data-carrying symbols.

Also note that in this Operational Practice, preamble symbols and PLC symbols and the like are modulated, and carry information (preamble symbols carry information about timing and frequency, for example, if not carrying information in their bits specifically), but these are categorized as non-data-carrying symbols because they do not contain any of the net bits.

Spectral efficiency, SE_{net} and SE_{gross}. This term is defined in Section 29.8.1 as the net bit rate (the information bits per second) divided by the channel bandwidth in Hz. The equation for spectral efficiency is $SE = bps_{net}/BW_{Hz}$ in Section 29.8.1. In this appendix we will call this spectral efficiency as "net spectral efficiency," so that net spectral efficiency is given by $SE_{net} = bps_{net}/BW_{Hz}$. Similarly, gross spectral efficiency is given by $SE_{gross} = bps_{gross}/BW_{Hz}$.

Symbol block rate. For orthogonal modulations, such as OFDM or S-CDMA, a "block" of symbols is created and presented to the channel together. The symbol block rate is the number of blocks presented by the modulator or transmitter every second. As examples, for OFDM with FFT period duration of FFT seconds, and cyclic prefix duration of CP seconds, there is a new block of symbols presented to the channel every FFT + CP seconds. The symbol block rate for OFDM is 1/(FFT + CP) blocks per second. For SC-QAM, there is only one symbol in each block, and the symbol block rate for SC-QAM is the same as the symbol rate, r_{sym}, when all the possible symbols are energized.

Symbol block duration, T_s. The symbol block duration is the inverse of the symbol block rate, denoted with T_s. For the OFDM example, the symbol block duration is FFT + CP seconds. For SC-QAM, the symbol block rate is the same as the symbol rate, so the symbol block duration is $T_s = 1/r_{sym}$ seconds.

Symbols per block, N_{mod}. The number of symbols per block, denoted N_{mod}, is the number of symbols in each block *which contain energy*. As an example, for OFDM, the excluded subcarriers are not counted in N_{mod}.

Silent symbols per block, N_{silent}. The silent symbols are the symbols which do not contain energy, but are available to contain energy. It is the choice of the modulation scheme to not put energy into those symbols (they are "unused"). As an example, for OFDM, the number of excluded subcarriers in each symbol block is N_{silent}.

Modulation Efficiency, ME. The Modulation Efficiency, ME, is the number of symbols presented to the channel every second divided by the channel bandwidth in Hz. The equation for this is $ME = r_{sym}/BW_{Hz}$. An equivalent expression for the Modulation Efficiency is obtained by noting that the modulator presents N_{mod} symbols to the channel every T_s seconds, so $ME = N_{mod}/(BW_{Hz} * T_s)$. As an example, for SC-QAM we have noted that N_{mod} is unity, and we have seen in previous sections that the raised cosine shaping factor α relates T_s to BW_{Hz} as $BW_{Hz} = (1 + \alpha)/T_s$. Thus, for SC-QAM, $ME = 1/(1 + \alpha)$.

Information Ratio Efficiency, IRE. The Information Ratio Efficiency is the ratio of information bits to total bits applied to the modulator, but *here no bits are counted in the denominator* for non-data-carrying transmitted (energized) symbols. This is different from the accounting for s_{b_gross} and bps_{gross}, which is explained above. There are never any information bits, or net bits, counted for non-data-carrying symbols, but there are gross bits (channel bits) assigned for non-data-carrying symbols in the computation of s_{b_gross} and bps_{gross}. The value for the ratio of information bits-to-total bits, neglecting the non-data-carrying symbols, is assigned as R for this appendix, but is also the value for IRE.

IRE = R = (information bits/total bits), neglecting the non-data-carrying symbols.

Note that the FEC code rate is typically denoted by r, and the total bits are comprised of the information bits plus the parity bits. The FEC layer may involve additional parity checks (i.e., CRC) usually for additional error detection. This is the case with D3.1 downstream FEC, for example. The additional CRC parity check bits would normally be included in the denominator in the calculation of r. The computation of R includes more categories of "overhead" bits in the denominator than just the FEC, so $R \leq r$. Additional bits included in the denominator for R *may* include framing bits or header (or tail) bits, as an example, especially where the framing is a key part of the PHY layer. This is the case in DOCSIS 3.0 SC-QAM, as one example.

In DOCSIS 3.1 OFDM the PHY link channel (PLC) and the next codeword pointers (NCPs) do not contain information bits (that is the convention for accounting); for this appendix we consider these as non-data-carrying symbols, so they do not contribute to the IRE.

It is often a "matter of taste" about how MAC messages are accounted, whether they would be considered "overhead" bits or information bits, but in this appendix we count the MAC data as information bits. In any treatment of information bit rate, efficiency, and similar, a clear explanation of accounting of what constitutes information bits and what are not information bits should be provided. It is certainly the case that in a thorough system analysis, the proportion of MAC traffic compared to User Data is an important consideration, but the most common practice is to consider MAC traffic as a portion of the information data, and it is a "traffic analysis" and not a PHY layer matter to consider how much MAC traffic is required in a system.

Powered Data Efficiency, PDE. Powered Data Efficiency is the ratio of data-carrying symbols-to-total transmitted (energized) symbols. In a given long duration of time (long enough for stability of the ratio we will be computing) the number of data-carrying symbols is N_d and the number of non-data-carrying (but energized) symbols is N_p. The PDE is given by PDE = $(N_d)/(N_d + N_p)$. Examples of non-data-carrying symbols are pilots, preamble symbols, and in this appendix, for DOCSIS 3.1 the PLC subcarriers and the NCP subcarriers are considered non-data-carrying symbols. It is not necessarily the case that the sum $(N_d + N_p)$ equals N_{mod}, the number of energized symbols per block; it could be that the ratio for PDE requires multiple symbol blocks to stabilize. The SC-QAM example where there is only one symbol per symbol block ($N_{mod} = 1$) is precisely such an example. Then, in the upstream SC-QAM it is common that there are un-energized symbols as guard-time, so it does take a number of symbol blocks to converge PDE in the case of DOCSIS upstream SC-QAM.

Bit Stream Efficiency, BSE. Bit Stream Efficiency is the ratio of information bits (net bits) to gross bits in the transmitted signal. This is given in equation form as BSE = bps_{net}/bps_{gross}. However, there is a complicating factor which we addressed above, and repeat here, with an "understanding." The bit stream for a modulation and coding system is analyzed for its ratio of information bits to total bits, where the total bits include FEC parity bits and perhaps other "overhead" bits, as explained in the discussion above for Information Rate Efficiency. Symbols which contain energy but are not carrying information bits (net bits), such as pilot symbols or preamble symbols, are categorized as non-data-carrying symbols, as explained in discussing PDE above. Symbols such as the subcarriers for DOCSIS 3.1 which are dedicated to the PLC and NCPs are also non-data-carrying symbols (discussed above and included in Powered Data Efficiency). The Bit Stream Efficiency, defined above, is also calculated as:

$$BSE = bps_{net}/bps_{gross} = (R * N_d)/(N_d + N_p) = IRE * PDE.$$

It is worth repeating that there is potential ambiguity in assigning symbols either as "overhead" (so they are included in reducing IRE) or as non-data-carrying symbols (so they are included in reducing PDE). These symbols such as DOCSIS 3.1 PLC and NCPs do not carry information bits, but are energized; they do carry necessary information for managing the PHY layer, but are definitively NOT MAC information nor are they information bits to be included in bps_{net}. It has been prior practice for D3.1 to include these particular subcarriers as non-data-carrying symbols in performing efficiency calculations, which is justified in part because they are not carrying any user data (none of this data passes out of the receiver or out of the PHY layer itself), and a different FEC is used than is applied to user data.

Symbol Utilization Efficiency, SUE. Symbol Utilization Efficiency, SUE, is the ratio of energized symbols to the sum of both the energized symbols and the non-energized (silent) symbols. As one example, for downstream DOCSIS, SC-QAM SUE is unity, every symbol is energized. For upstream SC-QAM, there are guard-time symbols which are not energized, so the SUE for upstream DOCSIS SC-QAM will be less than unity. For both upstream and downstream DOCSIS OFDM the SUE is less than unity because of (at least) excluded band-edge subcarriers.

Time-Frequency Efficiency, TFE. The Time-Frequency Efficiency is given by computing the ratio of the combined number of silent symbols and modulated (energized) symbols, in each block of symbols, to the channel bandwidth in Hz, and dividing that ratio by the symbol block duration. The equation for Time-Frequency Efficiency is given by $TFE = [(N_{mod} + N_{silent})/BW_{Hz}]/T_s$.

Note that the Time-Frequency Efficiency can also be expressed as $TFE = [(N_{mod} + N_{silent})/T_s]/BW_{Hz}$. The combined modulated (or energized) symbols plus the silent symbols represent the totality of the symbol "opportunities." This form of the expression for TFE illustrates that the rate of generation of symbol opportunities (per second) divided by the channel bandwidth (in Hz), is an inefficiency factor to the extent it is less than unity. The form in the preceding paragraph illustrates that to the extent that the symbol block duration, T_s, is larger than the ratio of the number of symbol opportunities generated with (divided by) a given bandwidth, that this is a view of inefficiency in the time domain. It is the same inefficiency, either way.

The TFE is a measurement of the number of symbol opportunities created in a given time-frequency space, and when this number is smaller than the number of (complex) dimensions[90] in the time-frequency space, which equals the product of the time duration and channel bandwidth, it represents an inefficiency.

[90] A channel of bandwidth B Hz, and a block of time of T seconds, occupies a "signal space" or time-bandwidth space of B ∗ T, which is unitless. In N blocks of such time-bandwidth space, N ∗ B ∗ T, as N gets arbitrarily large, the number of independent complex symbols (see Section 29.2) which *can be* generated, approaches N ∗ B ∗ T. This is equivalently stated as a channel with B Hz bandwidth can support a symbol rate (per second) arbitrarily close to B symbols per second (N ∗ B ∗ T symbols in N ∗ T seconds is B symbols per second). Note that for system designs where the symbol rate approaches the limit of B symbols per second, each symbol endures (far) longer than 1/B seconds. The B ∗ T time-bandwidth space is said to have B * T complex dimensions, since this is the (approachable) upper bound for the number of complex symbols which can be generated every T seconds, constrained to B Hz bandwidth.
For a signal waveform design which supports (provides) the symbol rate approaching the upper bound of B symbols per second, the duration (time span) of any one symbol will be large, generally much larger than 1/B seconds. The system will be encumbered with larger latency than a system with a more modest symbol rate in the same bandwidth. There are implementation and performance considerations for a system design with efficiency such that the symbol rate (per second) approaches B (Hz), including increased latency.

L.3 System Efficiency as a product of four factors

We now show that Modulation Efficiency is the product of Symbol Utilization Efficiency and Time-Frequency Efficiency. This is demonstrated by multiplying SUE ∗ TFE:

$$SUE * TFE = [(N_{mod})]/(N_{mod} + N_{silent})] * [(N_{mod} + N_{silent})/BW_{Hz}]/T_s.$$

Canceling the common term in numerator and denominator, SUE ∗ TFE = $(N_{mod})/(BW_{Hz} * T_s)$, and this is the equation directly resulting from the definition of Modulation Efficiency. **Thus, ME = SUE ∗ TFE.** As another note, in Section 29.8.2 the term S_{eff} is defined as "QAM-independent system efficiency," given by the equation $S_{eff} = SE/s_b$, but with our notation of this appendix, we use the definition for S_{eff} such that the equation is given by $S_{eff} = SE_{net}/s_{b\_gross}$. Also, in Section L.2 we have defined

$SE_{net} = bps_{net}/BW_{Hz}$, and we showed that
$s_{b\_gross} = bps_{gross}/r_{sym}$.

Substituting these expressions results in $S_{eff} = (bps_{net}/BW_{Hz})/(bps_{gross}/r_{sym})$.

Rearranging this expression for S_{eff}, we find $S_{eff} = (bps_{net}/bps_{gross})*(r_{sym}/BW_{Hz})$.

Now we remind readers that we defined Bit Stream Efficiency as BSE = (bps_{net}/bps_{gross}), and we defined Modulation Efficiency as ME = (r_{sym}/BW_{Hz}), so we recognize that we have shown

$$\mathbf{S_{eff} = (bps_{net}/bps_{gross}) * (r_{sym}/BW_{Hz}) = BSE * ME}.$$

So, we have shown that the "QAM-independent system efficiency" of Section 29.8.2 is the product of the newly defined, and intuitively satisfying, Bit Stream Efficiency (ratio of net bits to gross bits in the transmitted stream, with the adjusted values of Section L.2 accounting for energized symbols that don't carry data) and the newly defined Modulation Efficiency, which is the ratio of the symbol rate (in seconds) divided by the channel bandwidth (in Hz).

We have further shown (in this section, above) that we can decompose the expression for Modulation Efficiency into a product of the Symbol Utilization Efficiency (accounting for silent symbols) and the Time-Frequency Efficiency (accounting for the rate of blocks or symbols presented to the channel being slower than the channel bandwidth). Thus, we can substitute this product in place of the Modulation Efficiency, and yield QAM-independent system efficiency as a product of three terms:

$$S_{eff} = BSE * SUE * TFE$$

Writing out the equations,

$$\begin{aligned}\mathbf{S_{eff} = BSE*ME} &\mathbf{= (bps_{net}/bps_{gross}) * (r_{sym}/BW_{Hz}) = (bps_{net}/bps_{gross}) * (N_{mod})/(BW_{Hz} * T_s)} \\ &\mathbf{= (bps_{net}/bps_{gross}) * [(N_{mod})]/(N_{mod} + N_{silent})] * [(N_{mod} + N_{silent})/BW_{Hz}]/T_s} \\ &\mathbf{= BSE * SUE * TFE.}\end{aligned}$$

This is a very satisfying result for a "system efficiency"; it is a product of three factors which are fundamental: 1) the ratio of information bits to the *adjusted* total bits in the transmitted stream; 2) the ratio of energized symbols to the total symbol "opportunities"; and 3) the ratio of the total symbol opportunities to the time-frequency space complex dimensions (*so, a "time domain inefficiency" or a "frequency domain inefficiency," or both, but in truth, these are two sides of the same coin*).

It is definitely worth noting that we could have further factored the efficiencies involved in computing the System Efficiency, S_{eff} = BSE ∗ SUE ∗ TFE, such that S_{eff} = IRE ∗ PDE∗ SUE ∗ TFE. Inasmuch as the PDE is unity for DOCSIS SC-QAM downstream, and is notably less than unity for DOCSIS OFDM, similar to SUE in this respect, it is a noteworthy additional factorization.

$$\mathbf{S_{eff} = BSE*ME = BSE * SUE * TFE = IRE * PDE * SUE * TFE}$$
$$\mathbf{= R * [(N_d)/(N_d + N_p)] * [(N_{mod})]/(N_{mod} + N_{silent})] * [(N_{mod} + N_{silent})/BW_{Hz}]/T_s.}$$

This is also a very satisfying result for "system efficiency"; it is now a product of four factors which are fundamental: 1) the ratio of information bits to total bits, in the data-carrying symbols (non-data-carrying symbols are not counted); 2) the ratio of data-carrying symbols to the total number of energized symbols (data-carrying plus non-data-carrying); 3) the ratio of energized symbols to the total symbol "opportunities"; and 4) the ratio of the total symbol opportunities to the time-frequency space complex dimensions.

Writing out the four factors in a "word equation":

S_{eff} = (information bits per data-carrying symbol/total bits per data-carrying symbol)
∗ (data-carrying symbols/energized symbols)
∗ (energized symbols/all possible symbols) ∗ (all possible symbols per second/BW Hz).

It is obvious we can add "per second" to the numerator and denominator of each of the first three terms without changing the result. And we can cancel the numerator of the latter two terms with the denominator of the preceding term. Performing these operations:

S_{eff} = (information bits per data-carrying symbol/total bits per data-carrying symbol)
∗(data-carrying symbols per second/BW Hz)
= R * (N_d per second/BW Hz).

This is another expression for S_{eff} as a product of just two terms, each physically apparent.

S_{eff} is a unitless value, as are all the "efficiencies" in this section. It is one point of view that S_{eff}, being a product of four fundamental measures of efficiency, is accurately described as "system efficiency" without the qualifier of "QAM-independent."

L.4 Symbol rate and efficiency examples, OFDM and SC-QAM

For SC-QAM it is straightforward that one new symbol is presented to the channel every T_s seconds. For the DOCSIS downstream SC-QAM all the symbols are energized. In upstream DOCSIS some of the symbols are used for time-domain guard-time and thus are not energized. For downstream SC-QAM the symbol block rate is T_s seconds; the number of modulated (energized) symbols during the symbol block duration is one, so N_{mod} is one. It is seen that r_{sym} = ($1/T_s$) symbols per second.

The Bit Stream Efficiency for SC-QAM is called the ratio of the net bits to the gross bits, but this is adjusted to account for energized symbols which don't carry any of the data stream bits (as given in Section L.2). Redundant bits in FEC are not information bits, nor are bits classified as "header" bits. The percentage of information bits compared to all (gross) bits is determined for the modulation and coding scheme. The adjusted ratio represents a percentage of the energized symbols which is allocated or dedicated to “information” and not other uses. Then the symbols (if any) that are energized but don’t carry any bits are accounted as explained in Section L.2. The Bit Stream Efficiency is the result of the information bits fraction of the gross bits, but normalized to one gross bit per symbol; adding to the denominator the number of symbols which contain energy but NO bits completes the determination of the Bit Stream Efficiency. This was explained via equation in Section L.2.

For downstream DOCSIS SC-QAM the Symbol Utilization Efficiency is unity, SUE = 1, with every symbol “opportunity” being energized and no silent symbols in the continuous downstream.

For downstream DOCSIS SC-QAM the Time-Frequency Efficiency is

$$TFE = [(N_{mod} + N_{silent})/BW_{Hz}]/T_s,$$
$$TFE = [(1 + 0)/BW_{Hz}]/T_s = [(1 + 0)/[(1 + \alpha)/T_s]]/T_s = [1/(1 + \alpha)] * (T_s)/T_s = 1/(1 + \alpha).$$

For SC-QAM, ME = SUE ∗ TFE = 1 ∗ [1/(1 + alpha)] = 1/(1 + alpha).

For DOCSIS SC-QAM, recall that PDE = 1, because there are no non-data-carrying symbols.

The overall System Efficiency for DOCSIS SC-QAM is given by:

$$\begin{aligned} \mathbf{S_{eff} = BSE * ME} &\mathbf{= (bps_{net}/bps_{gross}) * (r_{sym}/BW_{Hz}) = (bps_{net}/bps_{gross}) * (N_{mod})/(BW_{Hz} * T_s)} \\ &\mathbf{= BSE * SUE * TFE} \\ &\mathbf{= (bps_{net}/bps_{gross}) * 1/(1 + alpha).} \\ &\mathbf{= IRE * PDE * 1/(1 + alpha).} \\ &\mathbf{= R * 1 * 1/(1 + alpha) = R/(1 + alpha).} \end{aligned}$$

For OFDM the Bit Stream Efficiency calculation is much as for SC-QAM. A subcarrier with energy is equivalent to a SC-QAM symbol with energy, in calculating Bit Stream Efficiency. Subcarriers which are pilots have a weighting of 0% for their net information. As another example, subcarriers in the PLC do not carry payload (user) data, so they too are weighted as 0% net information content. The subcarriers which are energized and carry data are weighted with the percentage of information bits that were calculated for the bit stream (called R in Section L.2). The data-carrying subcarriers are combined with the non-data subcarriers, as shown in Section L.2, to compute the adjusted bps_{net} and bps_{gross}, and the ratio of those is the BSE. This BSE is directly analogous to the calculations for SC-QAM and as explained in Section L.2.

For OFDM the Symbol Utilization Efficiency is calculated as SUE = $[(N_{mod})]/(N_{mod} + N_{silent})]$.
Note that the example in Section 29.8.3 has a value N_{CH}, which is the number of subcarriers in the OFDM channel, and this can be taken as the value for $N_{mod} + N_{silent}$. Basically, we are not “counting” the subcarriers which are excluded but are *outside* the channel’s assigned spectrum. N_{silent} includes the internal excluded subcarriers, and also the excluded guard-band subcarriers, the latter which are denoted as N_{GB} in Section 29.8.3.

The OFDM subcarriers within the defined channel spectrum are divided into the following three proper subsets: 1) data subcarriers, 2) excluded subcarriers (no energy), and 3) non-zero-valued subcarriers which are not carrying data.

The sum of #1 and #3 equals N_{mod}. N_{mod} is not limited to data subcarriers.

For OFDM the Time-Frequency Efficiency is calculated as follows.

FFT period duration = FFT seconds, and cyclic prefix duration = CP seconds. A new block of symbols is presented to the channel every FFT + CP seconds. Thus, the T_s = FFT + CP. The number of symbols in each block is the number of subcarriers in the channel which contain energy, N_{mod}.

For the OFDM channel we have the relationship $(N_{mod} + N_{silent})*(1/FFT) = BW_{Hz}$.

This is manipulated to form: $(N_{mod} + N_{silent})/BW_{Hz} = FFT$.

The Time-Frequency Efficiency for OFDM is then:

$$TFE = [(N_{mod} + N_{silent})/BW_{Hz}]/T_s = FFT/(FFT + CP).$$

This is a very intuitively pleasing result, where we know the time *inefficiency* is CP/(FFT + CP).

For OFDM, $ME = SUE * TFE = [(N_{mod})]/(N_{mod} + N_{silent})] * [FFT/(FFT + CP)]$.

The overall System Efficiency for DOCSIS OFDM is given by:

$$\begin{aligned} \mathbf{S_{eff} = BSE * ME} &\mathbf{= (bps_{net}/bps_{gross}) * (r_{sym}/BW_{Hz}) = (bps_{net}/bps_{gross}) * (N_{mod})/(BW_{Hz} * T_s)} \\ &\mathbf{= BSE * SUE * TFE = (bps_{net}/bps_{gross}) * [(N_{mod})]/(N_{mod} + N_{silent})] * [FFT/(FFT + CP)].} \\ &\mathbf{= IRE * PDE * [(N_{mod})]/(N_{mod} + N_{silent})] * [FFT/(FFT + CP)].} \\ &\mathbf{= R * [(N_d)/(N_d + N_p)] * [(N_{mod})]/(N_{mod} + N_{silent})] * [FFT/(FFT + CP)].} \end{aligned}$$

Comparison with the results for OFDM in Section 29.8.3 show that the value for S_{eff} above agrees exactly term-by-term, with some reorganization.

L.5 Conclusion

The System Efficiency in this appendix is the same as the “QAM-independent system efficiency” of Section 29.8.2, but is generalized to apply to any modulation and coding system. The System Efficiency is also shown to be factored into a product of newly defined “efficiencies” which are fundamental and heuristically satisfying. It is demonstrated in this appendix that the System Efficiency is defined as a measure of the utilization of the time and frequency resource dedicated to the system operation; the efficiency is a measure of the percentage of the time-frequency resource which is allocated or utilized for transmitting “information,” or information bits (also called in this document, net bits). The complement of this efficiency is properly viewed as the “inefficiency” of the system in utilizing resources (in utilizing the frequency allocated, during the time of operation).

New definitions of different but heuristically attractive “efficiencies” are introduced, and their relationships are developed and enlightening. This appendix provides a decomposition of System Efficiency (the “QAM-independent system efficiency” of Section 29.8.2) into a product of four intuitively satisfying factors. The approach is generalized, such that it is not restricted or customized for OFDM or SC-QAM, or any particular modulation.

The same decomposition applies to DOCSIS SC-QAM and DOCSIS OFDM.

The four factors which multiply to generate an overall measure of System Efficiency are Information Rate Efficiency, Powered Data Efficiency (the product of these two is Bit Stream Efficiency), Symbol

Utilization Efficiency, and Time-Frequency Efficiency (the product of the latter two is Modulation Efficiency):

- The Time-Frequency Efficiency is a measure (percentage) of the generation of symbol "opportunities" which are created by the system out of the number possible within a frequency (channel) and time allotted.

- The Symbol Utilization Efficiency is a measure (percentage) of how many of those symbol "opportunities" are energized and transmitted through the channel, as opposed to left silent.

- The Powered Data Efficiency (PDE) is the measure (percentage) of how many energized symbols are carrying information, i.e., the percentage of data-carrying symbols to the sum of the total number of energized symbols (data-carrying plus non-data-carrying).

- The Information Rate Efficiency (IRE) is the measure (percentage) of how much of the data-carrying symbols is dedicated to "information" bits, as opposed to "overhead" bits and/or FEC parity bits, etc.

The first three efficiency terms listed above, of the four which multiply to create the overall System Efficiency, S_{eff}, are an example of a "telescoping" series of factors: The numerator of one term "cancels" the denominator of an adjacent term in the order listed above.

The Bit Stream Efficiency is the product of the IRE and the PDE, and is a measure (percentage) of how much of the energy in the transmitted symbols is dedicated to information, rather than overhead uses such as error correction redundancy, pilots and/or preamble, protocol headers, etc.

The Modulation Efficiency is the product of Symbol Utilization Efficiency and Time-Frequency Efficiency. The Modulation Efficiency is the measure (percentage) of the number of energized symbols compared to the number possible based on the channel bandwidth, in any duration of time. The Modulation Efficiency is also the symbol rate divided by the channel bandwidth, in Hz.

Collecting the two pairs of efficiency factors as shown in the two preceding paragraphs, it follows that the System Efficiency is the product of Bit Stream Efficiency and Modulation Efficiency.

We also showed that the System Efficiency could be expressed as yet another product of just two terms, each physically apparent:

$$
\begin{aligned}
S_{eff} &= (\text{information bits per data-carrying symbol/total bits per data-carrying symbol}) \\
&\quad * (\text{data-carrying symbols per second/BW Hz}) \\
&= R * (N_d \text{ per second/BW Hz}).
\end{aligned}
$$

